Irreversibility and symmetry in quantum theory

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THE ROYAL SOCIETY

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Outline

- Motivations.
- Limitations of traditional accounts.
- Conservation laws and asymmetry.
- Coherence in thermodynamics.
- Irreversibility & non-commutativity

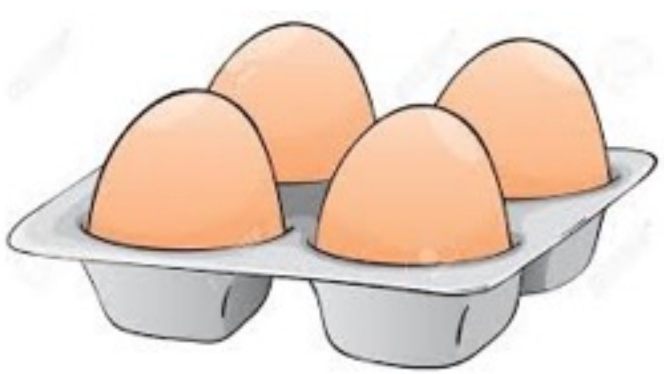
Traditional Thermodynamics

$$E_{\rm in} = E_{\rm out}$$

1st Law: “Energy is conserved microscopically.”

2nd Law: “Order is non-decreasing in time.”

??

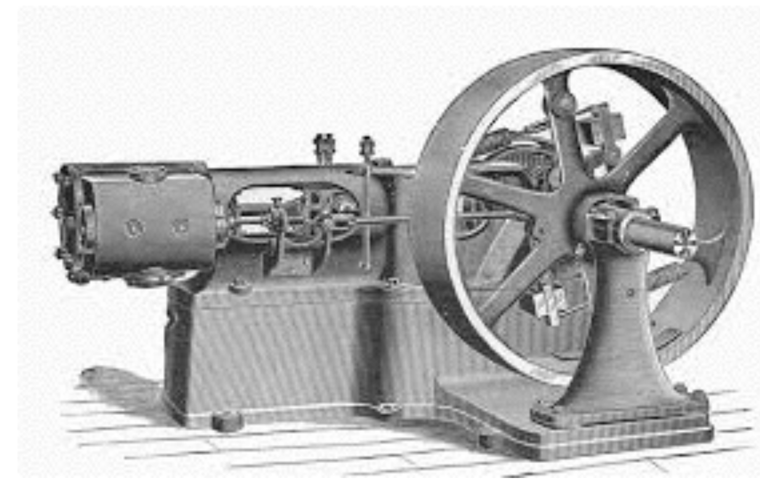


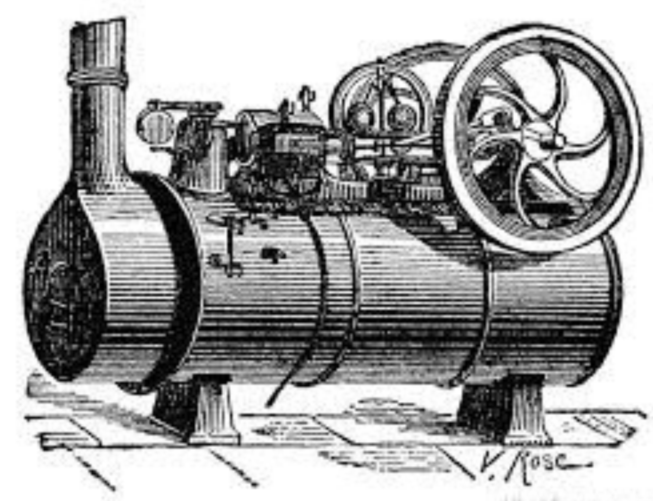
2nd Law of Thermodynamics

- “It is impossible to construct a device who’s sole effect is the extraction of work from heat.”
- “It is impossible to construct a device who’s sole effect is the erasure of a bit.”
- “It is impossible to see inside a furnace, solely by the light of the furnace.”

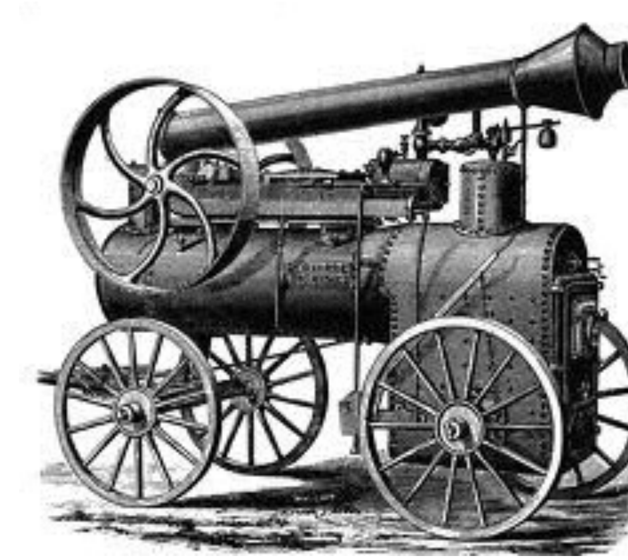
* *Bennett (1987)*

Limitations of existing thermodynamics





The Thermodynamic Limit



- *"Thermodynamics means the thermodynamic limit."*

The Thermodynamic Limit

- *“Thermodynamics means the thermodynamic limit.”*
- (Except it doesn’t)

The Thermodynamic Limit

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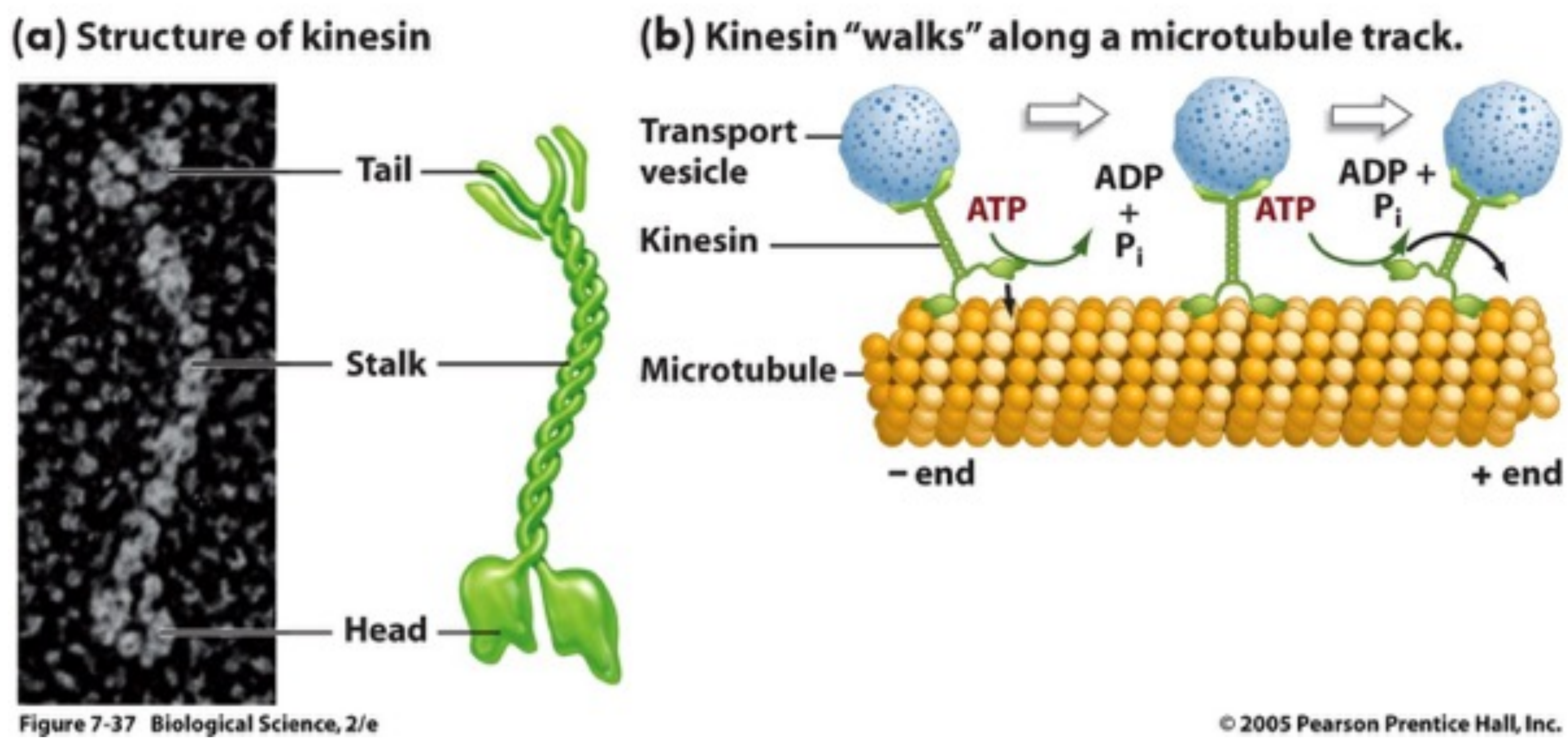
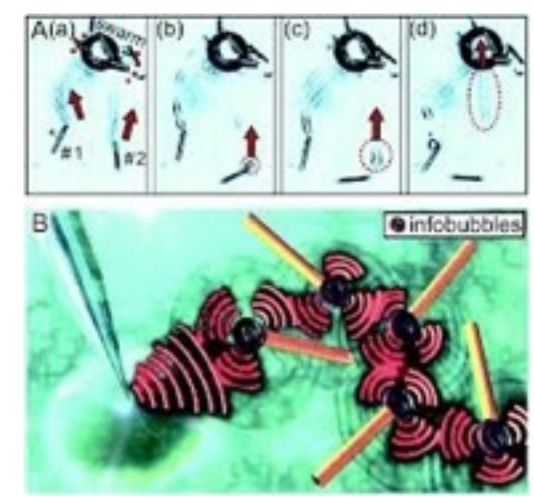


Motivation

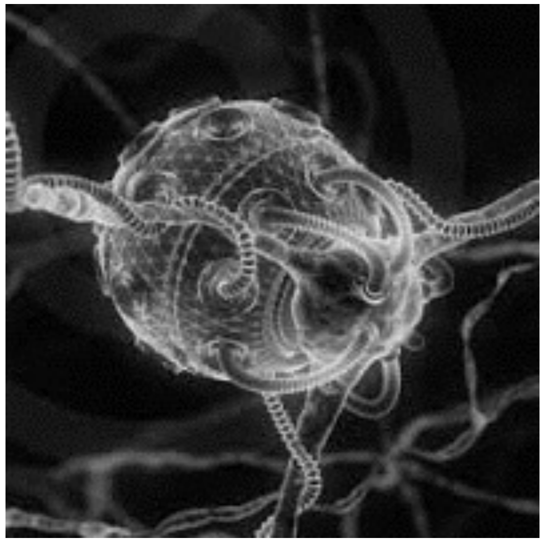
- Active work to develop nanoscale thermodynamic machines.

- Nanotechnology ~$6 billion (currently)



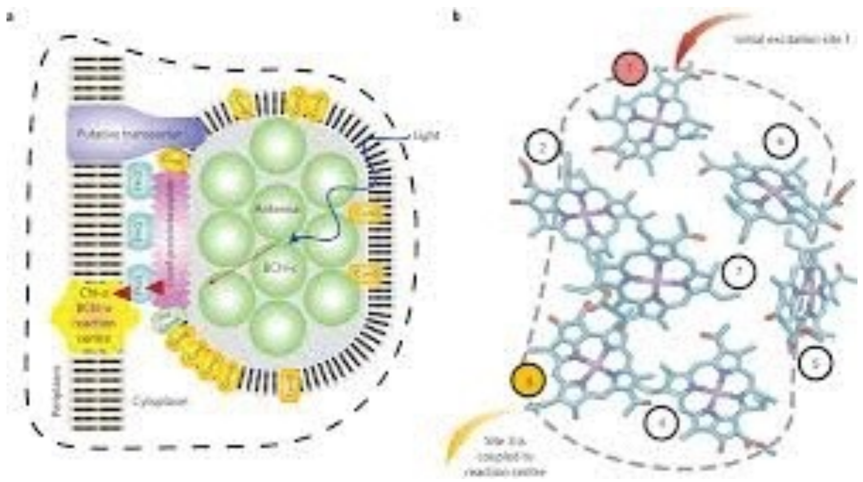
Motivation

Q: What thermodynamic laws operate at micro/nano/pico/… scales?



Q: How do coherent superpositions extend thermodynamic processes?

What laws describe irreversibility beyond the thermodynamic limit?



Axiomatic Analysis

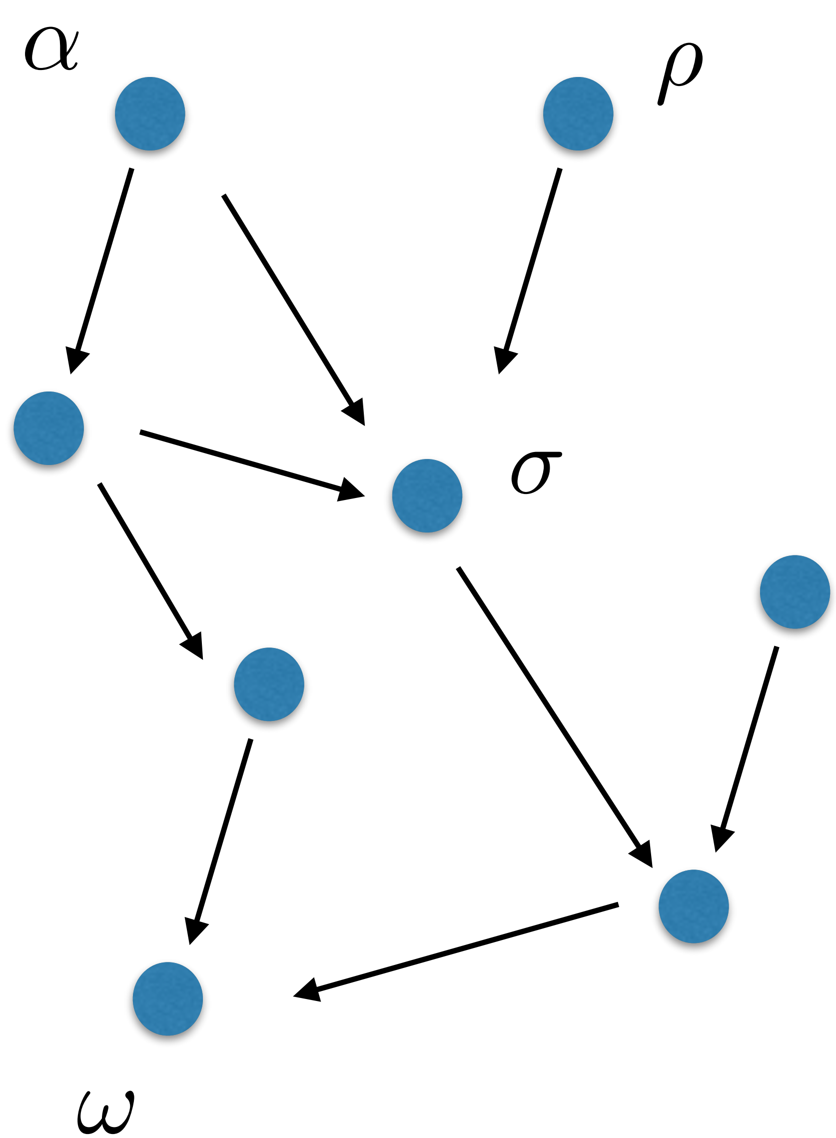
- "*Heat*", "*temperature*" — ambiguous/complex/ indirect.

- **Giles (1964)**: *thermodynamics ultimately concerns the accessibility/inaccessibility of one physical state from another.*

"processes = primitives"

**"The mathematical foundations of thermodynamics", R. Giles (1964)*

Ordering of States



Q: When does the thermodynamic ordering of states admit a unique entropic formulation?

$$\rho \to \sigma$$
$$\Leftrightarrow S(\rho) \leq S(\sigma)$$

Theorem (Lieb & Ingvason 1999):

A unique additive entropy exists
if and only if the following 7 conditions hold:

Reflexivity	$\rho \to \rho$
Transitivity	$\rho \to \sigma \text{ and } \sigma \to \tau \text{ implies } \rho \to \tau$
Consistency	$\rho_1 \to \sigma_1 \text{ and } \rho_2 \to \sigma_2 \text{ then } (\rho_1, \rho_2) \to (\sigma_1, \sigma_2)$
Scale invariance	$\rho \to \sigma \text{ then } \rho^{\otimes t} \to \sigma^{\otimes t} \text{ for } t \geq 0$
Splitting	$\rho \leftrightarrow (\rho^{\otimes t}, \rho^{\otimes (1-t)})$
Stability	$(\rho, \epsilon_1) \to (\sigma, \epsilon_2) \text{ then } \rho \to \sigma$
Comparability	$\text{if } \alpha \to \rho \text{ and } \beta \to \rho \text{ then } \alpha \to \beta \text{ or } \beta \to \alpha$

