Times Higher
15 March 2002

By Tony Hoare

Conversations with a Mathematician : math, art, science and the limits of reason

By Gregory J. Chaitin

Springer, 158 pp, £15.00 hardback

ISBN 1852335491

Published 2002

Randomness is a concept that seems to be strongly in conflict with the most basic ideals of mathematics, -- the pursuit of complete precision and of absolute certainty checked by mathematical proof. Many mathematicians have been reluctant to accept randomness, just as many physicists have been reluctant to accept the uncertainty that lies at the heart of quantum theory. Gregory Chaitin's achievement has been to discover a precise mathematical definition of randomness, based on ideas taken from theoretical computer science. His informal explanation is as follows.

Choose your favourite general-purpose programming language, -- it does not matter which, as long as there is no restriction on the range of numbers that a program can print, one digit at a time. Each program itself consists of a certain number of characters, which have to be read into the computer before the program starts. Some programs in the language print numbers which are shorter than their own length, some longer, and some the same. For example, it is easy to write a very short program that prints the digit 1 followed by a million zeroes. Such a number would hardly be considered random. After a few hundred zeroes, it is too easy to guess correctly what the next digit will be. But a different program that just prints the number ten is rather longer than two characters. The shortest such program is something like "print(10)", which has nine characters . So ten could be regarded as a random number. Certainly, even given that the first digit is 1, there are no good grounds to guess what the second digit will be.

Chaitin defines a number to be random if it is shorter than all the programs that will print it out. According to this definition, very small numbers (like ten) are all random. On the other hand, the number consisting of the first million digits of the decimal expansion of the real number pi is *not* random; there already exist quite short programs that will print it. But that is an exception. By far the majority of numbers longer than a million digits are random according to this definition.

An interesting feature of the definition is that it does not depend on the concept of probability. Furthermore, it gives an effective way of proving that a particular number is non-random: just write a program shorter than it which outputs that number. But Chaitin has shown that there is absolutely no way of giving a checkable mathematical proof that a large number is actually random. This he proved to be impossible for any number that is longer than the computer program that would be capable of checking the proof.

These ideas are simply and engagingly explained in the first section of Chaitin's "Conversations with a Mathematician", a transcript of a talk given in 1999 at Umass-Lowell. They justify the phrase "the limits of reason" which appear in the subtitle of the book. The technical ideas are repeated and extended in two further lecture transcripts. They define Chaitin's famous number called omega as the probability that a probabilistically generated program (e.g., one typed by monkeys at a keyboard) will ever terminate. This number is so random that no program can ever be written to print out even just only one of its digits.

[But there is worse to come. Even if a mathematician knew or firmly believed that the seventeenth digit is a 1, it would be impossible to write a checkable proof of this proposition. Even if every digit except the seventeenth were known, it would still be impossible for a mathematician to find the seventeenth digit. And the same for any other digit. And although each digit is forever unknowable, Chaitin has proved that exactly half of them (in the limit) are even digits, and half of them are odd. Technically, propositions about the value of each digit constitute an infinite set of independently undecidable propositions. These and related results are a generalisation and explanation of the pioneering discoveries of Godel and Turing.

[Chaitin's undecidability results are defined in terms of computers and probabilities, subjects which many mathematicians are happy to ignore. So Chaitin has translated his result right into the core of mathematical practice, which has always been the solution of algebraic equations. Some equations are known to have no solutions (for example $x = x + 1$); some have just one solution (for example $2x = 4$); some have a finite number of solutions (for example $x^2 - 3x + 2 = 0$); and some have an infinite number of solutions (for example $(x + 1)^2 = x^2 + 2x + 1$). Chaitin has constructed an equation for which it is mathematically undecidable whether it has finite or an infinite number of solutions. Actually he got a computer to write out the equation, since is about two hundred pages long.

But again, there is worse to come. The equation has an extra unknown, which can be set to any whole number you like. If you set it to seventeen, the remaining equation will have an infinite number of solutions just if the seventeenth digit of Chaitin's omega is a 1 . Similarly for all the other digits. Since the digits of omega are undecidable, Chaitin has given us an infinite set of independently undecidable propositions, lurking right in the central core of mathematics.]]

All these ideas are explained (with some overlap) in three sections of the book. The remaining eight sections are much shorter. They are mainly transcripts of interviews with Chaitin conducted by journalists in various countries over the last ten years. As the author admits, there is a lot of repetition here, but it is compensated by the immediacy and the liveliness of the spoken word, and the drama of human interaction. We learn about his father, his childhood in Manhattan and Buenos Aires, and his first engagement with the problem of randomness at the age of fifteen. His early absorption with mathematics made him shy with girls, but since then he has learnt to enjoy life to the full. He talks of street carnivals, cafes, champagne, hill climbing and beautiful women; and asserts that his pleasure in mathematics is similar to these. There are brief analogies drawn with physical science, but almost nothing on art or music, and no pictures or songs.

He tells stories of his meetings with other contemporary mathematicians, his failure to meet Godel, and his recollection of reading of the deaths of Russell and Einstein. He repeatedly drops the names of earlier mathematicians, -- Hilbert, Gauss, Borel, Hardy, Pascal, ... In some cases he briefly describes their work, or recounts an anecdote about their lives. Many of the anecdotes are interesting, but most of the references will be significant only to those who are already quite familiar with the name and the ideas associated with it.

In the preface, which was written specially for this collection, the author claims he would have loved reading this book as a teenager turning into a mathematician. He hopes to show that math and science are fun, and asks the reader to tell him whether he has succeeded. Yes, his enjoyment of the topics of his own research is infectious. This is a suitable bedside book for a teenager whose library already contains some more coherently written books in popular mathematics and science. It might encourage an extension of the library to include an earlier book by Chaitin himself.

Professor Sir Tony Hoare, FRS.

[The material in square brackets [or double square brackets]could be omitted]

Tony Hoare was attracted to a career in computing when he was a student of philosophy by reading the works of Turing and Quine. After eight years learning the trade as a programmer and manager in the computer industry, he spent thirty years as a University Professor, at Belfast and then at Oxford. His research has sought application of the techniques of formal logic to the problem of correctness of computer programs. On retirement from University, he took up a senior research position in Cambridge with the software Company Microsoft .

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But there is worse to come. Even if a mathematician knew or firmly believed that the 17th digit is a 1, it would be impossible to write a checkable proof of this proposition. Even if every digit except the 17th were known, it would still be impossible for a mathematician to find the 17th digit. And the same for any other digit. And although each digit is forever unknowable, Chaitin has proved that exactly half of them (in the limit) are even digits, and half of them are odd. Technically, propositions about the value of each digit constitute an infinite set of independently undecidable propositions. These and related results are a generalisation and explanation of the pioneering discoveries of Gödel and Turing.

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But again, there is worse to come. The equation has an extra unknown, which can be set to any whole number. If you set it to 17, the remaining equation will have an infinite number of solutions just if the 17th digit of Chaitin's omega is a 1. Similarly for all the other digits. Since the digits of omega are undecidable, Chaitin has given us an infinite set of independently undecidable propositions, lurking right in the central core of mathematics.

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Professor Sir Tony Hoare is a computer scientist, now working for Microsoft.

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Publisher - Springer
Price - £15.00
Pages - 158