# Sequential Calculus.

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**Summary.**

This paper presents an algebraic calculus like the relational calculus for reasoning about sequential phenomena. It provides a common foundation for several proposed models of concurrent or reactive systems. It is clearly differentiated from the relational calculus by absence of a general converse operation. This permits the treatment of temporal logic within the sequential calculus.

## 1 Introduction and general axioms.

The relational calculus has been remarkably successful in reasoning about possibly non-deterministic systems, provided that their behaviour can be fully characterised by observation of just their initial and final states. Many alternative models have been proposed for reactive systems, whose behaviour between initiation and termination is also of significance. A common feature of these calculi is that past observations cannot be cancelled or undone by what comes after. As a consequence, the converse operation of the relational algebra must be abandoned. The purpose of this paper is to provide a common framework of laws applicable to many of these alternative models.

The general modelling technique is to represent each possible system as a set, whose elements represent single *observations* of a single experiment on the system described. In the relational calculus [10], the observations are pairs $(s, t)$, where $s$ and $t$ are drawn from the same set of states. In the calculus of intervals [1], these are required to be related by a total ordering ($s \leq t$). In regular expressions [7] the observations are finite sequences of letters drawn from an alphabet $A$. In the regularity calculus [2], the sequences are given the structure of a free group. In temporal logic e.g. [9, 11], the observations are functions from time intervals to states, where time is a total ordering and may be discrete or continuous, finite or infinite.

Any calculus for sets should start from consideration of the properties of their members; and we are primarily interested in properties shared by *all* the observation spaces in question. The most basic common property is the existence of an associative composition operator $(x; y)$, which makes a possibly longer observation from subobservations $x$ and $y$. For regular expressions, this is just concatenation of strings, and for free groups it is the group multiplication. In other cases, composition is a partial operator: in the relational calculus, the pair $(r, s)$ can be composed with $(s', t)$ only if $s = s'$; and when this equality holds, the intermediate state is omitted:

$$(r, s); (s, t) \;=\; (r, t).$$

Similarly, in temporal logic, composition is defined only when the end time and final state of the first operand are the same as the start time and initial state of the second operand. Then the two functions are compatible, so that their union is a function and can be taken as the result of the composition.

To help reasoning about the definedness of composition, we introduce two functions between observations. Each observation $x$ has a left unit $\overleftarrow{x}$ and a right unit $\overrightarrow{x}$, which satisfy the unit properties for composition:

$$\overleftarrow{x}\,; x \;=\; x \;=\; x;\, \overrightarrow{x}\,.$$

For example, in the relational and interval calculi

$$\overleftarrow{(s,t)} = (s,s) \text{ and } \overrightarrow{(s,t)} = (t,t).$$

In temporal logic $\overleftarrow{x}$ is the initial state and time, whereas $\overrightarrow{x}$ is the final state and time. In both cases, composition is defined just when the right unit of the left operand is the same as the left unit of the right operand:

$$x; y \text{ is defined iff } \overrightarrow{x} = \overleftarrow{y}\,.$$

In regular expressions (as in free groups) there is just a single unit for composition, the empty sequence. As a consequence $\rightarrow$ and $\leftarrow$ are constant functions, and composition is everywhere defined.

The unit functions have two additional properties: they map units to themselves, and they depend only on the left or right operands of composition:

$$\begin{array}{ccc} \overleftarrow{\overleftarrow{x}} = \overrightarrow{\overleftarrow{x}} = \overleftarrow{x} & \text{and} & \overleftarrow{\overrightarrow{x}} = \overrightarrow{\overrightarrow{x}} = \overrightarrow{x} \\ \overleftarrow{x;y} = \overleftarrow{x} & \text{and} & \overrightarrow{x;y} = \overrightarrow{y}\,. \end{array}$$

These properties endow the observation space with the structure of a small category. This gives some hope that our calculus may have even wider applications than those listed. But no acquaintance with category theory is needed for an understanding of this paper, or for application of its results.