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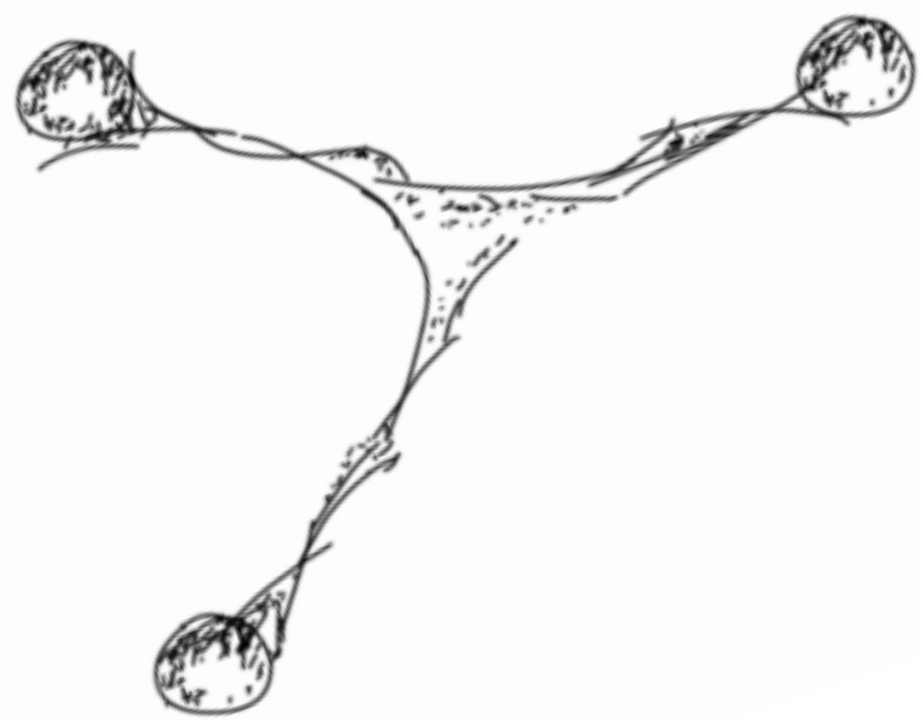
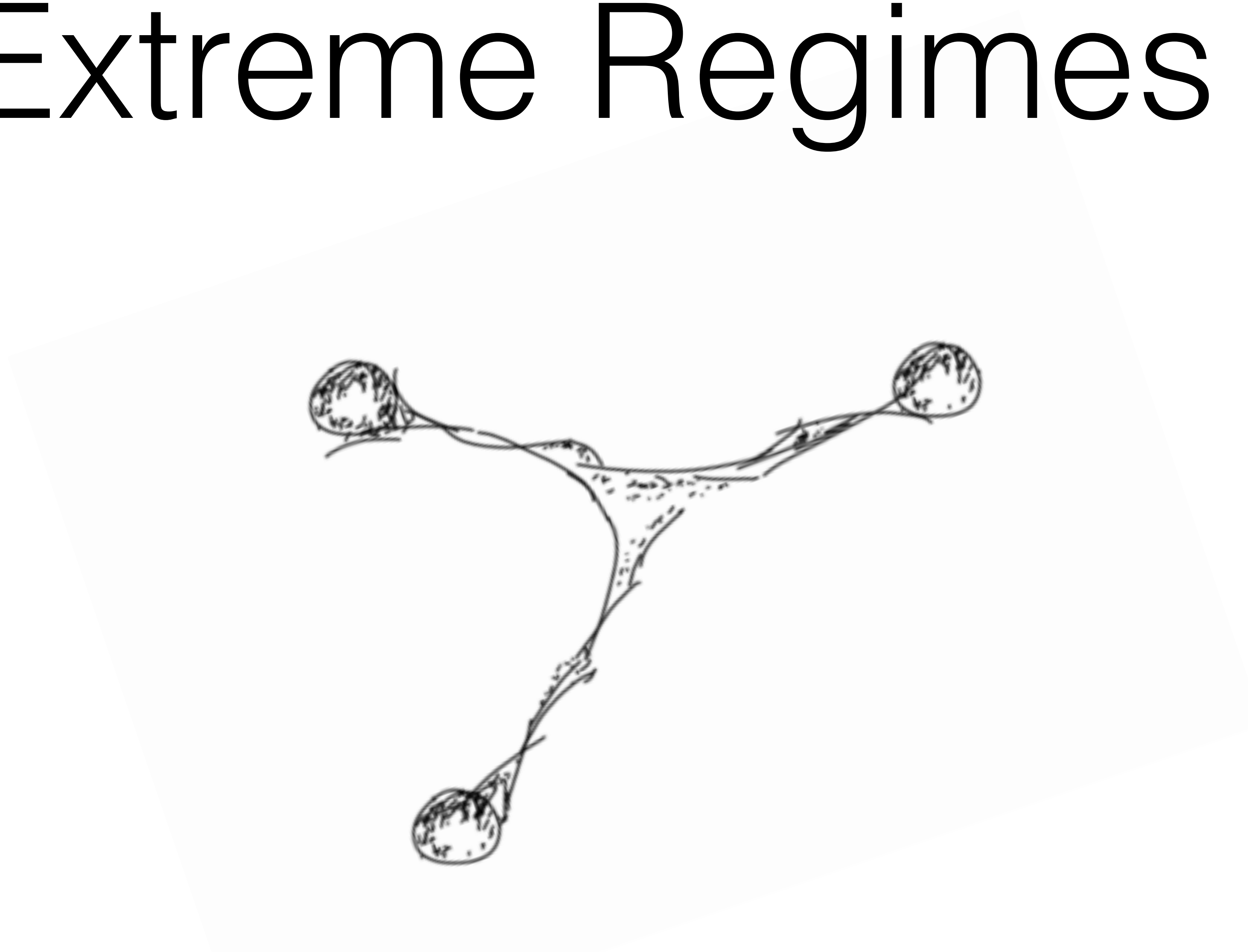
Splitting	$\rho \leftrightarrow (\rho^{\otimes t}, \rho^{\otimes (1-t)})$
Stability	$(\rho, \epsilon_1) \to (\sigma, \epsilon_2)$ then $\rho \to \sigma$
Comparability	if $\alpha \to \rho$ and $\beta \to \rho$ then $\alpha \to \beta$ or $\beta \to \alpha$

Extreme Regimes



- **Determine the thermodynamics of highly entangled quantum systems in extreme regimes.**

Extreme Regimes



- **Determine the thermodynamics of highly entangled quantum systems in extreme regimes.**

$\langle H \rangle$ and "ensemble of microstates": No!

Fluctuation Theorems?

- Arbitrarily violent dynamics on thermal state.
- Sharpening of 2nd Law to an **equality**.

Core structure: $\mathcal{P}_{\gamma_*}(x) = e^{-\beta x}\mathcal{P}_{\gamma}(-x)$

$$\beta = \frac{1}{kT}$$

Distribution for backward process

Distribution for forward process

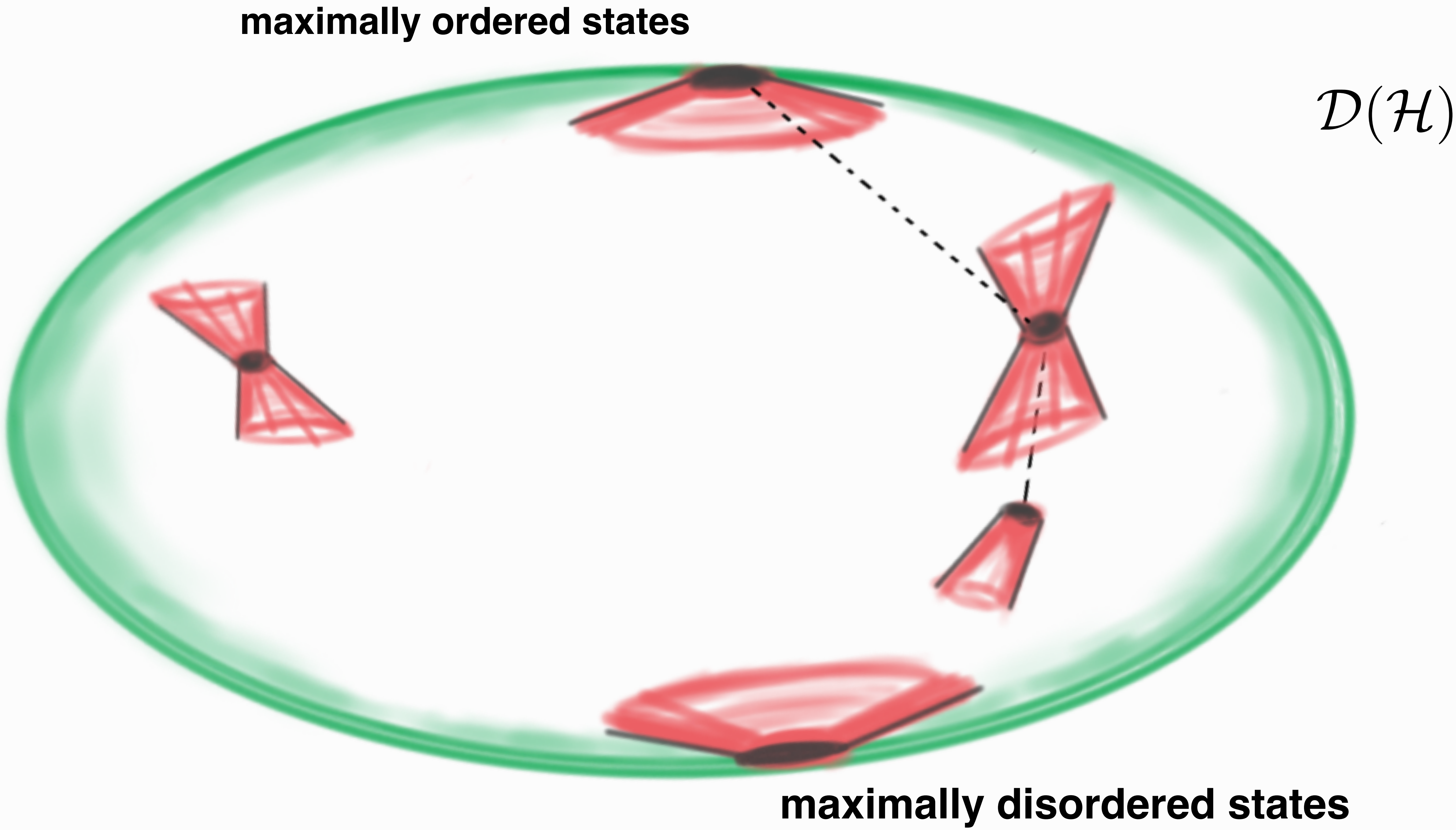
Fluctuation Theorems=Classical

$$\mathcal{P}_{\gamma_*}(x) = e^{-\beta x}\mathcal{P}_{\gamma}(-x)$$

The pairing of γ and γ_* forces us into a **classical** regime.

Poorly suited to handling **coherence** and **entanglement**.

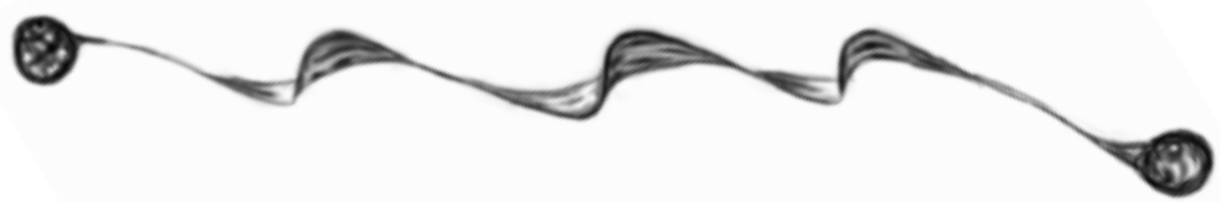
Thermodynamics



MINING FOR DEEPER INSIGHTS
I THINK WE BROUGHT THE WRONG TOOLS.
INSIGHT TEAM
SO WHAT?
BEEN THERE... SEEN THAT.
LIES... ALL LIES!
GOLD!
ANDERSON

Learn from Entanglement Theory?

Resource formulation

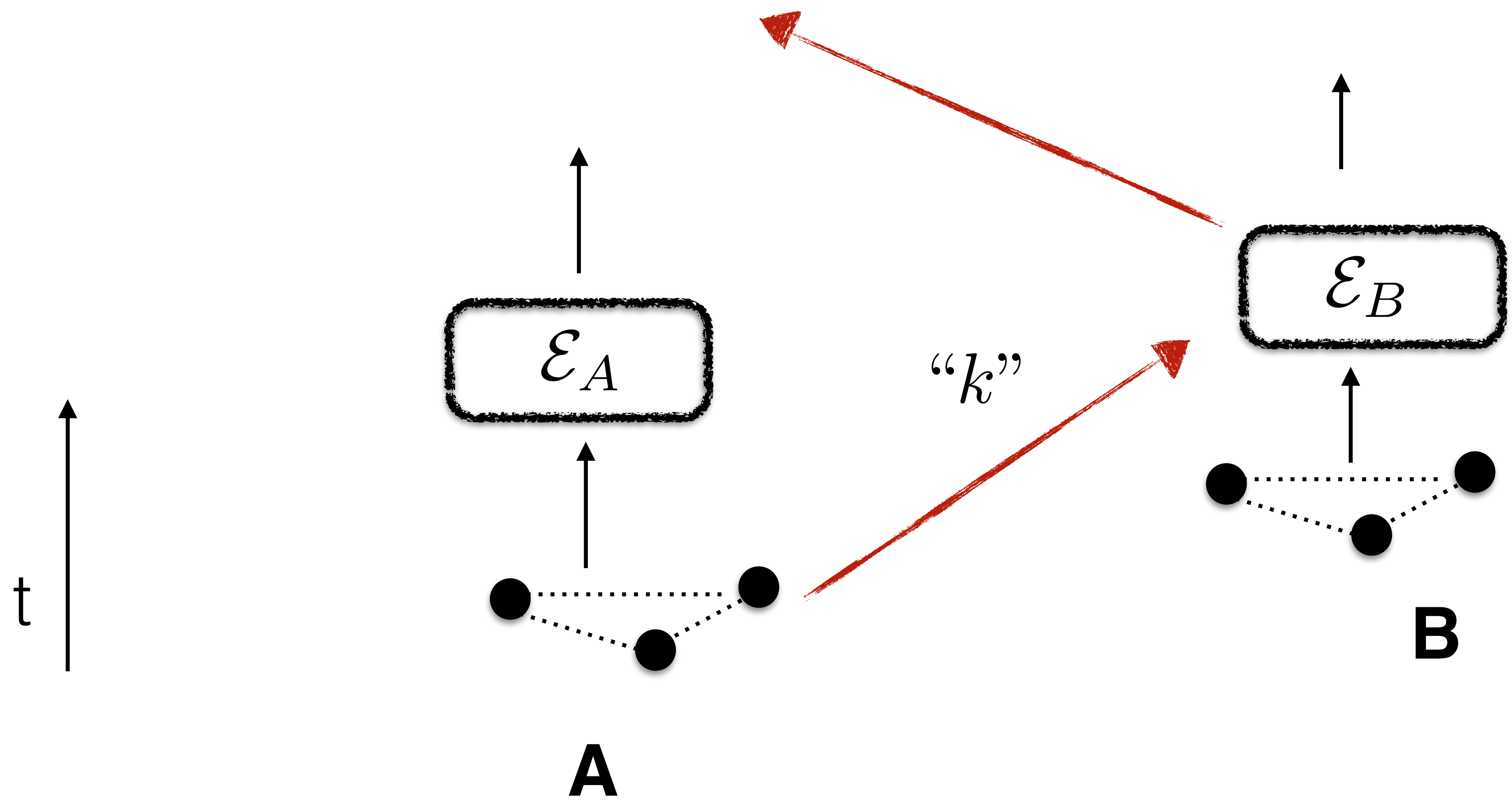
A  B

- **Entanglement** defined by what it's ***not***.
- Local algebra of observables at **A** and **B**.

Define a set of **"free quantum operations"**:

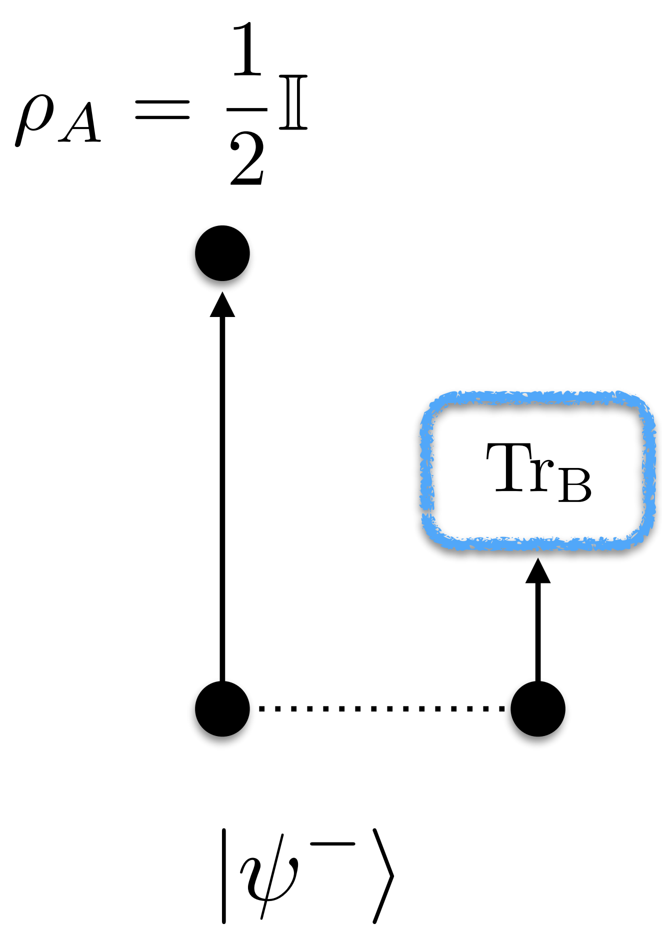
Local Operations + Classical Communications

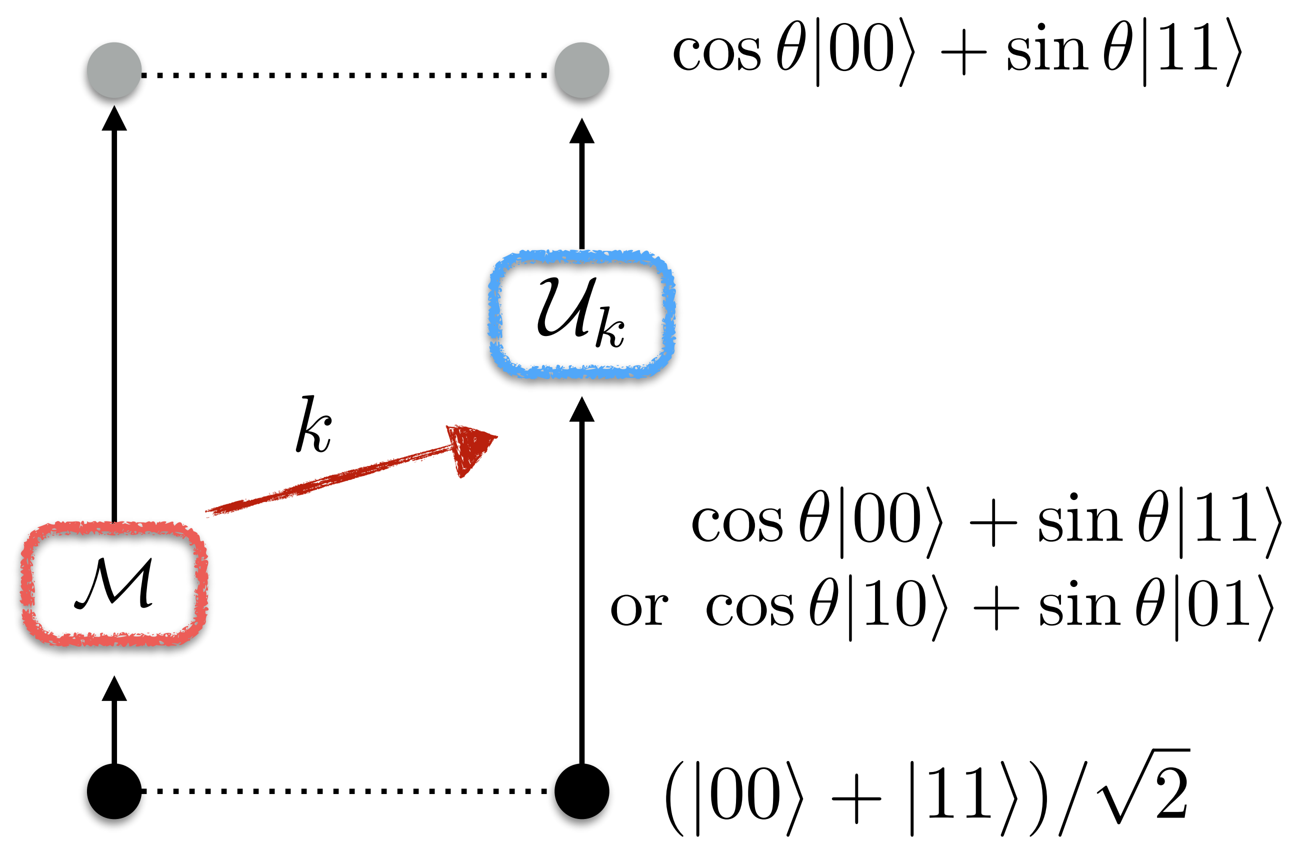
Free Operations



LOCC = "Local operations + Classical Communications"

