18 May 1994

Dear Tony,

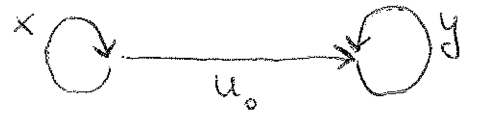
I am enclosing my current draft of the short paper. I included what proofs I could do at the calculus level (except for Dedekind's law, because that already appears elsewhere in the literature). More laws can be proved when we make use of the fact that U is a subset of a groupoid with the four properties (in particular, the laws relating composition to ~~posti~~ relative converse). Also several of the proofs given become simpler. But I agree with your suggestion that this should be the subject of a later paper.

Now the paper is slightly too long; please feel free to suggest where we should cut.

I like the intuition of observations as lines that overlay one another, but did you notice the following irregularity? A "line" may properly contain itself, i.e.

$$u = x;u;y \not\Rightarrow x = \overleftarrow{u} \wedge y = \overrightarrow{u}$$

A counterexample is an observation space spanned by 3 elements



$$U = \{x^n \mid n \geq 0\} \cup \{y^m \mid m \geq 0\} \cup \{u_i \mid i \in \mathbb{Z}\}$$

with
$$x^n ; x^m = x^{n+m}$$
$$y^n ; y^m = y^{n+m}$$
$$x^n ; u_i = u_{i+n}$$
$$u_i ; y^n = u_{i-n}$$

Your ordering among arrows still works (as it must) and gives $u_i \leq u_j \iff i \leq j$.

Yours sincerely

Burghard

P.S. A nice property of your construction (that you probably already noticed) is that it also works backwards.

Assume G is a groupoid with a partial order $\leq$, satisfying

(i) $a \leq b \Rightarrow \overleftarrow{a} = \overleftarrow{b}$

(ii) $a \leq b \wedge \overrightarrow{x} = \overleftarrow{a} \Rightarrow xa \leq xb$

(iii) $a \leq b, c \leq d \Rightarrow b \leq c$ or $c \leq b$

Then $R = \{a \mid \overleftarrow{a} \leq a\}$ satisfies our four properties.

Date: Wed, 7 Sep 1994 10:24:25 +0100
From: Burghard von Karger <bvk@de.d400.uni-kiel.informatik>
To: jane.ellory@comlab
Subject: New axiom for sequential calculus

Dear Tony,

We seem to agree on the thesis that antisymmetry

(not J)(not J) contained in (not J)

should not be one of the basic axioms of sequential algebra. However, it would be feeble to just scratch it, without replacement. Consider

(1) $$J/U = U\backslash J$$

This certainly holds in every groupoid (where, in fact, both sides are equal to U), but also in every cancellative observation space.

(Proof: Suppose x in J/U, say xu in J. Then xux = x and cancellativity forces ux in J, whence x in U/J)

I do not believe that (1) follows from the axioms we already have.

Assuming cancellativity, the set J/U is certainly very interesting, as it describes the set of invertible elements, i.e. the largest groupoid contained in U.

Subsets of J are called conditions; what name could we invent for subsets of J/U ? Perhaps "events", since an invertible observation cannot consume time. The new axiom I propose is

For all events E:

(2) $$(P/E)Q = P(E\backslash Q)$$

This would also establish that the set of all events is a relation algebra. So events have converses and using those, (2) becomes an instance of the associativity axiom.

It is easy to show that (2) holds in every cancellative observation space. I can show that (1) follows from (2), but I could not show the converse. I have not tried very hard.

Yours,
Burghard

Date: Mon, 5 Sep 1994 22:49:22 +0100
From: Burghard von Karger <bvk@de.d400.uni-kiel.informatik>
To: jane.ellory@comlab
Subject: Re: Sequential Calculus

Dear Tony,

Thank you for proposal for a redraft of the IPL paper. I agree that this is a very good way to tell as much of the story as possible in the given page limit.

On the other hand, the present article contains almost all the information you propose it should, and we have also made it quite clear that antisymmetry is used only once, very near to the end. So I would be happiest if I did not have to redraft it (it is already the fifth draft anyway).

The one thing I failed to mention, and which I agree is a very nice remark is the fact that the associativity

$$(P/Q)R \;=\; P(Q\backslash R)$$

brings us right back to the calculus of relations.

I propose we add this law as the set level version of invertibility on page 6, four lines from the bottom.

I do not understand your remark on Tarski's law distinguishing regularity from relational calculus. It holds in both, doesn't it? I am not sure the two calculi *can* be distingusished algebraically, because the regularity calculus *is* a relational calculus. Simply identify every subset A of the free group G with the relation

{ (x, xa) | x in G and a in A }.

This is an injective homomorphism of relational algebras.

I am slightly surprised that you prefer the split law over double cancellation. True, it is simpler and more often directly applicable. However, it has the defect of not being symmetric. And adding both the left and right version of split as axioms is silly, because in the presence of (single) cancellation, they are equivalent.

I am not quite sure about the reference to Rustan Leino you asked for. Let me recall the facts:

After you had posed the repetition-never as a problem, Leino and Hofstee together wrote a little note which failed to solve the problem (wrong formalization of the problem in the regularity calculus).

Later Hofstee, not Leino, did produce a solution (at the calculus level!) Comparing our solution to his shows that we have made enormous progress. Hofstee's proof is more than twice as long than ours, involves quantifications (both universal and existential), needs Dijkstra's [..] and < .. > operators and relies on three axioms one of which is false in our setting, namely

P;Q = empty   implies   P = empty or Q = empty

and another one of which goes over three lines and involves 8 variables.

One must admit that Hofstee's proof is very ingenious, and foreshadows the double cancellation law. He had a very hard time stating it without using the relative converses. Had he known of the relative converses, I think he would have found the solution.

Neither of the two notes mentions the three diamond law itself, but only the weak version of it that actually can be proved algebraically. There was certainy no point-level proof of it (as you seem to remember). I did one myself, in the Procos report entitled "Observation spaces" of this January. Anyway, the point-level proof is a trivial affair, not worth mentioning.

Now, please feel free to make any changes or corrections that you feel are appropriate (Jane Ellory has the LaTeX file), or ask me to do them.

Yours,

Burghard