## 2 A reduced relational calculus.

Our overall goal is to formalise a calculus of sets of observations drawn from a domain $U$ satisfying the axioms described in the previous section. Our first step is to restore as much as possible of the standard theory of relational algebra. The axioms formalising a (complete) Boolean algebra are obviously inherited by a complete powerset of any carrier set of observations; we can therefore concentrate on the specifically relational laws, involving composition and units.

Relational composition is just a lifted form of the composition of observations; and our more general composition is defined similarly:

**Definition of ;** $\quad R; S \overset{def}{=} \{z \mid \exists x \in R, y \in S \,.\, x; y = z\}.$

This has the advantage of being everywhere defined; it is still associative, and its unique identity element is the set of all unit observations

**Definition of $J$** $\quad J \overset{def}{=} \{x \mid \overleftarrow{x} = x = \overrightarrow{x}\}$

**Identity** $\quad R; J = R = J; R.$

The absence of a general converse forces us to define a *relative* converse $T;^{\cup}S$, to play the same role that $T; \breve{S}$ plays in the relational calculus. Each observation of $T;^{\cup}S$ is obtained from an observation of $T$ by cutting from the end something that is an observation of $S$:

**Definition of $;^{\cup}$** $\quad T;^{\cup}S \overset{def}{=} \{x \mid \exists\, z \in T\,,\, y \in S \,.\, x; y = z\}.$

This satisfies some of the familiar laws of the relational calculus.

**Left exchange** $\qquad R;S \subseteq \neg T \quad \text{iff} \quad T;^{\cup}S \subseteq \neg R.$

**Converse-unit** $\qquad T;^{\cup}J = T.$

The exchange law is also known as Schröder equivalence. — Sequential calculus enjoys a perfect symmetry between left and right (past and future). For example, there is also a right exchange law

**Right exchange** $\qquad R;S \subseteq \neg T \quad \text{iff} \quad R\,{}^{\cup}\!;T \subseteq \neg S.$

involving the mirror image ${}^{\cup}\!;$ of $;^{\cup}$. Wherever necessary, we assume that the reader has provided symmetric versions of definitions and laws stated here. — We use the convention that unary operators (complement and the modal operators introduced below) bind tightest and set operators ($\cap$ and $\cup$) bind loosest, with composition and relative converse in between.

The exchange laws also imply that ; and $;^{\cup}$ distribute over arbitrary unions. In the relational calculus, they are equivalent to the Dedekind law, which has the advantage of being a single inequation, rather than an equivalence between two inequations. We have the same situation in the sequential calculus, except that there are now three variations instead of one:

**Dedekind laws**

$$\begin{array}{rcl} (R \cap P;^{\cup}S);S & \supseteq & R;S \cap P \\ R\,{}^{\cup}\!;(S \cap R;P) & \supseteq & R\,{}^{\cup}\!;S \cap P \\ (R \cap P;S);^{\cup}S & \supseteq & R;^{\cup}S \cap P. \end{array}$$

*Proof.* We show only the first inequation, using indirect inequality.

$$\begin{array}{ll} & (R \cap P;^{\cup}S);S \subseteq \neg X \\ \equiv & \{\text{ left exchange }\} \\ & X;^{\cup}S \subseteq \neg(R \cap P;^{\cup}S) \\ \equiv & \{\text{ shunt } P;^{\cup}S \ \} \\ & X;^{\cup}S \cap P;^{\cup}S \subseteq \neg R \\ \Rightarrow & \{\text{ monotonicity }\} \\ & (X \cap P);^{\cup}S \subseteq \neg R \\ \equiv & \{\text{ left exchange }\} \\ & R;S \subseteq \neg(X \cap P) \\ \equiv & \{\text{ shunt } P \ \} \\ & R;S \cap P \subseteq \neg X. \end{array}$$

■

*Exercise* 1. Show that the two exchange laws are equivalent to the Dedekind laws.