LOCC Examples

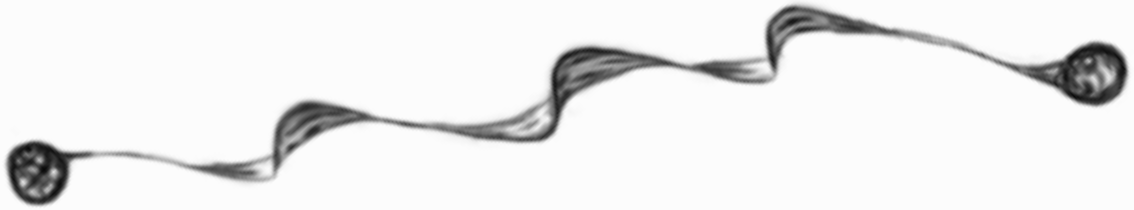




Entanglement Theory

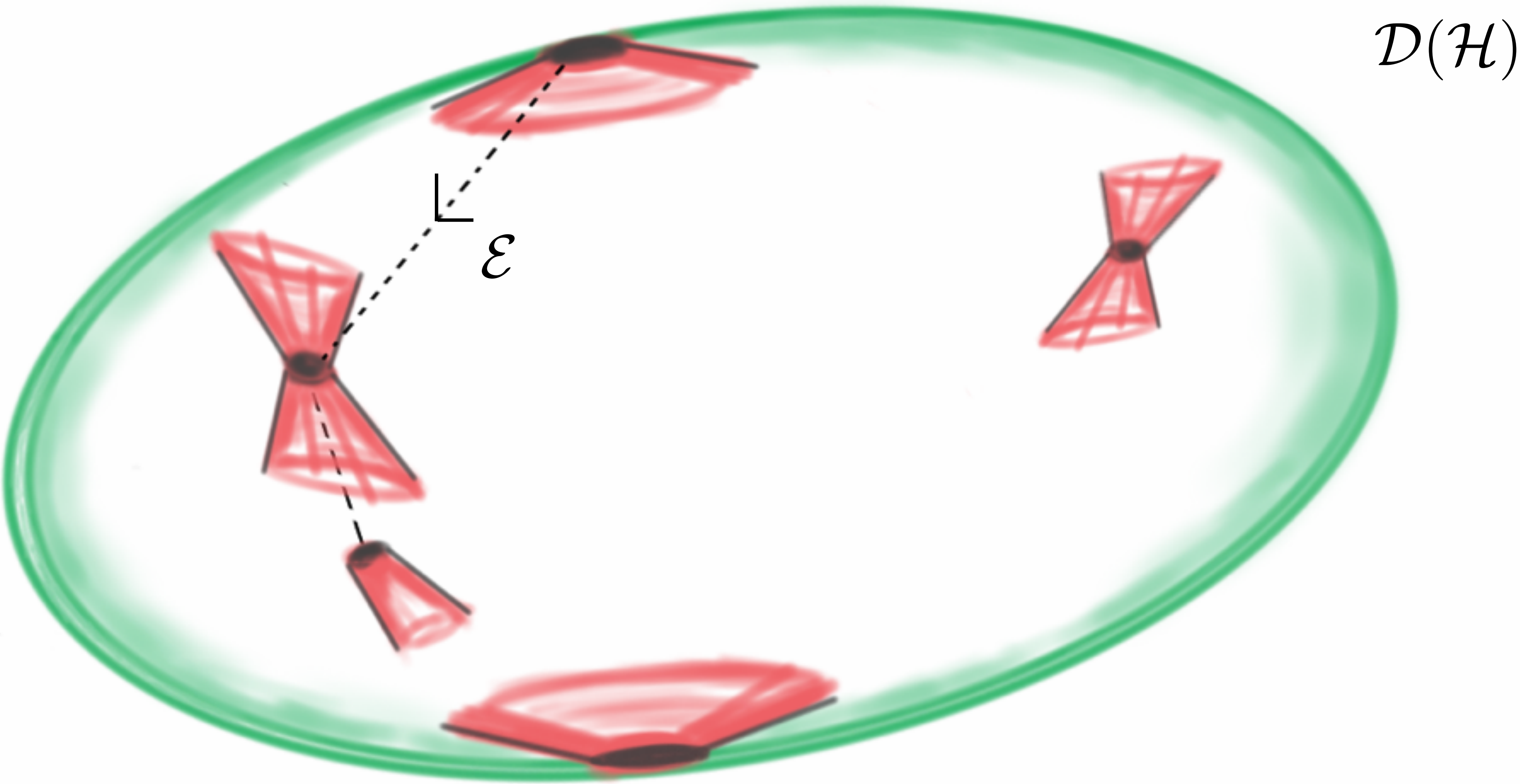
- **Resource state** ρ_{AB}: anything that **cannot** be created under LOCC.

E.g. $$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$$

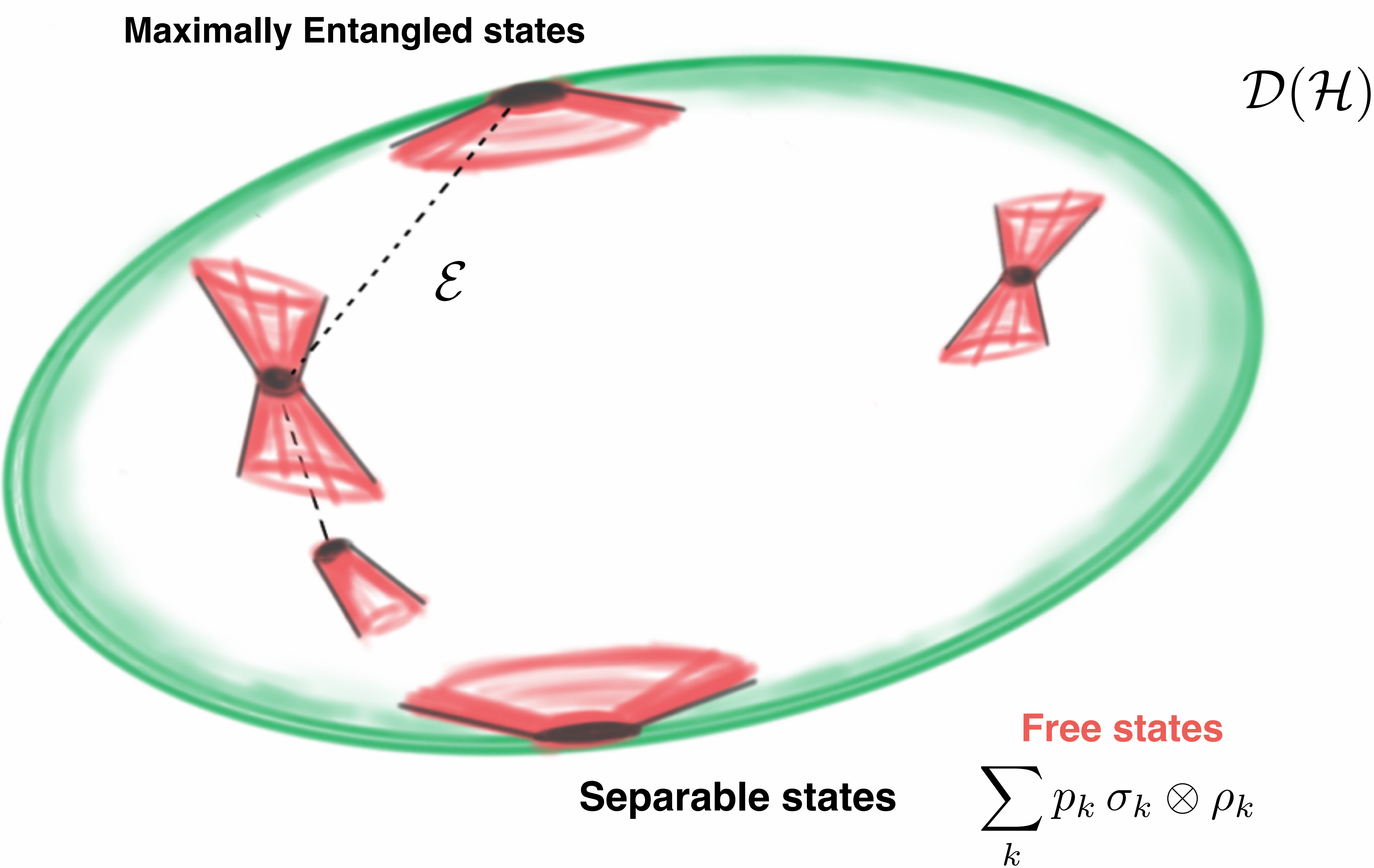


LOCC induces **partial order** on set of all quantum states.

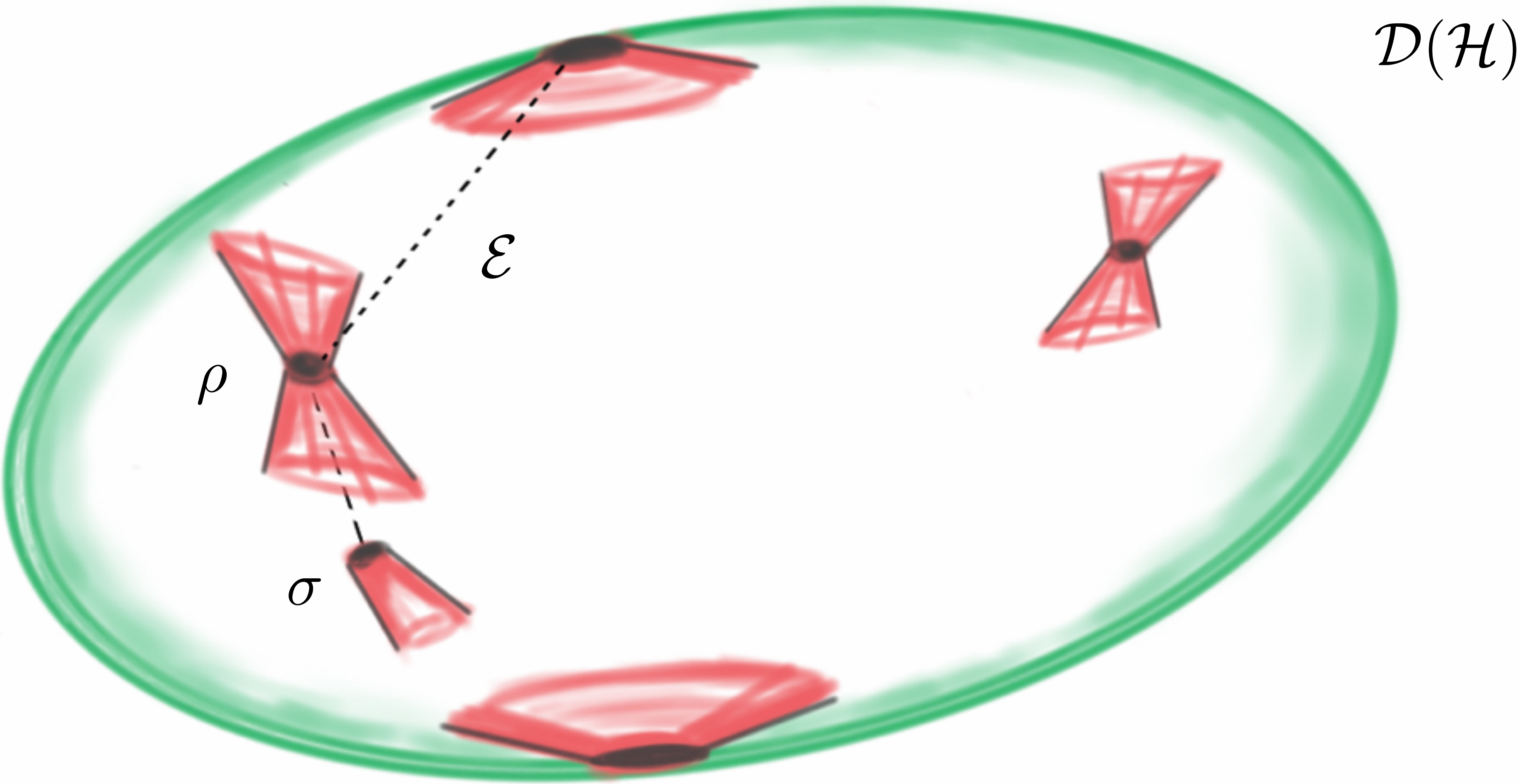
Entanglement Theory



Entanglement Theory



Entanglement Theory



$M(\rho) \geq M(\sigma)$

Resource formulation of thermodynamics

Recent developments

L. del Rio et al, Nature 474 (2011)

I. Marvian, R. Spekkens, Nature Comm 5 (2014)

Toyabe et al, Nature Physics, (2010)

M. Horodecki, Oppenheim, Nature Comm 4 (2013)

F. Brandao et al, Phys. Rev. Lett. 111 (2014)

F. Brandao et al, Nature Phys. (2014)

J. Aberg, Nature Comm (2013)

F. Brandao, M. Plenio, Nature Physics (2010)

1. **Lostaglio, DJ, Rudolph, Nature Communications (2015)**
2. **Lostaglio, Korzekwa, DJ, Rudolph, Physical Review X (2015)**
3. **Korzekwa, Lostaglio, Oppenheim, DJ, New Journal of Physics (2015)**
4. **Cirstoiu, DJ (soon!)**

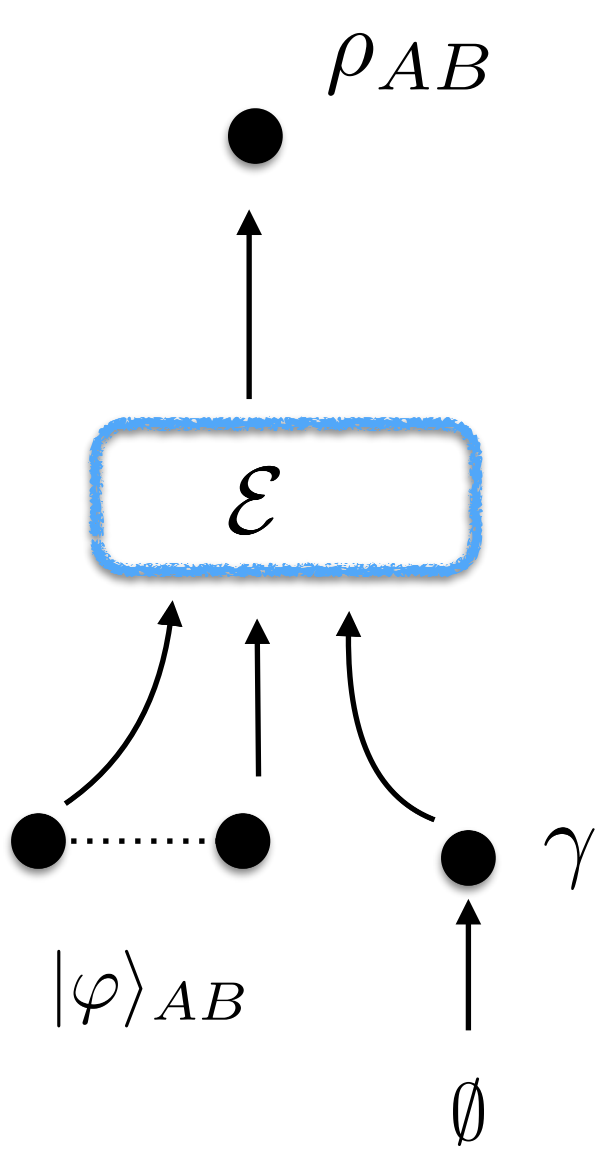
Resource Theory of Thermodynamics

1. Allow single **free state**: $\gamma = e^{-\beta H}/Z$

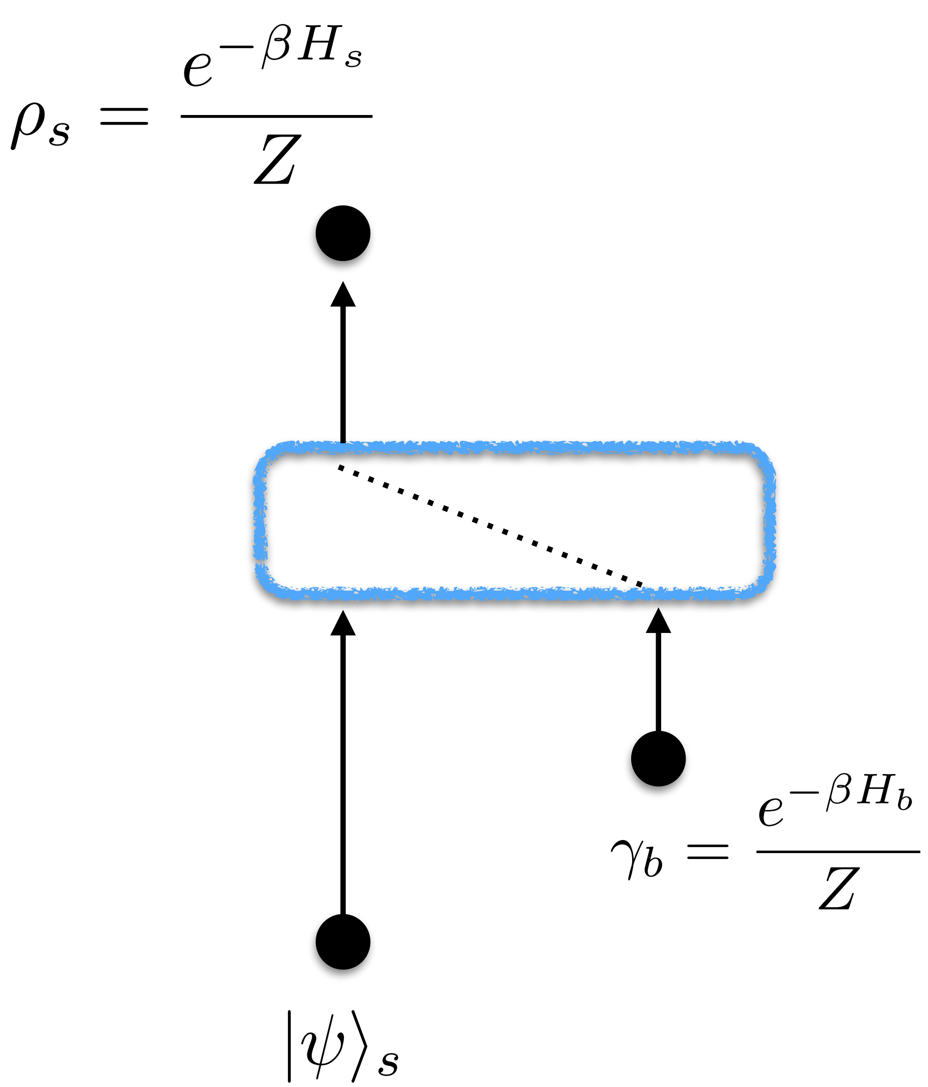
2. **Free quantum operations:**

$$\mathcal{E}(\rho) = \mathrm{tr_b}[\mathrm{U}(\rho \otimes \gamma_\mathrm{b})\mathrm{U}^\dagger]$$

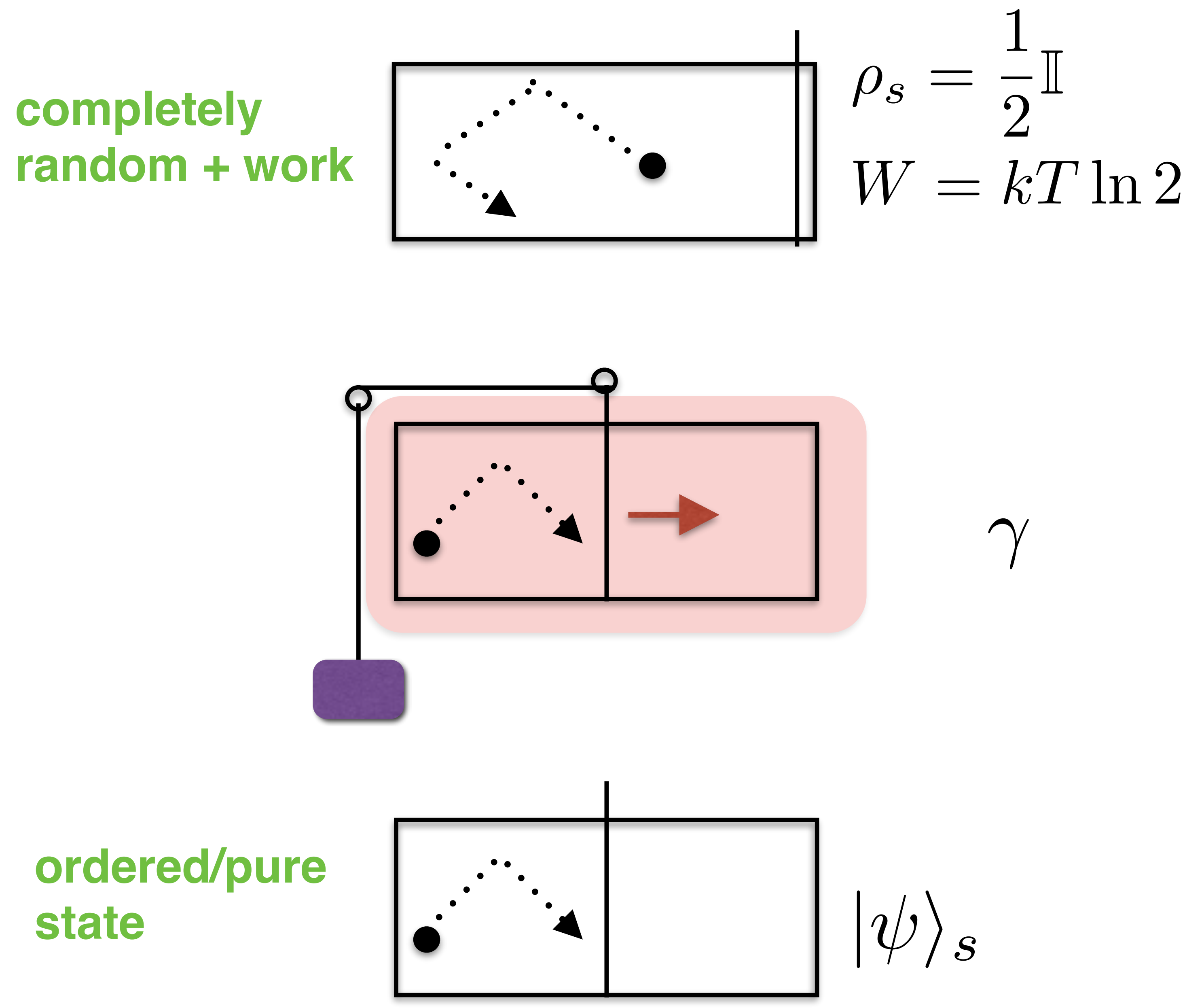
$$[U, H_{\mathrm{tot}}] = 0$$



Thermal Examples

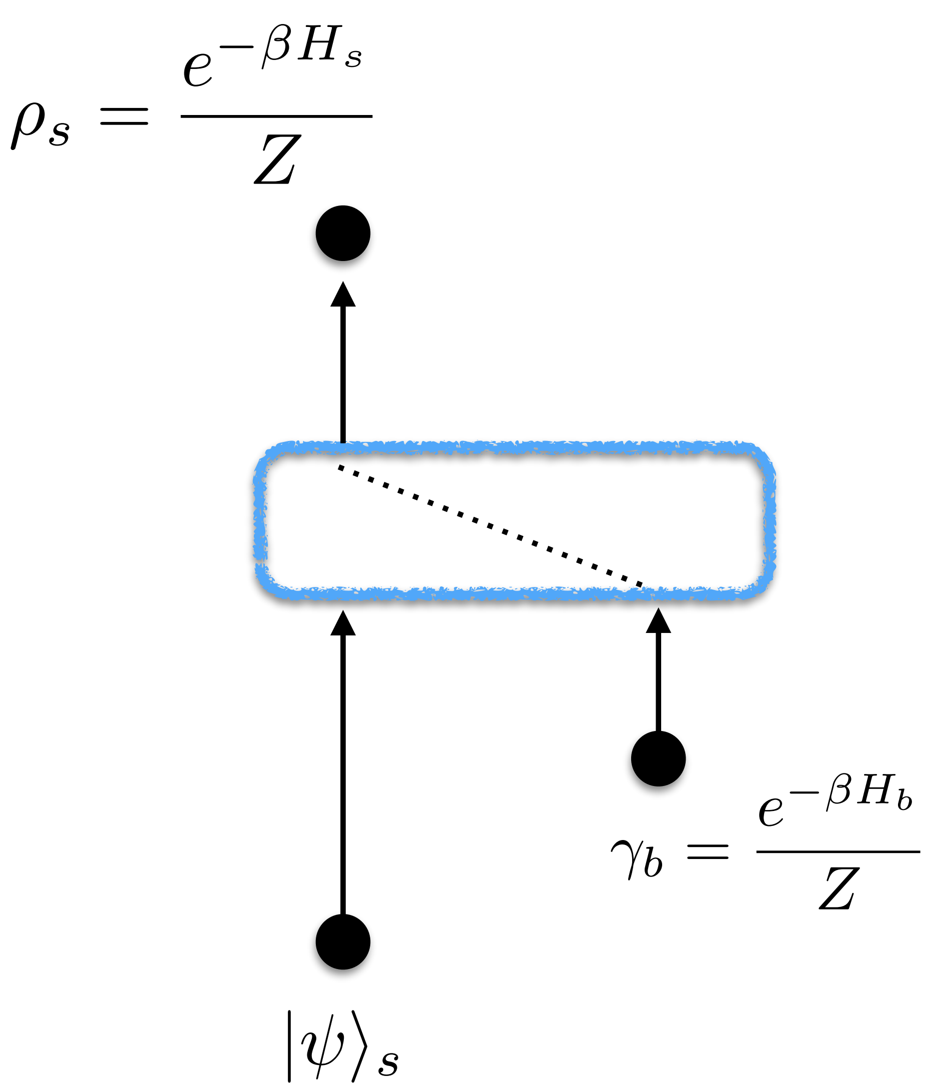


Thermalization

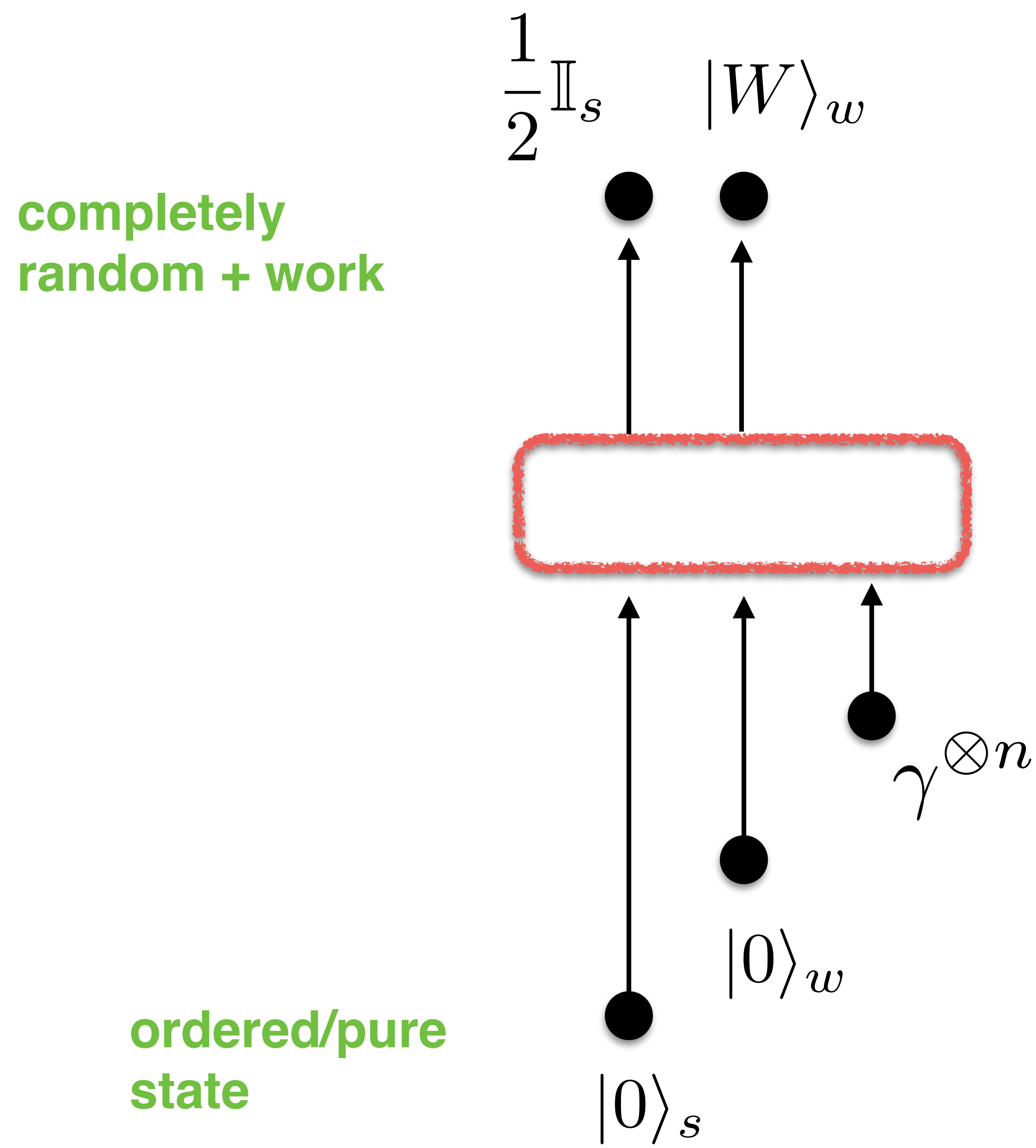


Work Extraction

Thermal Examples

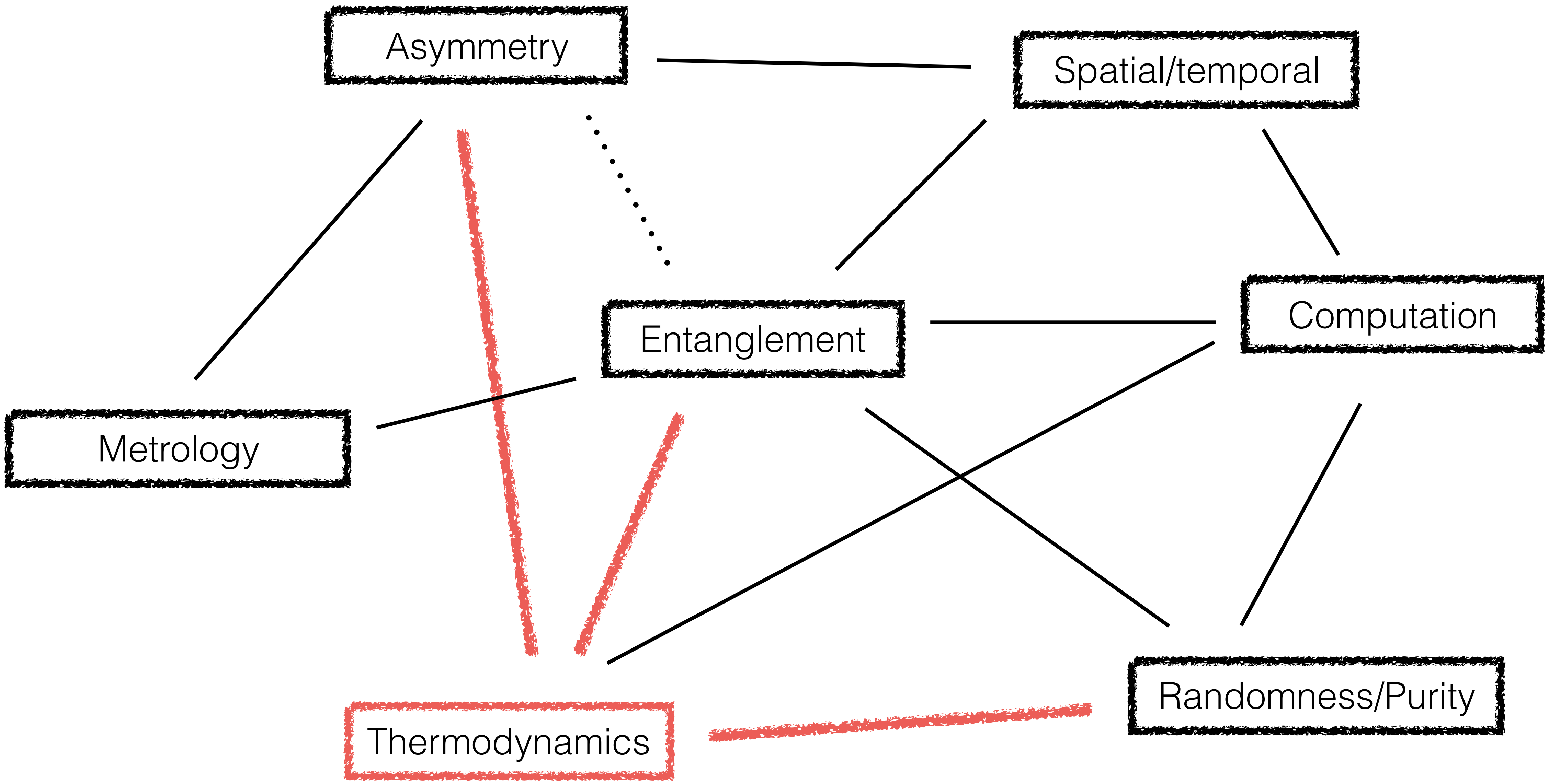


Thermalization

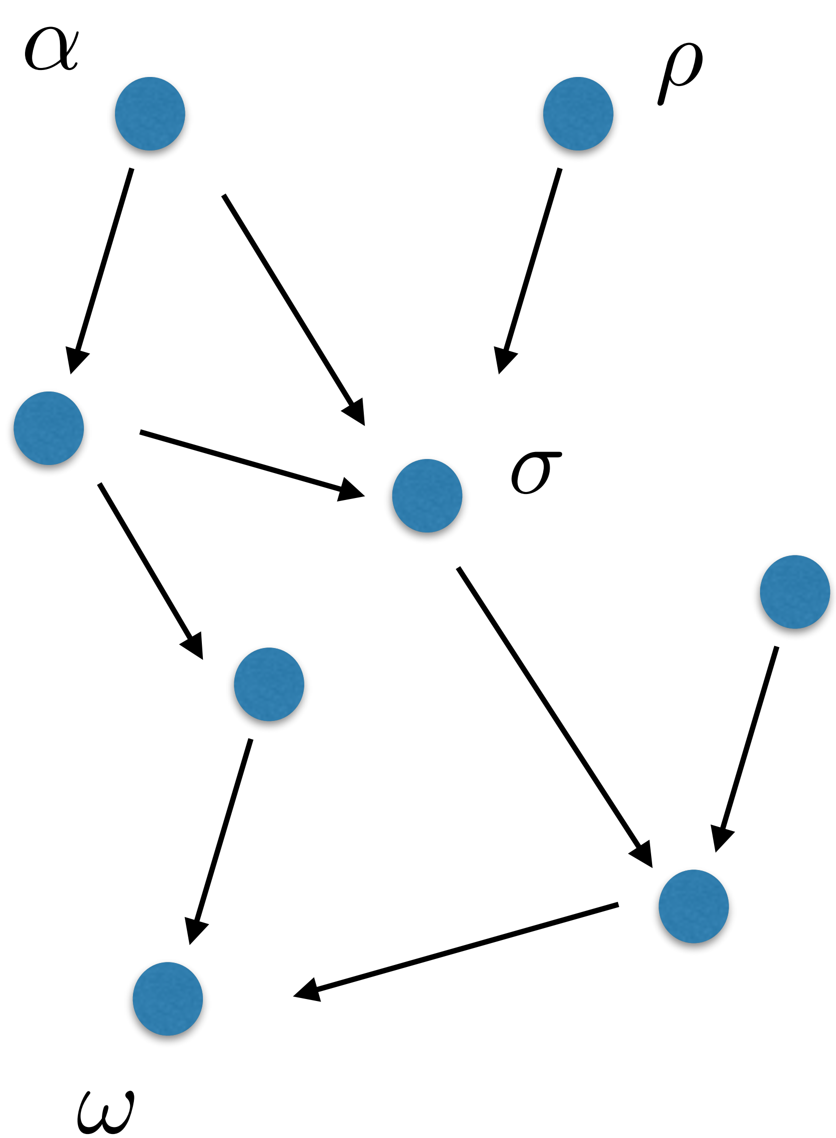


Work Extraction

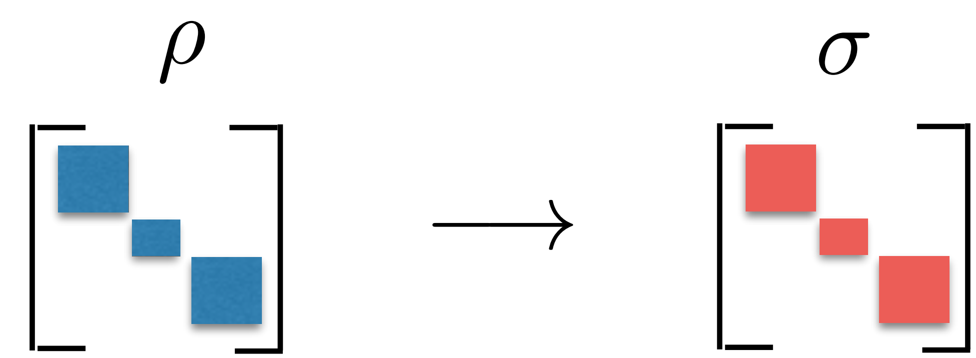
Information-Theoretic Components



Ordering of States?



Zero-coherence states:



Q: Does the ordering of states admit an entropic formulation?