## 3 Conditions

In the relational calculus, a state $s$ can be represented by the observation $(s, s)$. So, a *condition* on states may be given as a set of observations $x$ with $\overleftarrow{x} = \overrightarrow{x}$. Generalising this idea to the sequential calculus, we define a condition as a subset of the identity $J$. Let $B$ and $C$ always denote conditions; we will now prove some of their basic properties.

**Superdistributivity** $\qquad B;(P \cap Q) \;=\; B;P \;\cap\; Q.$

*Proof.*

$$\begin{array}{ll}
 & B;P\cap Q \\
\subseteq & \{\ \text{Dedekind}\ \} \\
 & B;(P\cap B \mathbin{\overset{\cup\!\!\shortmid}{;}} Q) \\
\subseteq & \{\ B \text{ is a condition}\ \} \\
 & B;(P\cap J \mathbin{\overset{\cup\!\!\shortmid}{;}} Q) \\
= & \{\ \text{converse-unit}\ \} \\
 & B;(P\cap Q) \\
\subseteq & \{\ \text{monotonicity}\ \} \\
 & B;P\cap B;Q \\
\subseteq & \{\ B \text{ is a condition}\ \} \\
 & B;P\cap Q.
\end{array}$$

■

Taking $P = J$ and $Q = C$ we find that the composition of two conditions is just their conjunction. — Just like in the relational calculus, conditions are invariant under transposition:

**Transpose condition** $\qquad B;P \;=\; B \mathbin{\overset{\cup\!\!\shortmid}{;}} P.$

*Proof* (by indirect equality).

$$\begin{array}{ll}
 & B \mathbin{\overset{\cup\!\!\shortmid}{;}} P\subseteq X \\
\equiv & \{\ \text{right exchange}\ \} \\
 & B;\neg X\subseteq \neg P \\
\equiv & \{\ \text{shunt } P\ \} \\
 & P\cap B;\neg X = \emptyset \\
\equiv & \{\ \text{superdistributivity}\ \} \\
 & B;P\cap\neg X = \emptyset \\
\equiv & \{\ \text{shunt } X\ \} \\
 & B;P\subseteq X.
\end{array}$$

■

In relational algebra, sets of states may also be encoded as *vectors*: relations $P$ with $P = P;U$ where $U$ denotes the universal relation. In the sequential calculus, vectors have to be characterised by

$$P \;=\; (P;U)\ \cup\ (P;^{\cup}U)$$

which seems less attractive than the definition of conditions. Another advantage of conditions over vectors is their symmetry. However, the set of all conditions is not closed under complementation. We therefore have to define the negation of a condition by

$$\overline{B} \overset{def}{=} J\cap\neg B.$$

All vectors are of the form $B;U$, and the complement of a vector is itself a vector.

**Vector negation** $\qquad \neg(B;U) \;=\; \overline{B};U.$

*Proof.* To prove that two sets are each other's complements, we have to check that their intersection is empty, and that their union is the universe. So:

$$\begin{array}{ll}
 & B;U\cap\overline{B};U \\
= & \{\ \text{superdistributivity}\ \} \\
 & (B\cap\overline{B});U \\
= & \\
 & \emptyset
\end{array}
\qquad\qquad
\begin{array}{ll}
 & B;U\cup\overline{B};U \\
= & \{\ \text{distributivity}\ \} \\
 & (B\cup\overline{B});U \\
= & \\
 & U.
\end{array}$$

■

To every $R$ we associate a condition $\overleftarrow{R}$, the *domain* of $R$.

**Definition of domain** $\qquad \overleftarrow{R} \overset{def}{=} \{ \overleftarrow{x} \mid x \in R\} \ = \ J \cap R;{}^{\cup}R.$

Here is an alternative definition which is more useful in calculations.