The Second Laws of Thermodynamics

Theorem: For zero coherence states, the transformation $\rho \to \sigma$ is possible

if and only if

$$F_\alpha(\rho) \geq F_\alpha(\sigma) \qquad \forall \alpha$$

Renyi-divergences:

$$D_\alpha(\rho||\sigma) = \frac{1}{\alpha - 1} \log\left[\mathrm{tr}(\sigma^\kappa \rho \sigma^\kappa)^\alpha)\right] \qquad \kappa = \frac{1-\alpha}{2\alpha}$$

$$F_\alpha(\rho) := D_\alpha(\rho||\gamma)$$

* *Brandao et al, PNAS (2015)*

Rough ingredients

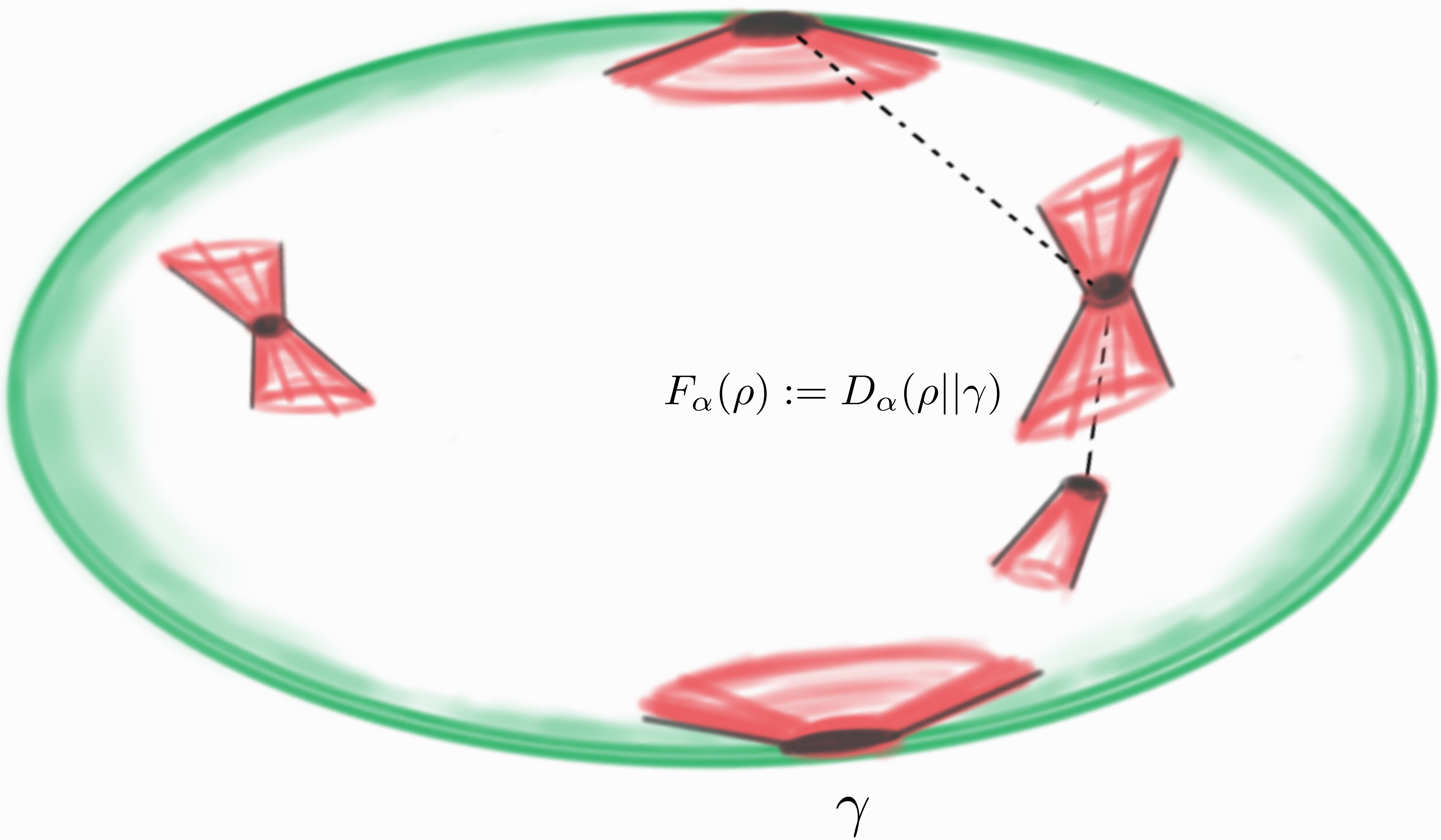
1. "Essentially classical states"
2. Thermal operations —> bistochastic maps
3. Bistochastic maps —> majorization relation
4. Majorization relations <—> entropic measures

1. $$\rho = \mathrm{diag}(\boldsymbol{x})$$

2. $$y_k = \sum_j A_{kj} x_j$$

3. $$x \prec y$$

4. $$x \prec y \Leftrightarrow \{S_\alpha(x) \leq S_\alpha(y)\}_\alpha$$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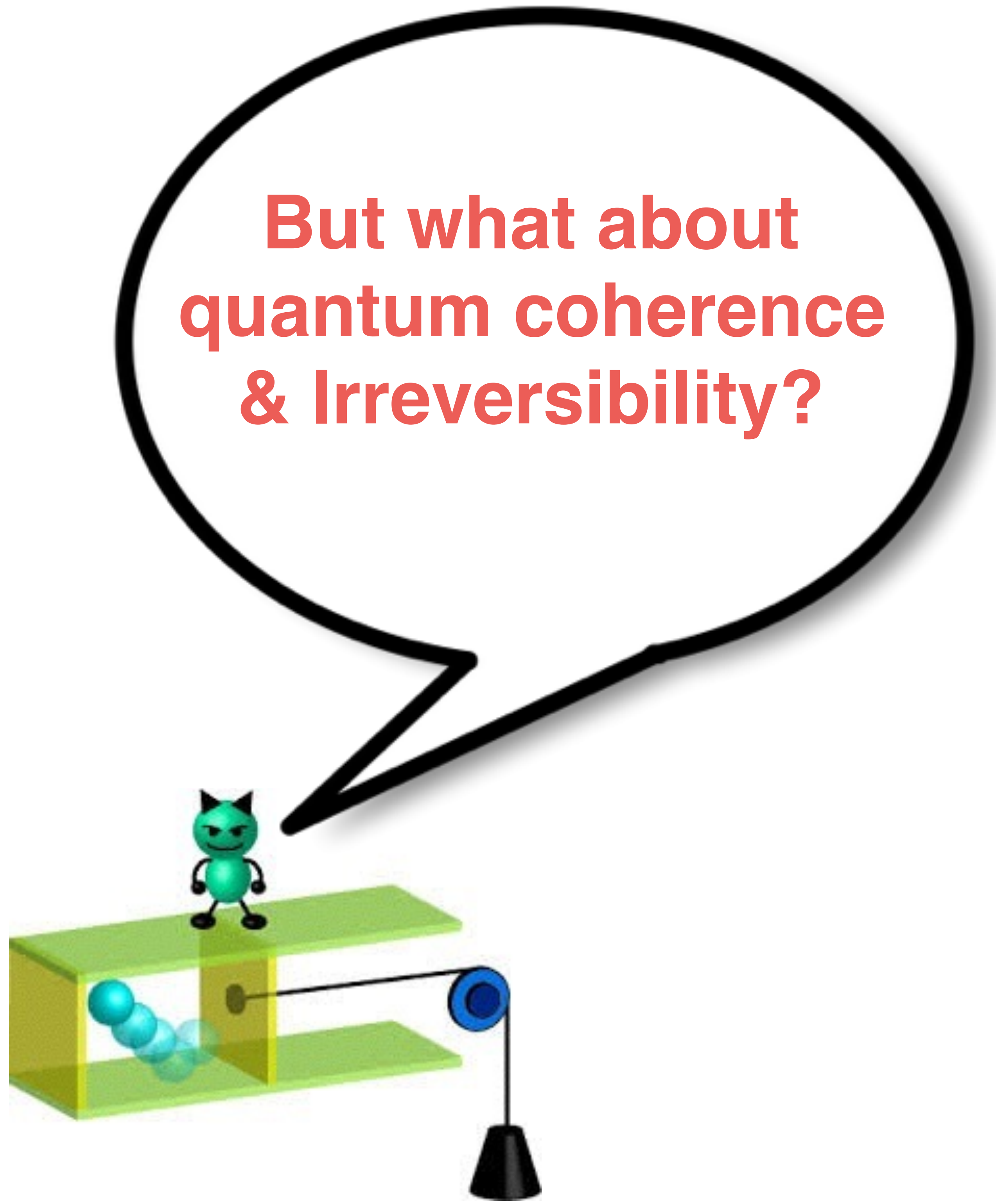


* Lostaglio, DJ, Rudolph, Nature Comm. (2015)

Korzekwa, Lostaglio, DJ, Rudolph, Phys. Rev. X (2015)

Symmetry & the 1st Law of Thermodynamics

- Traditional form: $dE = dQ + dW$

- Microscopic energy conservation (system+bath).

Quantum Mechanical **Symmetry:**

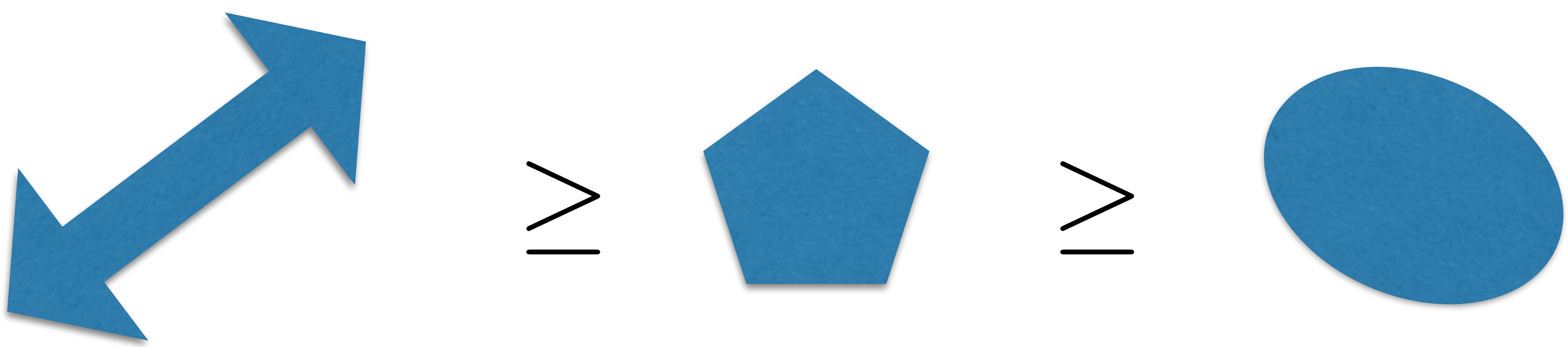
$$[U, H_{\text{tot}}] = 0$$

$$t \mapsto e^{-itH_{\text{tot}}}$$

Constrains **non-conservation** of **two** quantities:

(a) System energy
(b) System "coherence"

When is A is more asymmetric than B?



A theory of asymmetry

* *I. Marvian,R. Spekkens Phys. Rev. A 90, (2014)*
* *I. Marvian, R. Spekkens, New J. Phys. 15, (2013)*
* *M. Ahmadi, DJ, T. Rudolph, New. J. Phys. 15 (2013)*
* *Bartlett et al Rev. Mod. Phys. 79, (2007)*

A theory of asymmetry

- “Group-theoretic Anna Karenina Principle”:

A theory of asymmetry

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"all symmetric objects are alike; each asymmetric object can be asymmetric in its own way."

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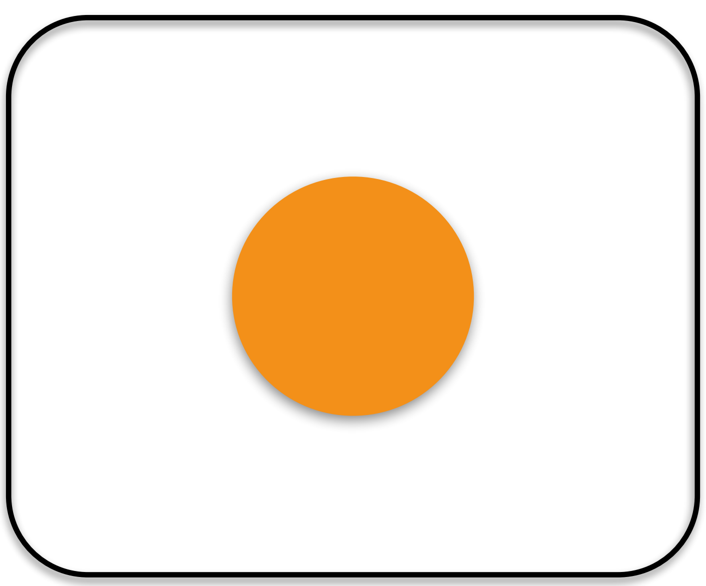
Asymmetry Examples, $G = SU(2)$

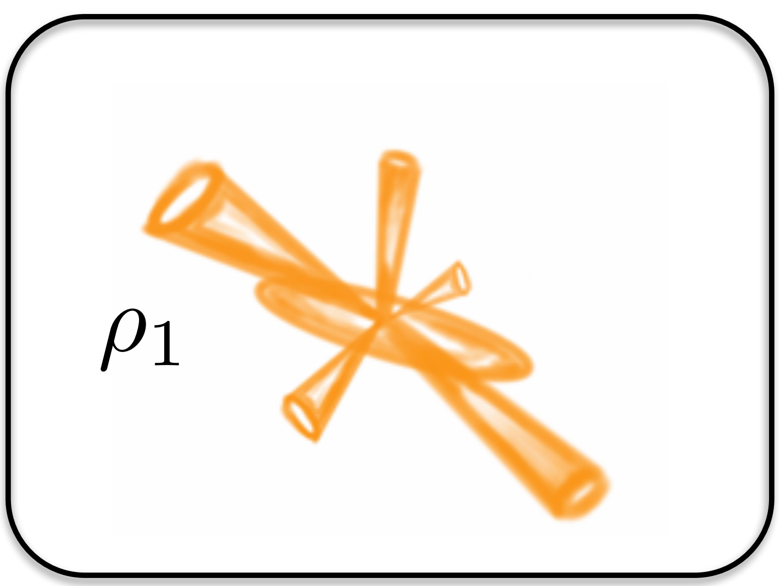
Rotationally invariant states

Pointy/Asymmetric states

$|\psi^-\rangle$, $\rho_s = \frac{1}{2}\mathbb{I}$

$\rho_s = |\uparrow\uparrow\rangle$, $\frac{1}{3}|l,l\rangle\langle l,l| + \frac{2}{3}|l,0\rangle\langle l,0|$,





A theory of asymmetry

- Symmetry group, with unitary representation on $\mathcal{H}$.

$$U : G \to \mathcal{B}(\mathcal{H})$$

$$\mathcal{U}_g(\rho) = U(g)\rho U(g)^\dagger$$

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- Symmetry group, with unitary representation on $\mathcal{H}$.

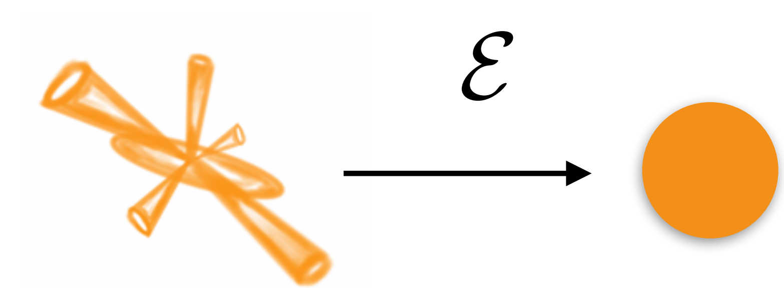
$$U : G \to \mathcal{B}(\mathcal{H})$$

$$\mathcal{U}_g(\rho) = U(g)\rho U(g)^\dagger$$

Free operations: **G-covariant maps** $[\mathcal{U}_g, \mathcal{E}] = 0$

Free states: **symmetric states** $\mathcal{U}_g(\rho) = \rho$

ρ is more asymmetric than σ if
$\sigma = \mathcal{E}(\rho)$ for some covariant $\mathcal{E}$



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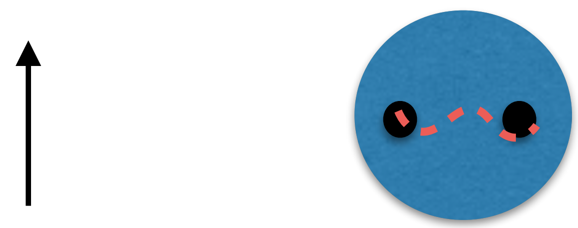
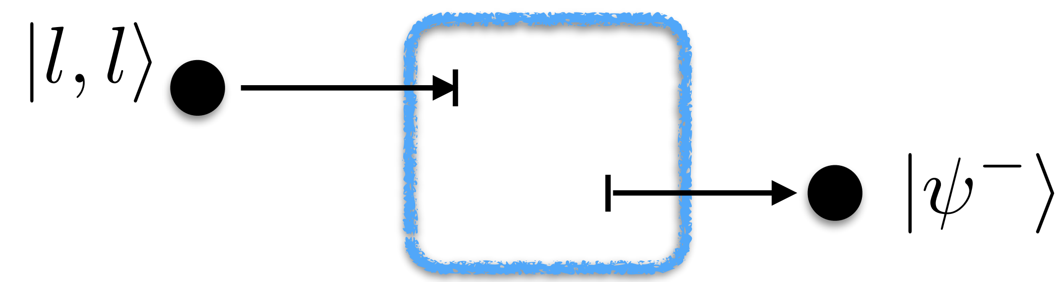
Examples:

E.g. 1



$$|l,l\rangle \longrightarrow |\psi^-\rangle$$

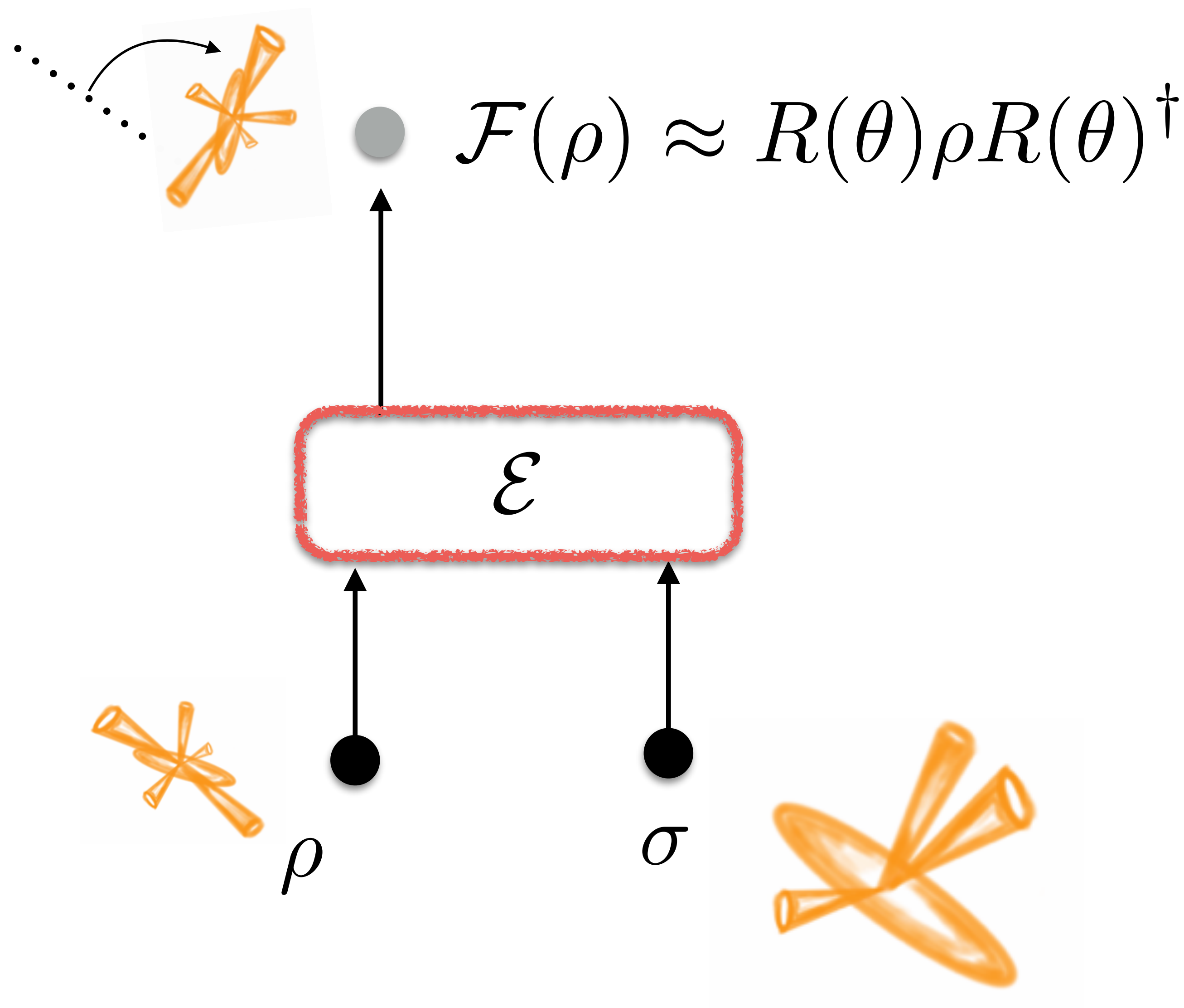


$$A_k = |\psi^-\rangle\langle k|$$

E.g. 2

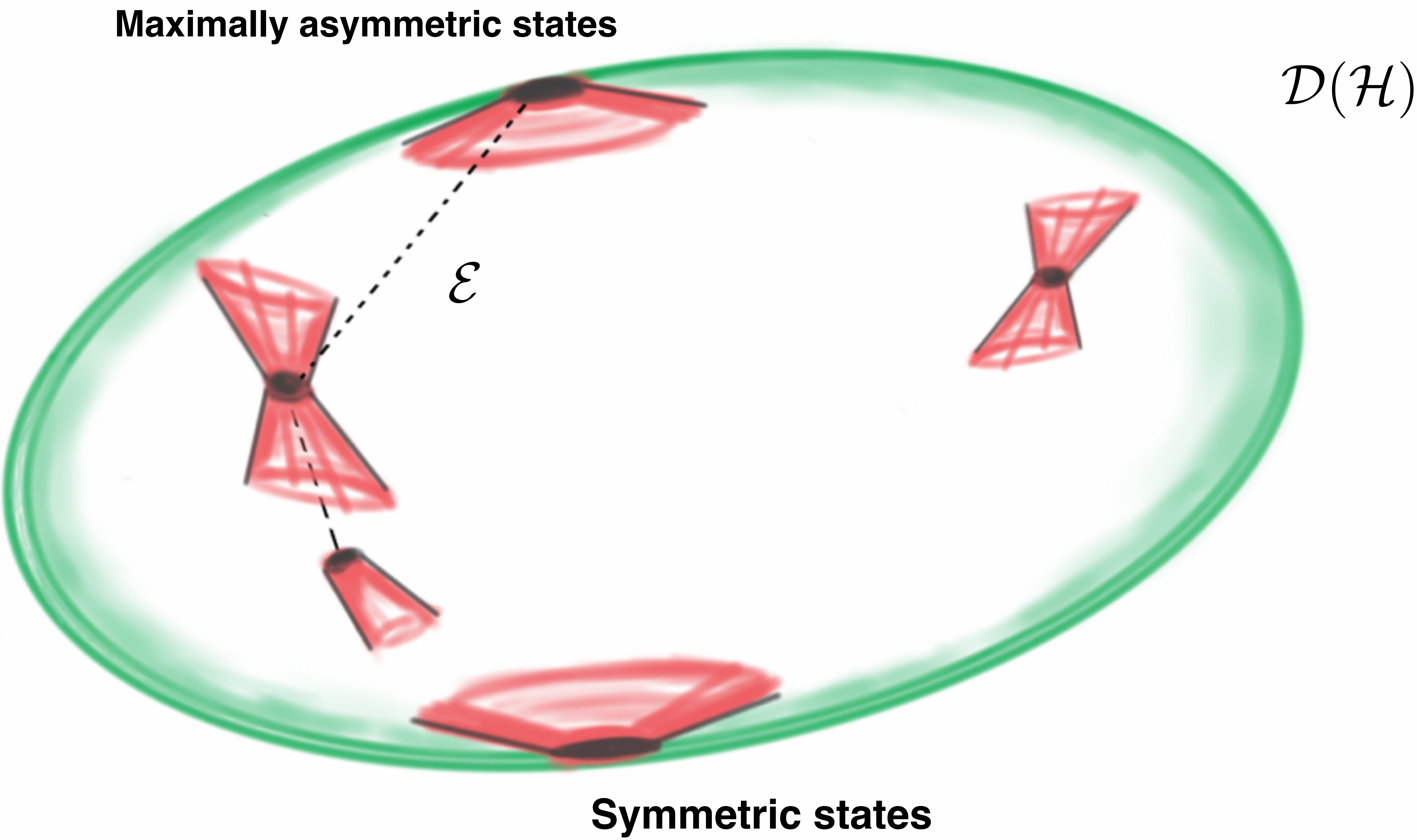
$$\rho \to p\rho + \frac{(1-p)}{l(l+1)} \sum_{\alpha=x,y,z} L_\alpha \rho L_\alpha$$

Use of resources: asymmetry



Spatial Rotation

A theory of asymmetry



Application:
the WAY-theorem

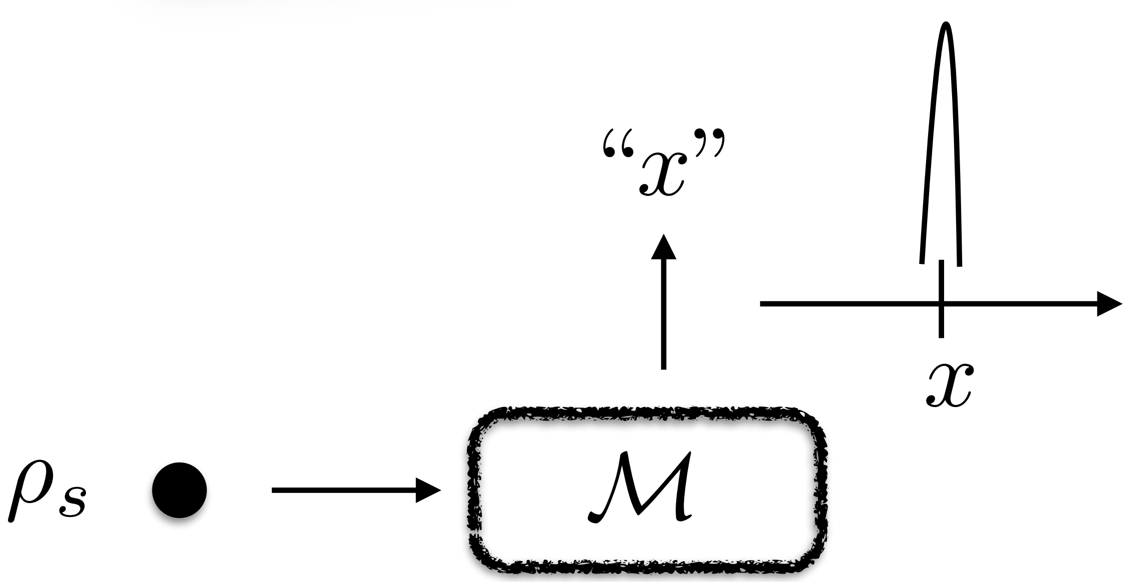
Application: the WAY theorem.

- **Theorem (Wigner-Araki-Yanase, 1952):**

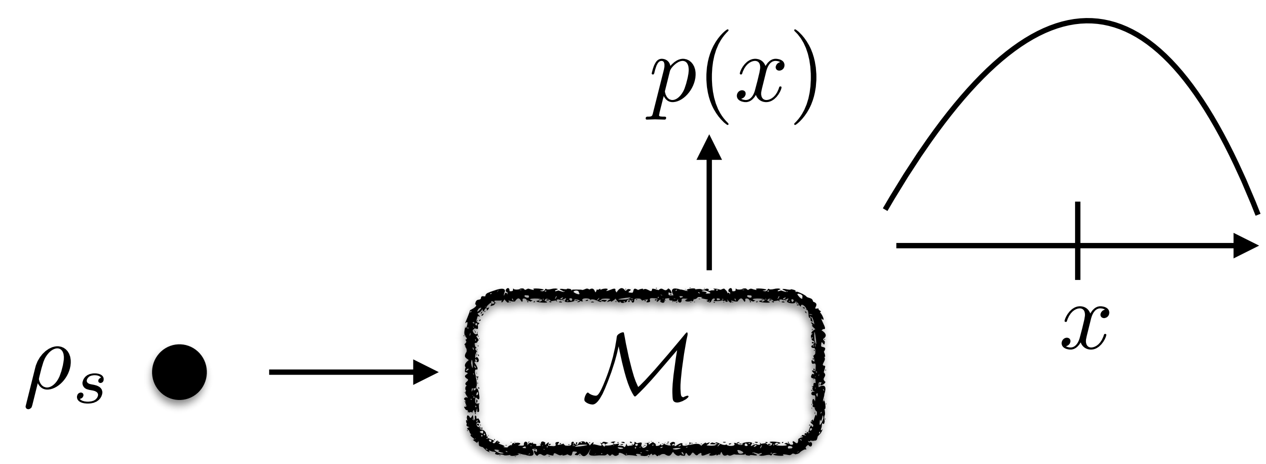
Observable P (e.g. momentum) conserved globally.

If $[X, P] \neq 0$

Then X cannot be sharply measured.



No Conservation Law.



Conservation Law present.

WAY-theorem: QI-view

- Define group action: $U(\theta) = e^{-i\theta P}$ on $\mathcal{H}$

Covariant CPTP maps $\mathcal{E} : \mathcal{B}(\mathcal{H}) \to \mathcal{B}(\mathcal{H})$

$$\mathcal{E}(U\rho U^\dagger) = U\mathcal{E}(\rho)U^\dagger$$

Measurement of X under conservation law	⟷	State discrimination of eigenstates of X, under covariance.

* *M. Ahmadi, DJ, T. Rudolph NJP 15 (2013)*

Proof:

State discrimination of eigenstates $\{\rho_1, \rho_2, \dots\}$ under covariance.

$\Leftrightarrow$

State discrimination of $\{\mathcal{G}[\rho_1], \mathcal{G}[\rho_2], \dots\}$ with **no constraint**.

$$\mathcal{G}(\rho) = \int d\theta U(\theta) \rho U(\theta)^\dagger$$

$\{\mathcal{G}[\rho_k]\}$ Perfectly distinguishable $\Leftrightarrow$ pairwise orthogonal supports

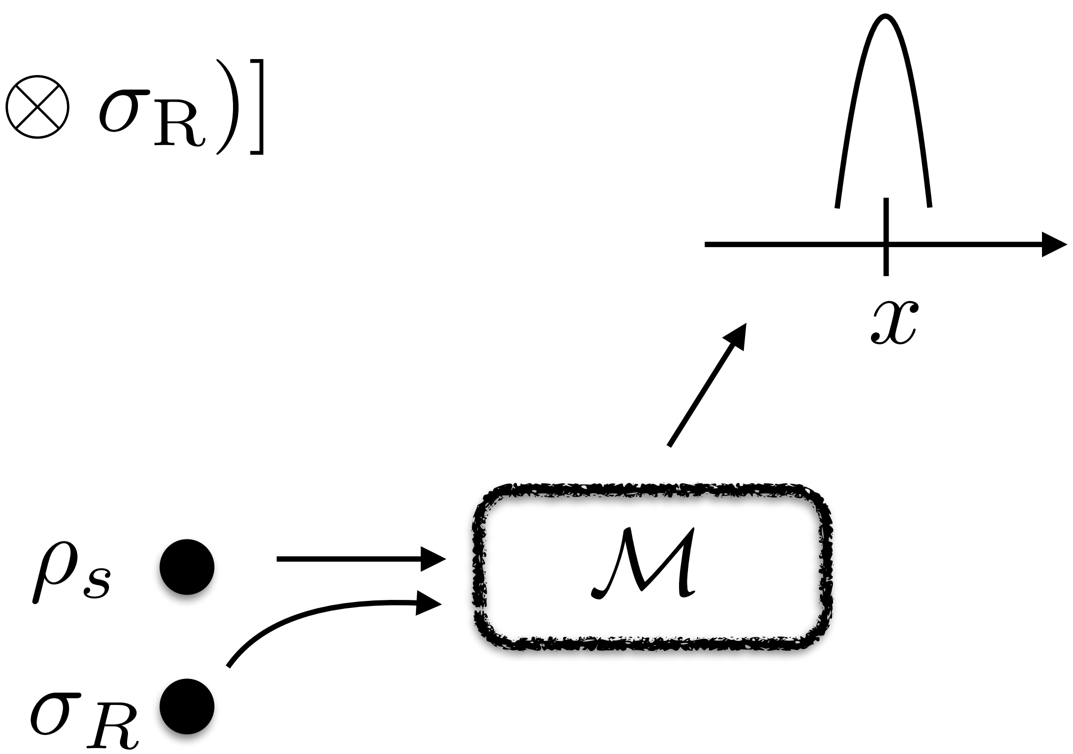
$$\Leftrightarrow \mathcal{G}[\rho_k] = \text{ rank-1} \Leftrightarrow \mathcal{G}[\rho_k] = \int d\theta U(\theta) \rho_k U(\theta)^\dagger = |\varphi_k\rangle\langle\varphi_k| = \rho_k$$

$$\Leftrightarrow [P, \rho_k] = 0 \Leftrightarrow [P, X] = 0 \quad \blacksquare$$

Asymmetric resource states

- Asymmetric σ_R state $\longrightarrow$ can "simulate" a *conservation-violating* operation $\tilde{\mathcal{E}}$

$$\tilde{\mathcal{E}}(\rho) = \mathrm{tr}_{\mathrm{R}}[\mathcal{E}(\rho \otimes \sigma_{\mathrm{R}})]$$



$$(|0\rangle + e^{i\theta_1}|1\rangle + \cdots + e^{i\theta_{n-1}}|n-1\rangle)$$

Conservation Law present.

Resources decay.

Symmetry & the 1st Law of Thermodynamics

- Traditional form: $dE = dQ + dW$
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Quantum Mechanical Symmetry:

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Constrains **non-conservation** of **two** quantities:

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(b) System "coherence"

↔ U(1)-asymmetry

Thermal $\subset U(1)$-covariant $\subset CPTP$-maps

1. Free state: $\gamma = e^{-\beta H}/Z$

2. Free quantum operations:

$$\mathcal{E}(\rho) = \mathrm{tr_b}[\mathrm{U}(\rho \otimes \gamma_{\mathrm{b}})\mathrm{U}^\dagger]$$
$$[U, H] = 0$$

1. Free states $\gamma = U(\theta)\gamma U(\theta)^\dagger$

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*M. Lostaglio, DJ, T. Rudolph, Nature Comm. (2015)
(Covariant Stinespring), M. Keyl, R. Werner J. Math. Phys. 40 (1999)

Applications of Framework:

1. The insufficiency of free energy relations.
2. Coherence “work-locking”.
3. General thermodynamic bounds on coherence.
4. Intrinsically-quantum 2nd law constraints.

*M. Lostaglio, DJ, T. Rudolph, Nature Comm. (2015)

M. Lostaglio, K. Korzekwa, DJ, T. Rudolph, Phys. Rev. X (2015)

M. Lostaglio, K. Korzekwa, J. Oppenheim, DJ, NJP (~2015)

(1). Insufficiency of free energies in thermodynamics.

Consider any set of functions $\{D_\alpha(\cdot)\}_\alpha$ that **"behave like free energies"**:

If $\rho \to \sigma$ then we have $\{D_\alpha(\rho) \leq D_\alpha(\sigma)\}_\alpha$

and $D_\alpha(\rho) \geq c||\rho - \gamma||$

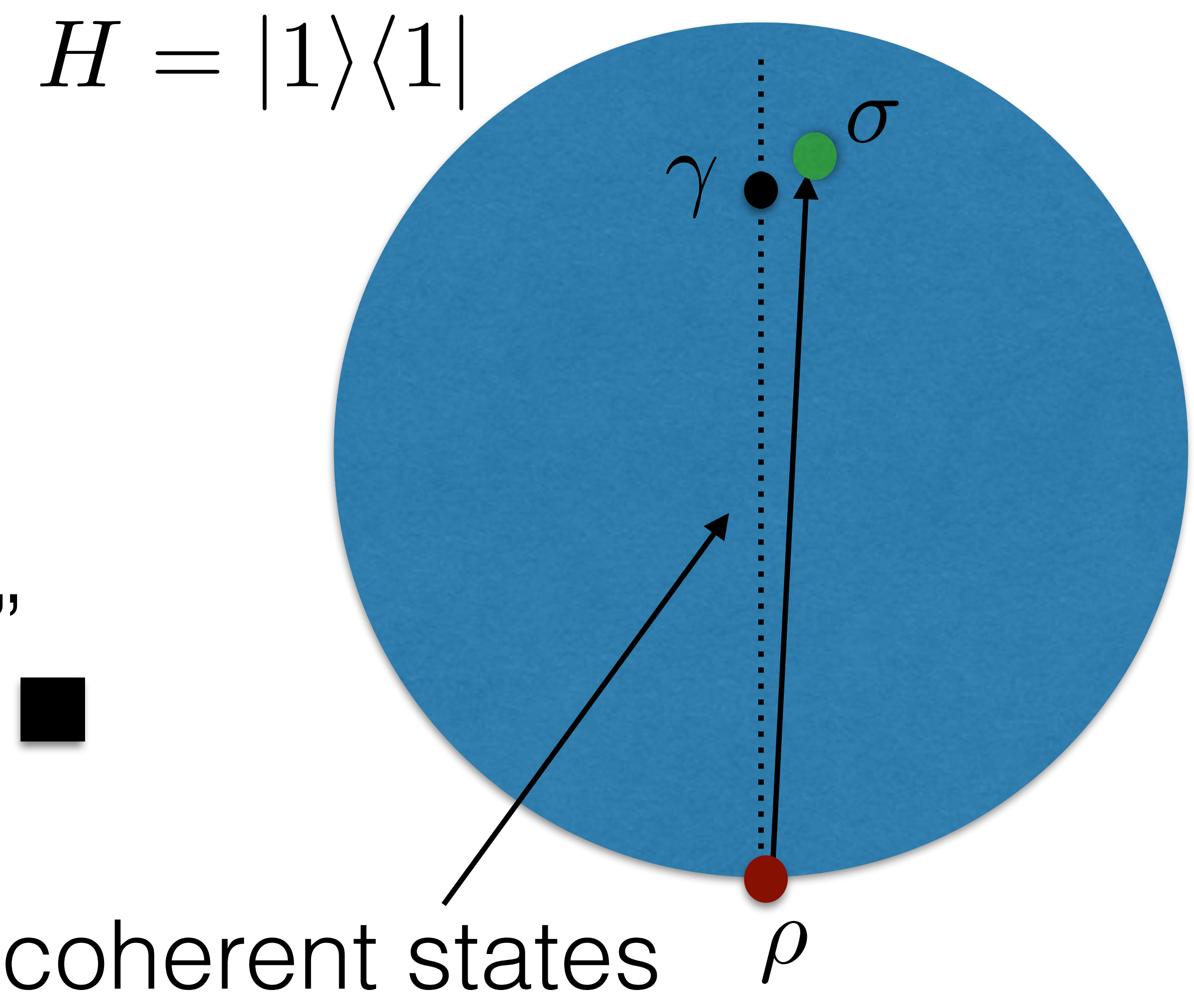
Then $\{D_\alpha(\cdot)\}_\alpha$ **cannot** provide a complete set of thermodynamic constraints.

(1). Insufficiency of free energies in thermodynamics.

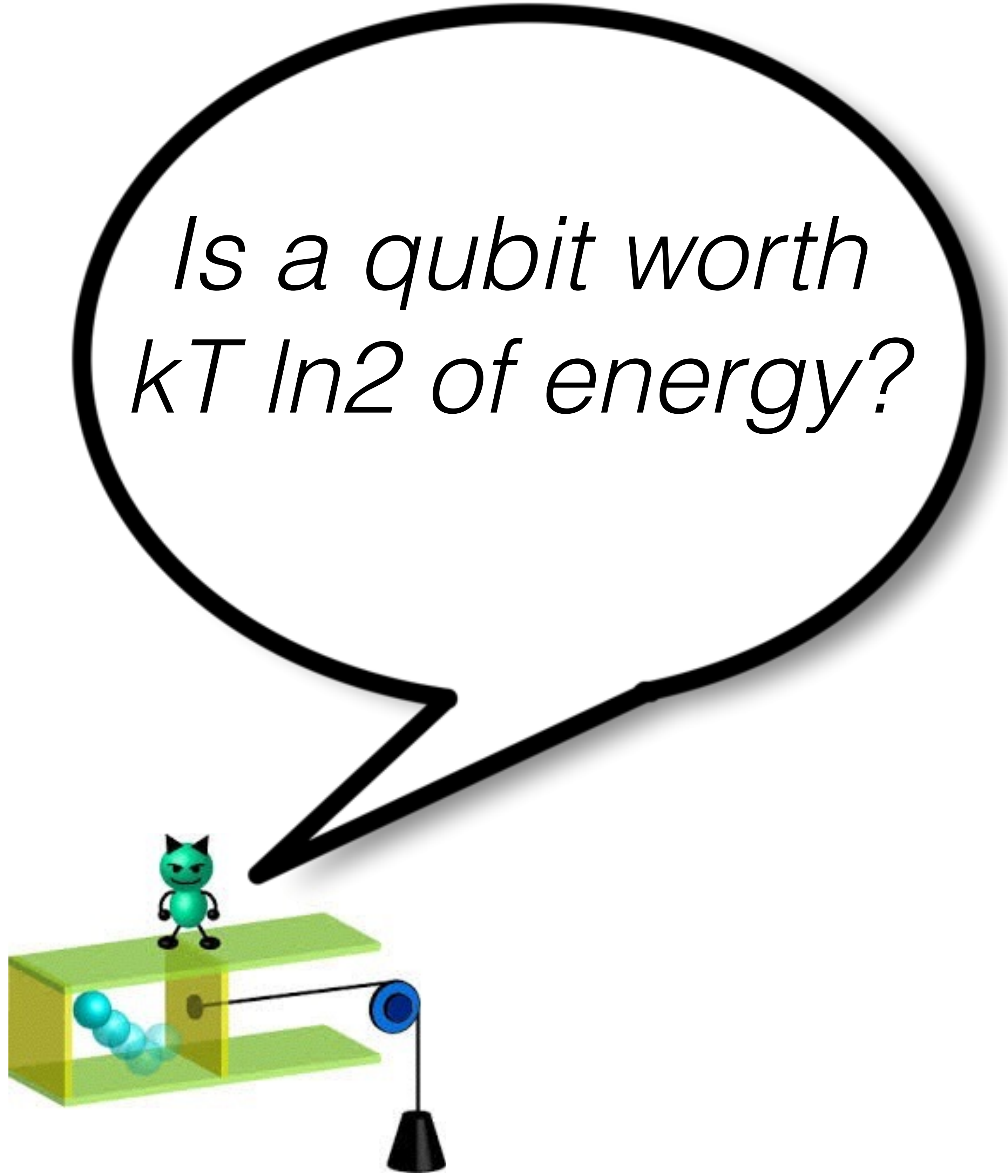
Proof:

D_α say "get closer to γ."