**Galois characterisation of $\overleftarrow{R}$** $\qquad \overleftarrow{R} \subseteq B \quad$ iff $\quad R \subseteq B;U.$

*Proof.* $R \subseteq B;U$

$\equiv$ { vector negation }

$\overline{B};U \cap R = \emptyset$

$\equiv$ { superdistributivity }

$\overline{B};R \cap R = \emptyset$

$\equiv$ { shunt $R$ and left exchange }

$R;{}^{\cup}R \subseteq \neg\overline{B}$

$\equiv$ { definition of $\overline{B}$ and shunt $J$ }

$J \cap R;{}^{\cup}R \subseteq B$

$\equiv$ { definition of domain }

$\overleftarrow{R} \subseteq B.$ ∎

The other properties of the domain operator and its symmetric counterpart follow from the Galois connection. As an example, take the single-sided unit laws

$$\overleftarrow{R};R \ = \ R \ = \ R;\overrightarrow{R}.$$

*Proof.* We need only prove the first equation. Applying the above Galois connection with $\overleftarrow{R}$ in place of $B$ yields

$$R \ \subseteq \ \overleftarrow{R};U.$$

Therefore $R \ = \ R \cap \overleftarrow{R};U \ = \ \overleftarrow{R};(R \cap U) \ = \ \overleftarrow{R};U$, by superdistributivity. ∎

## 4 Temporal Logic

In the relational and regularity calculi, the equation

$$x;a;y = b \qquad (a, b \text{ given observations})$$

always has a solution for $x$ and $y$. In the other calculi, the existence of a solution defines an interesting relation between $a$ and $b$, namely that $a$ is a *subobservation* of $b$. We define $\Diamond P$ (*sometime* $P$) as the set of all observations that have a subobservation in $P$.

**Definition of $\Diamond$** $\qquad \Diamond P \overset{def}{=} U;P;U.$

The dual modality (*always* $P$) is defined in the usual way

**Definition of $\Box$** $\qquad \Box P \overset{def}{=} \neg\Diamond\neg P,$

so $x \in \Box P$ iff all subobservations of $x$ are in $P$. These definitions can also be made in the relational calculus, but they are uninteresting, because with relations

$$\Diamond P = U \quad \text{unless } P = \emptyset.$$

This law is known as the Tarski rule. — $\Box$ distributes through arbitrary conjunctions and $\Diamond$ distributes through arbitrary disjunctions (so both are monotonic). Moreover

**$\Box$-$\Diamond$ basics** $\qquad \Box\Box P \;=\; \Box P \;\subseteq\; P \;\subseteq\; \Diamond P \;=\; \Diamond\Diamond P.$

Suppose every subobservation of a given observation $x$ lies in $P$. Then the same must be true for any observation obtained by cutting something off $x$.

**$\Box$-restrict** $\qquad (\Box P);^{\cup}R \;\subseteq\; \Box P.$

*Proof.*

$$\begin{aligned}
& (\Box P);^{\cup}R \subseteq \Box P \\
= \quad & \{ \text{ exchange, definition of } \Box \text{ } \} \\
& (\Diamond\neg P);R \subseteq \Diamond\neg P \\
\Leftarrow \quad & \{ \text{ definition of } \Diamond \text{ } \} \\
& U;R \subseteq U.
\end{aligned}$$

■

$R$ is *temporally importable* if $P;Q\cap R \subseteq (P\cap R);(Q\cap R)$ for all $P,Q$. Moszkowski argues that temporal importability is surprisingly common and allows 'very compositional' reasoning [9].

**Temporal import** $\qquad (P;Q) \;\cap\; \Box R \;\;\subseteq\;\; (P\cap\Box R)\;;\;(Q\cap\Box R).$

*Proof.*

$$\begin{aligned}
& P;Q\cap\Box R \\
= \quad & \{ \text{ Dedekind } \} \\
& (P\cap(\Box R);^{\cup}Q);Q \;\cap\; \Box R \\
\subseteq \quad & \{ \text{ } \Box\text{-restrict } \} \\
& (P\cap\Box R);Q \;\cap\; \Box R \\
\subseteq \quad & \{ \text{ Dedekind } \} \\
& (P\cap\Box R);(Q \;\cap\; (P\cap\Box R)^{\cup};(\Box R)) \\
\subseteq \quad & \{ \text{ } \Box\text{-restrict } \} \\
& (P\cap\Box R);(Q\cap\Box R).
\end{aligned}$$

■

If $P$ holds always and $Q$ at some time then $P$ and $Q$ must hold together at some time.