Symmetry says:
"asymmetry non-increasing." ∎

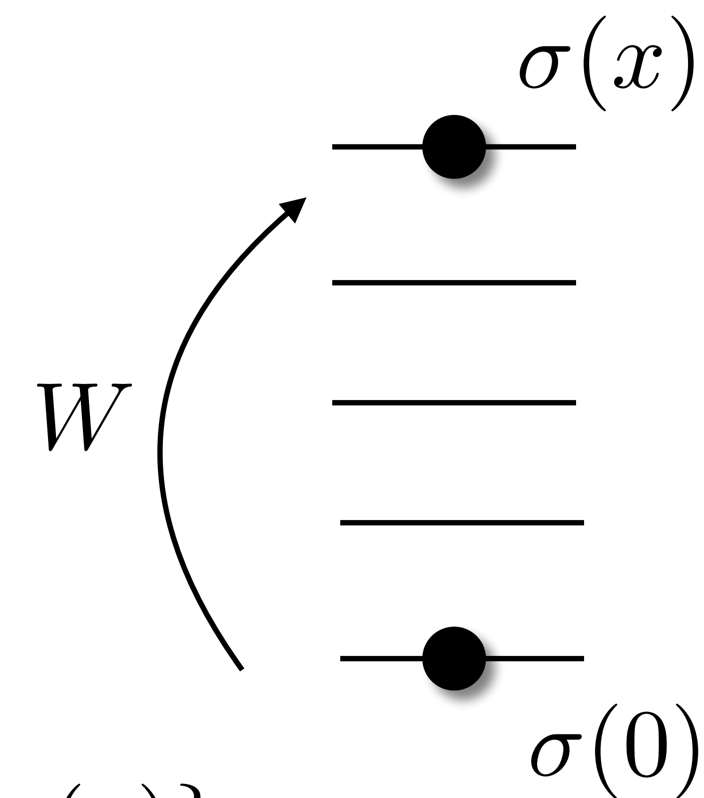


$$\sigma = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$


Is a qubit worth
kT ln2 of energy?

Work / Ordered Energy

Broad work definition:
"raising a weight up a ladder by height W"

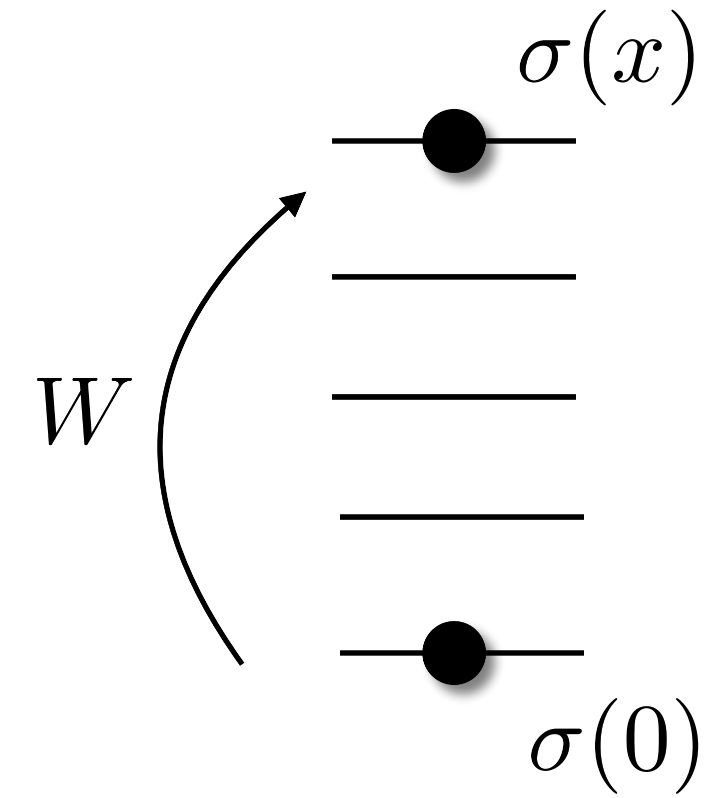


$$W := \sup\{\mathrm{x} : \mathcal{E} \text{ thermal \& sends } \rho \otimes \sigma(0) \to \sigma(\mathrm{x})\}$$

(2). Work-locked in coherence

Theorem:

if $\rho \to W$ then $\mathcal{D}(\rho) \to W$



where

$$\mathcal{D}(\rho) = \mathcal{G}_H(\rho) = \int dt e^{-itH} \rho e^{itH}$$

Follows directly from

$$[\mathcal{E}, \mathcal{U}_t] = 0 \Rightarrow [\mathcal{E}, \mathcal{D}] = 0$$

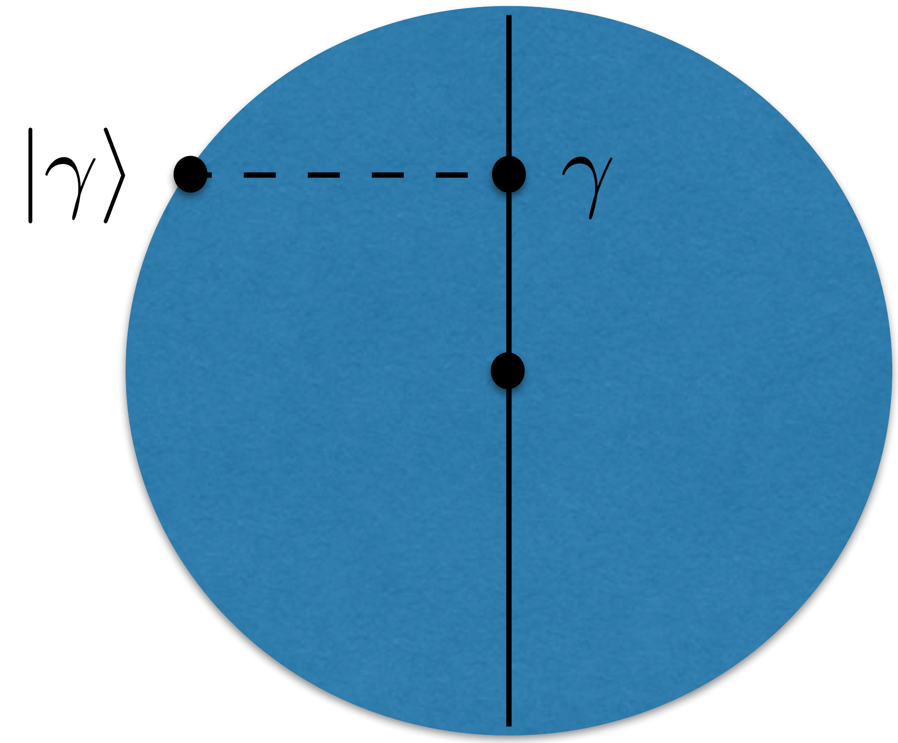
* *M. Lostaglio, DJ, T. Rudolph, Nature Comm. (2015)*

Szilard and coherence

Pure state $|\gamma\rangle$

$$\mathcal{D}(|\gamma\rangle\langle\gamma|) = \gamma$$

$\Rightarrow$ **No work** can be extracted from $|\gamma\rangle$ **on its own.**



Value of a qubit ? Non-trivial.

Requires “resource counting”.

Unlocking coherence for work.

- Must use additional coherent resources:

$$\mathcal{D}(|\gamma\rangle\langle\gamma|) = \gamma$$

$$\mathcal{D}(|\gamma\rangle\langle\gamma| \otimes \sigma_R) \neq \mathcal{D}(|\gamma\rangle\langle\gamma|) \otimes \mathcal{D}(\sigma_R)$$

(*relational* coherence protected)

σ_R acts as quantum reference frame for $|\gamma\rangle$

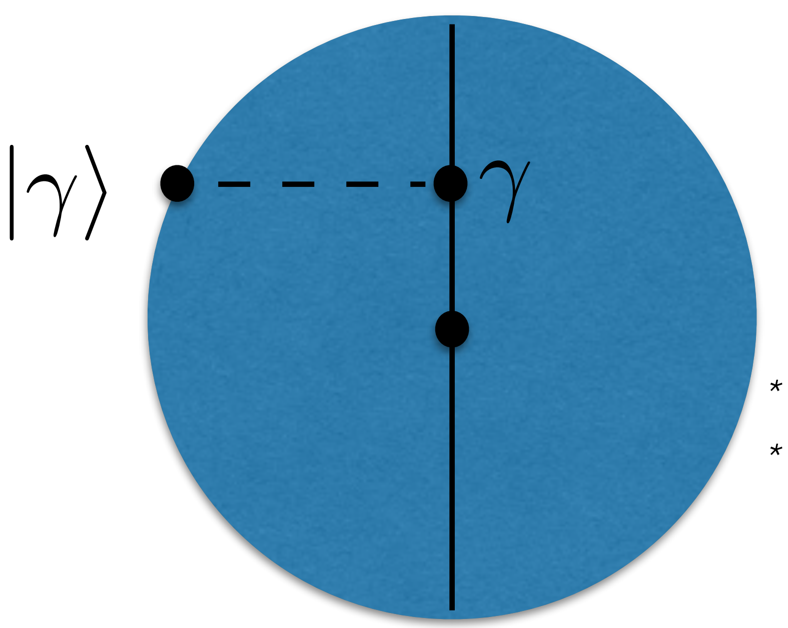
E.g.

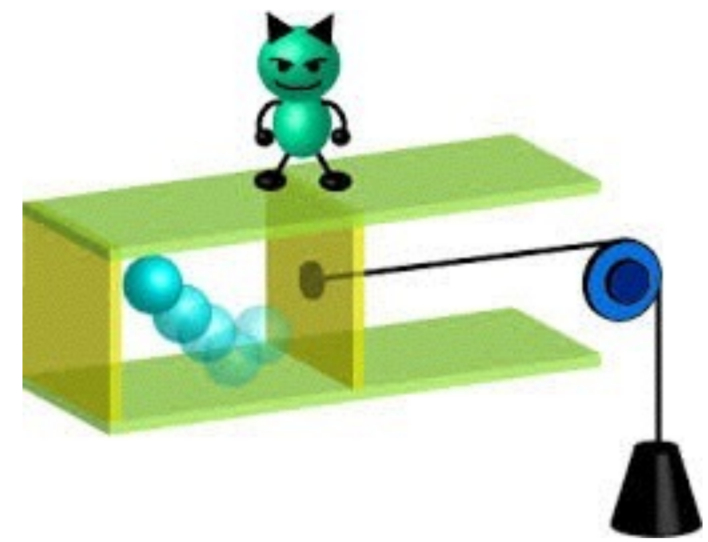
$$|\gamma\rangle \otimes |\gamma\rangle \to W \leq Z^{-1} e^{-\frac{E}{kT}} (E - 2kT \ln Z)$$
$$= kT \ln 2 \ (\text{for } E = 0)$$

A fully quantum Szilard engine

- Result: it is only for a particular *“classical”* regime that we can associate the free energy to every qubit state.

$$|\Psi\rangle \longrightarrow W = -\Delta F$$





M. Lostaglio, K. Korzekwa, J. Oppenheim, DJ,
“Extracting work from quantum coherence” NJP (2015)

Bounding Coherence

Mode operators

- Apply harmonic analysis to operators: irreps of group action.

$$\mathcal{B}(\mathcal{H}) = \bigoplus_{\nu} V_\nu$$

$$U(t)\rho^{(\nu)}U(t)^\dagger = e^{-i\nu t}\rho^{(\nu)}$$

$$\rho = \sum_{\nu=-d}^{d} \rho^{(\nu)}$$

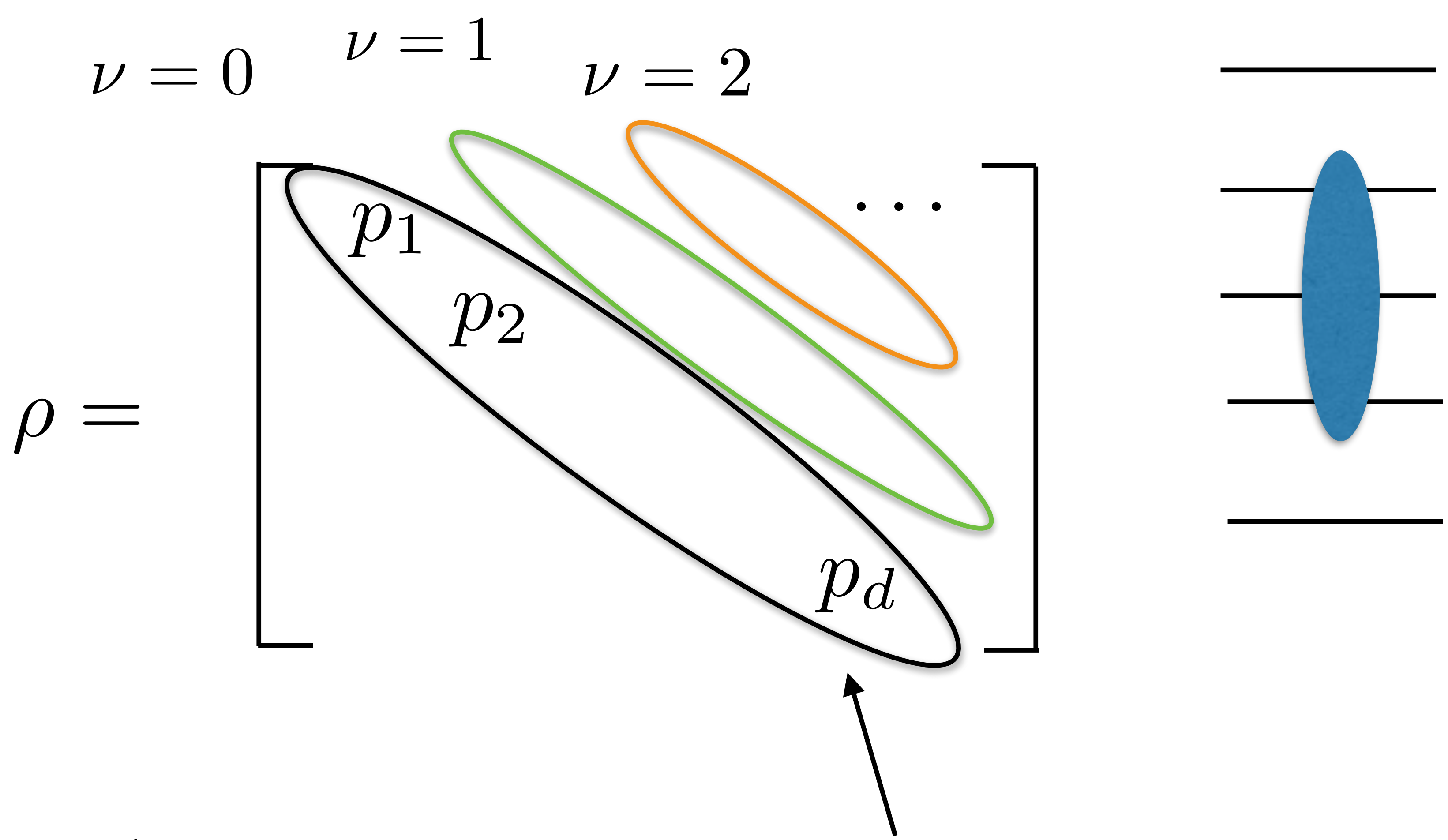
Thermal operations

$$[\mathcal{E}(\rho)]^{(\nu)} = \mathcal{E}(\rho^{(\nu)})$$

$$||\mathcal{E}(\rho)^{(\nu)}||_1 \leq ||\rho^{(\nu)}||_1$$

*M. Lostaglio, K. Korzekwa, DJ, T. Rudolph, Phys. Rev. X (2015)

State structure



$\rho \to \sigma$

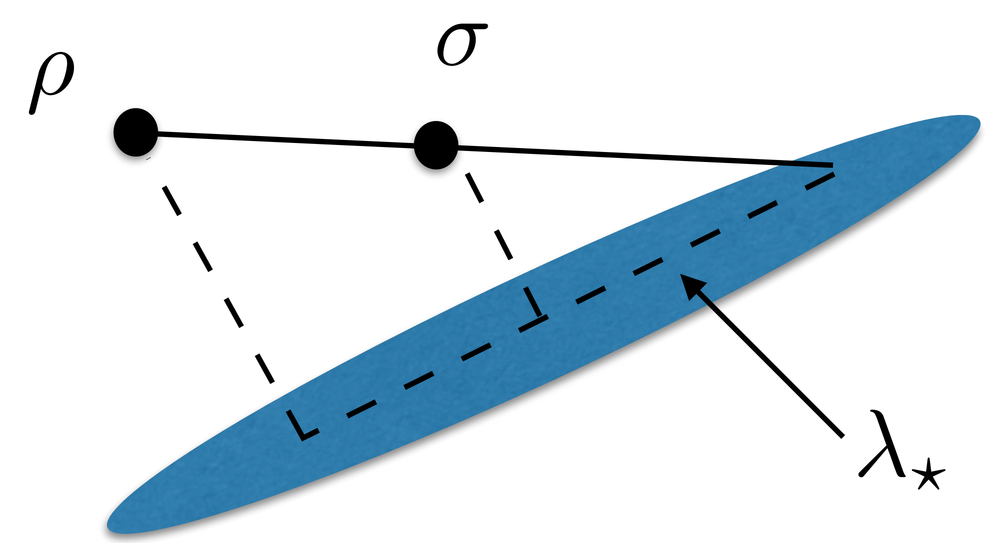
$\Delta F_\alpha \leq 0$

previous results relate to the "zero-mode"