**$\Diamond$-$\Box$-combine** $\qquad \Diamond P\cap\Box Q \;\;\subseteq\;\; \Diamond(P\cap Q).$

*Proof.*

$$\begin{aligned}
& \Diamond P\cap\Box Q \\
\subseteq \quad & \{ \text{ definition of } \Diamond\text{, temporal import } \} \\
& U;(P\cap\Box Q);U \\
\subseteq \quad & \{ \text{ } \Box\text{-}\Diamond \text{ basics, definition of } \Diamond \text{ } \} \\
& \Diamond(P\cap Q).
\end{aligned}$$

■

## 5 Additional axioms

The axiom of *local linearity* expresses the linear progress of time in any given observation. It holds in all models mentioned so far (though not in partial order models like [12]). Consider an observation that can be split into subobservations in two different ways:

$$x;y = x';y'.$$

The transition between $x$ and $y$ may occur at the same time as that between $x'$ and $y'$, or before or after. In all cases, there is an observation $w$, called a mediator, which fills the gap between the transitions (where an empty gap is "filled" by a unit observation). Thus we postulate:

$$x;y = x';y' \quad \Rightarrow \quad \exists\, w \,.\, (x;w = x' \;\wedge\; w;y' = y) \;\;\vee\;\; \exists\, w \,.\, (x';w = x \;\wedge\; w;y = y').$$

This axiom may be drawn as a pair of commuting diagrams, familiar in category theory, where they assert factorisation, usually unique.

$$\begin{tikzcd}
 & {} \arrow[dd, dashed, "w"] \arrow[dr, "y"] & \\
{} \arrow[ur, "x"] \arrow[dr, "x'"'] & & {} \\
 & {} \arrow[ur, "y'"'] &
\end{tikzcd}
\qquad \text{or} \qquad
\begin{tikzcd}
 & {} \arrow[dr, "y"] & \\
{} \arrow[ur, "x"] \arrow[dr, "x'"'] & & {} \\
 & {} \arrow[uu, dashed, "w"] \arrow[ur, "y'"'] &
\end{tikzcd}$$

As an example, the string *abc* can be decomposed in two different ways:

$$a;bc \;=\; ab;c.$$

In this case, $b$ is the mediator $w$. Note that $b$ is the only mediator satisfying the first diagram, and there is no mediator satisfying the second diagram. In the relational calculus, the equation

$$(r,s);(s,t) = (r,s');(s',t)$$

has two mediators, namely $w = (s,s')$ satisfying the first diagram and $w = (s',s)$ satisfying the second diagram. Similarly, in the regularity calculus, the equation

$$x;y = x';y'$$

has mediators $x^{-1};x'$ and $x'^{-1};x$. In these last two calculi, all non-trivial equations: $x;y = x';y'$ with $(x,y) \neq (x',y')$ have two mediators, one in each direction.

We formulated local linearity as a property of individual observations, but our main objective is to design a calculus of sets of observations, just as the relational calculus applies to sets of pairs. Fortunately, there is a set level version of local linearity.

**Double exchange** $\qquad R;S \subseteq \neg(X;Y) \;\text{ iff }\; X;(Y;^{\cup}S) \subseteq \neg R \;\text{ and }\; (R^{\cup};X);Y \subseteq \neg S$

*Exercise* 2. Show that double exchange and local linearity are equivalent.

The axiom of *cancellation* is another property shared by all our models of reactive systems. It states that any equation of the form $x;b = c$ or $b;x = c$ has at most one solution for $x$. A consequence of cancellativity is that each of the two mediators in the local linearity axiom is unique (when it exists). We do not have a set-level version of this axiom; the relational calculus also has to do without one.

We now introduce a pair of alternative axioms which clearly differentiate the relational and regularity calculi from all the others. These two calculi apply to systems in which each observation $x$ has an inverse $\breve{x}$, which cancels its effect:

$$\breve{x};x = \overrightarrow{x} \quad \text{and} \quad x;\breve{x} = \overleftarrow{x}\,.$$

This is the defining property of a groupoid [3]. Note that the existence of inverses implies cancellativity. At the set level, the existence of inverses can be expressed by the axiom

$$(P;^{\cup}Q);R \;=\; P;(Q\,{}^{\cup}\!;R)$$

On the other hand, in reactive systems, it is not possible to backtrack or rewrite history by an inverting operation. The only action that can be undone is the trivial action that has not actually done anything, i.e. a unit $\overleftarrow{z}$. So we require that units are *indivisible*

$$x;y = \overleftarrow{z} \quad \text{implies} \quad x = y = \overleftarrow{z}\,.$$

This can be translated to the set-level into an axiom without variables

$$(\neg J)(\neg J) \;\subseteq\; \neg J.$$

The following variant is equivalent but better suited to calculations

**Indivisibility of units** $\qquad (P;Q) \cap B \;=\; P \cap Q \cap B.$

*Exercise* 3. Prove that the two preceding laws are equivalent.

## 6 Some Exercises

In this section we illustrate the use of indivisibility of units and local linearity. Space constraints prevent us from giving the proofs; they are left as exercises. — We start with laws that enable elimination of temporal operators.

*Exercise* 4. $\qquad J \cap \Diamond P \;=\; J \cap P \;=\; J \cap \Box P.$

Note that taking $P = J$ gives $J = \Box J$, which is yet another way of stating the indivisibility of units. — Another corollary states that $P$ holds always in some subobservations iff $P$ holds in some unit subobservation:

*Exercise* 5. $\qquad \Diamond\Box P \;=\; \Diamond(J \cap P).$

The following law shows a subtle difference to point-based temporal logic [8] where $\Diamond\Box\Diamond P = \Box\Diamond P$.

*Exercise* 6. $\qquad \Diamond\Box\Diamond P \;=\; \Diamond\Box P.$

A major difficulty of calculating with negation is its failure to distribute in any way over composition. Sometimes the following helps:

*Exercise* 7. $\qquad (\neg\Diamond P);(\neg\Diamond Q) \;\subseteq\; \neg\,((\Diamond P);(\Diamond Q)).$

There are two ways of cutting something from the right of $P;Q$. The cut may be placed either before or after the transition between $P$ and $Q$. This alternative is expressed as follows.

**Right split** $\qquad (P;Q);^{\cup}R \;=\; P;(Q;^{\cup}R) \,\cup\, P;^{\cup}(R;^{\cup}Q).$

*Exercise* 8. Prove that the right split law is equivalent to the double exchange law. Deduce that the right split law is (in the presence of the single exchange laws) equivalent to its time-wise dual.

# 7 Conclusion and future work

The sequential calculus combines the essence of the calculus of relations with interval temporal logic. By restricting the nature of observations we may specialise it to other important models such as regular expressions or CSP.

A desirable objective in the design of a calculus is to ensure that all true facts expressible in the calculus can be derived from the axioms alone, without any resort to reasoning about individual observations. Our experience so far suggests that this is almost always possible, just like in the relational calculus. However, here is a challenge. Consider

$$P;Q;R \ \cap \ \Diamond X \quad \subseteq \quad (P;Q \ \cap \ \Diamond X);R \ \cup \ P;(Q;R \cap \Diamond X) \ \cup \ \Diamond(X \cap \Diamond Q).$$

On the observation level this states that if $p;q;r$ has a subobservation $x$ then either $x$ is a subobservation of $p;q$, or of $q;r$, or covers $q$ — and that follows easily from local linearity and cancellativity. However, we could not find a proof at the set-level. (Hofstee and Leino give a very ingenious proof of this law, based on stronger assumptions [5, 4].)

All axioms of sequential calculus except the indivisibility of units are valid in the relational calculus. It would be nice to derive the sequential calculus by restricting the relational calculus appropriately. Recent research shows that this is indeed possible [6].

Another line of current research is the application of the sequential calculus to point-based (rather than interval-based) temporal logic.

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