(3). General Bounds on Coherence

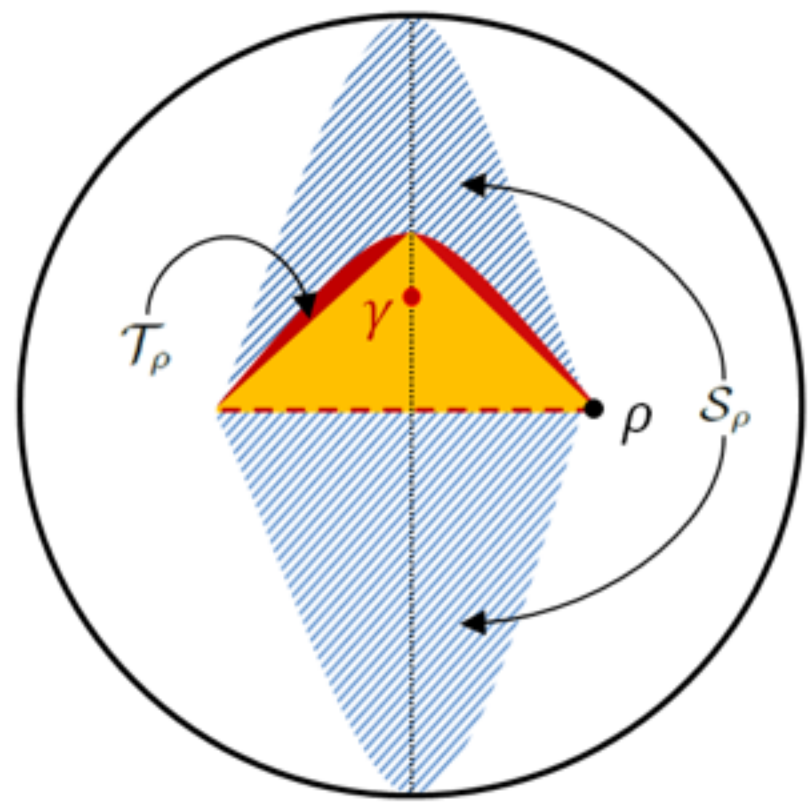
Lower bound:

$$\sigma^{(\nu)} = \lambda_\star \rho^{(\nu)}$$



Upper bound:

$$|\sigma_k^{(\nu)}| \leq \sum_{c:\omega_c \leq \omega_k} |\rho_c^{(\nu)}| e^{-\beta\hbar(\omega_k - \omega_c)} + \sum_{c:\omega_c > \omega_k} |\rho_c^{(\nu)}|$$

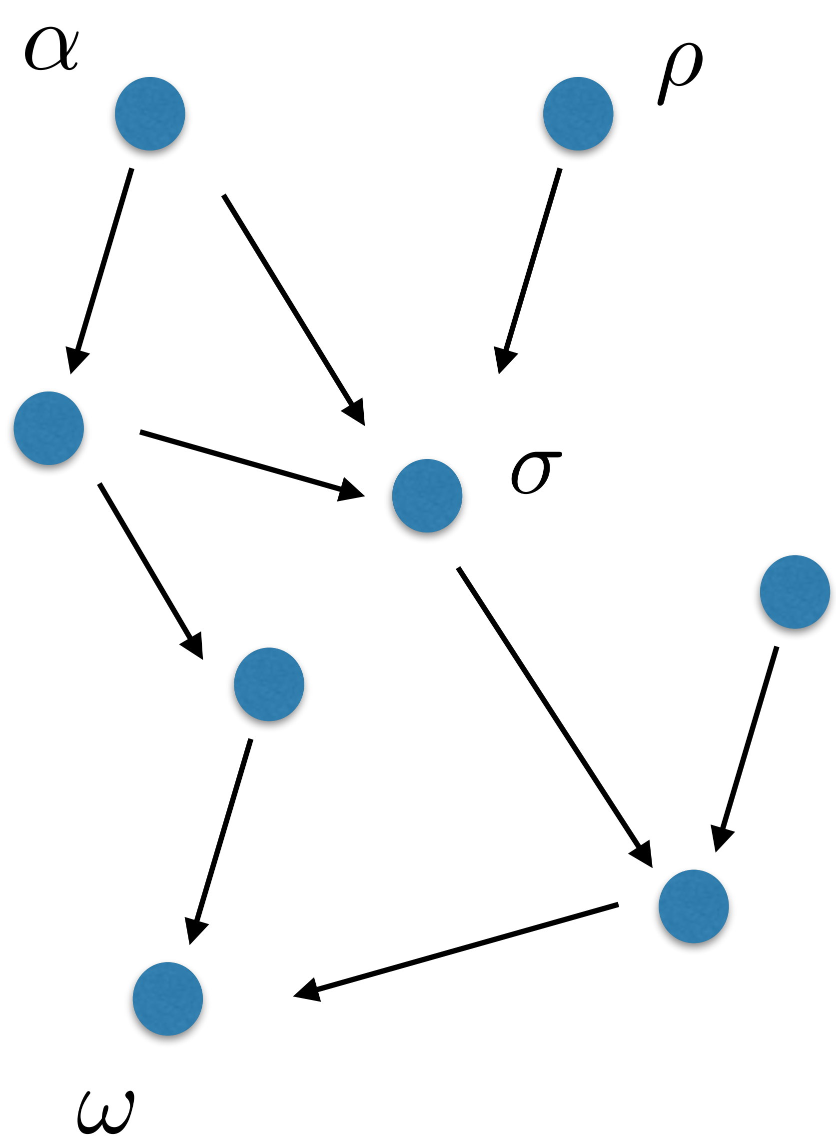


Previous bound:

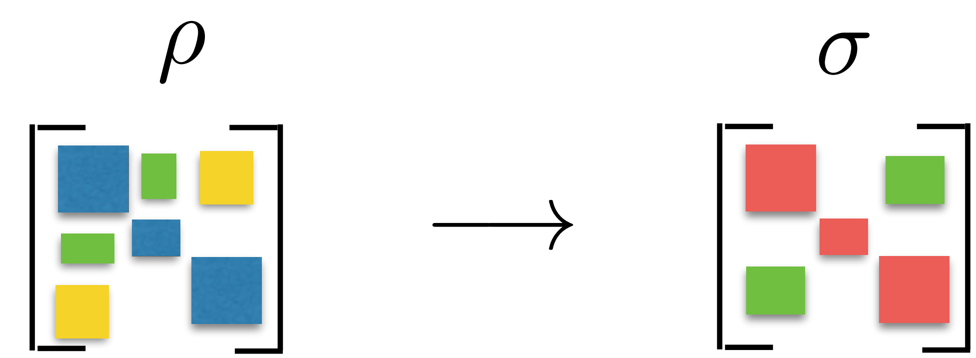
$$|\sigma_{nm}| \leq |\rho_{nm}| \sqrt{p_{n|n} p_{m|m}}$$

* Cwiklinski, Studzinski, Horodecki, Oppenheim, arxiv (2014)

(4). The full thermodynamic ordering of states?



coherence present:



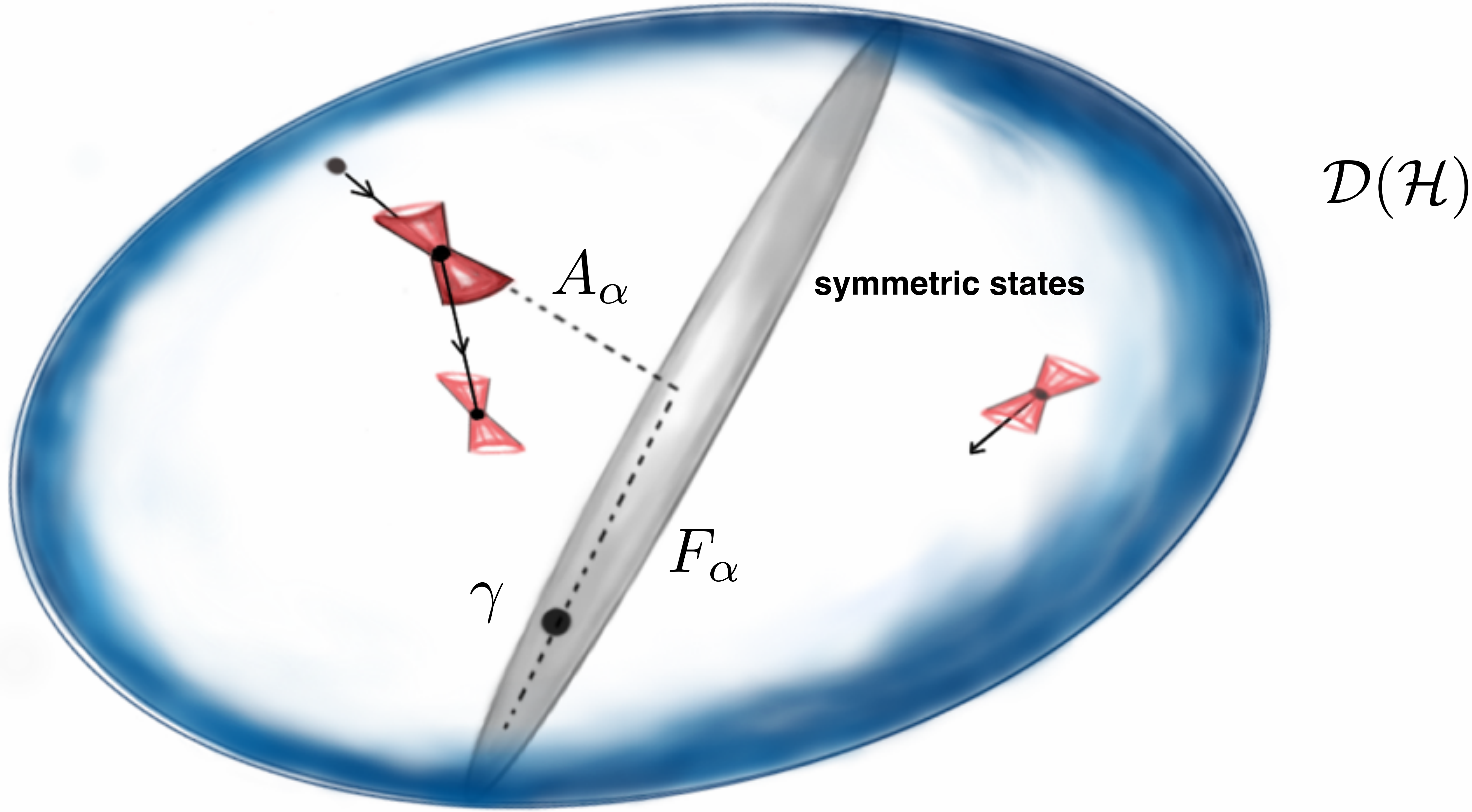
Q: Does the ordering of states admit an entropic formulation?

Thermodynamic structure

- Entanglement theory ~ non-local resources.
- Asymmetry theory ~ asymmetry resources.
- Thermodynamics ~ **ordered energy + asymmetry**

Work

Quantum coherence



$\mathcal{D}(\mathcal{H})$
A_α
symmetric states
F_α
γ

(4). Necessary entropic constraints

Theorem: For arbitrary quantum states, the thermodynamic transformation $\rho \to \sigma$ is possible provided

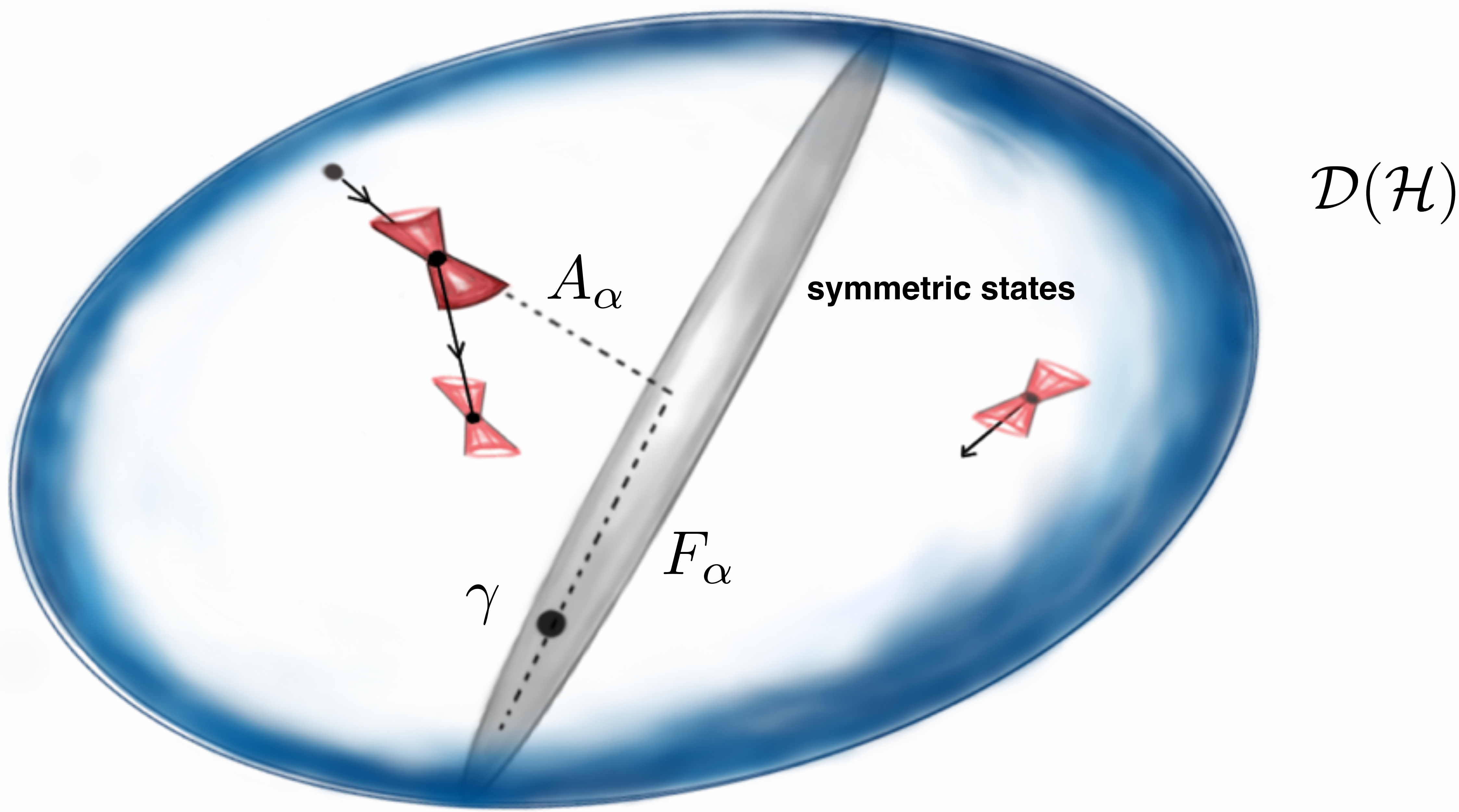
$$F_\alpha(\rho) \geq F_\alpha(\sigma) \qquad \forall \alpha \geq 0$$
$$A_\alpha(\rho) \geq A_\alpha(\sigma)$$

Monotones:

$$A_\alpha(\rho) := D_\alpha(\rho || \mathcal{G}(\rho))$$

$$\mathcal{G}(\rho) = \int_G dg \, U(g) \rho U(g)^\dagger$$

* M. Lostaglio, DJ, T. Rudolph, Nature Comm. (2015).



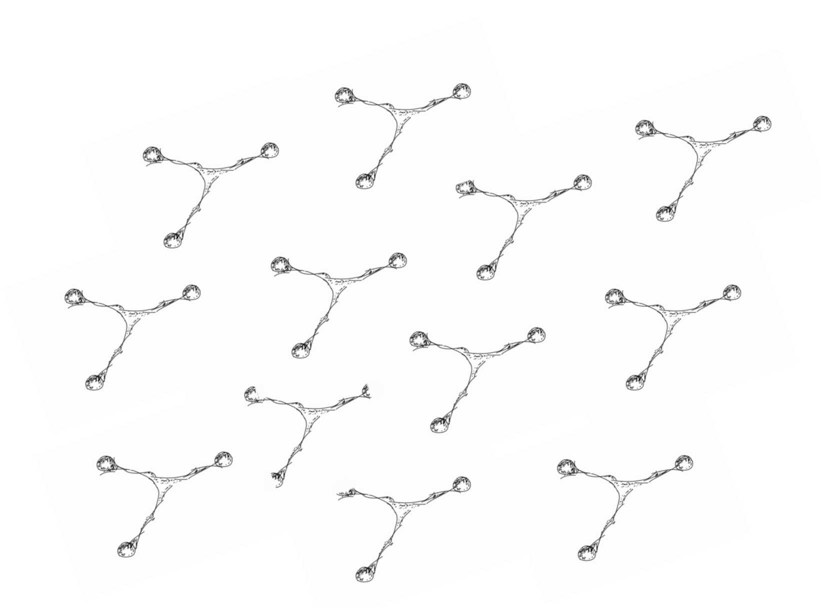
$\mathcal{D}(\mathcal{H})$
A_α
symmetric states
F_α
γ

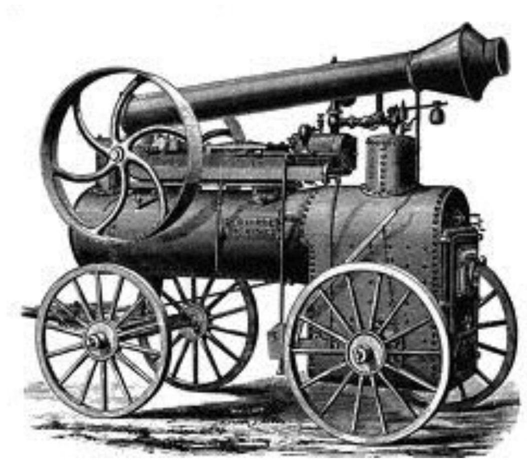
Macroscopic regime

- **Theorem:** for any $\rho \in \mathcal{B}(\mathcal{H})$ we have

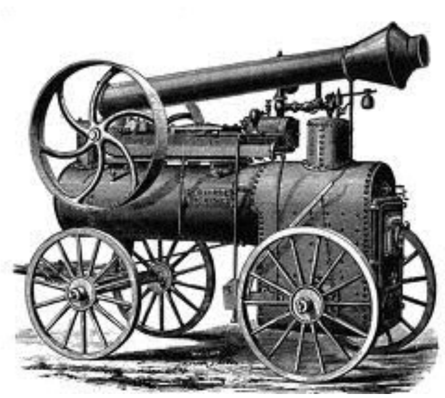
$$\lim_{n\to\infty} \frac{1}{n} \begin{bmatrix} F_\alpha(\rho^{\otimes n}) \\ A_\alpha(\rho^{\otimes n}) \end{bmatrix} = \begin{bmatrix} F(\rho) - F(\gamma) \\ 0 \end{bmatrix}$$

$$F = \langle H \rangle - TS$$





Current perspective

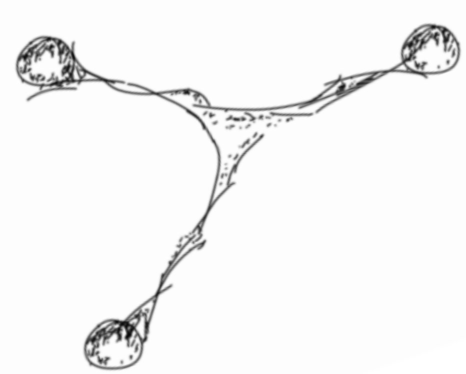


Essentially unique entropy.

$$\rho \to \sigma \;\Leftrightarrow\; S(\rho) \leq S(\sigma)$$

$$\langle e^{-\beta(W-\Delta F)} \rangle = 1$$ (incomplete)

$$\rho \to \sigma \;\Leftrightarrow\; D_\alpha(\rho||\gamma) \leq D_\alpha(\sigma||\gamma)$$



$$\begin{bmatrix} F_\alpha(\rho) \\ A_\alpha(\rho) \end{bmatrix} \leq \begin{bmatrix} F_\alpha(\sigma) \\ A_\alpha(\sigma) \end{bmatrix} \quad +$$

Beyond Thermodynamics — Irreversibility & non-commutativity

Broad notions of Irreversibility

Reversible		**Irreversible**
Newtonian Dynamics	⟶	Classical Thermodynamics
Unitary Dynamics	⟶	Quantum Thermodynamics
Gauge dynamics	⟶	??

Gauge (field) theories

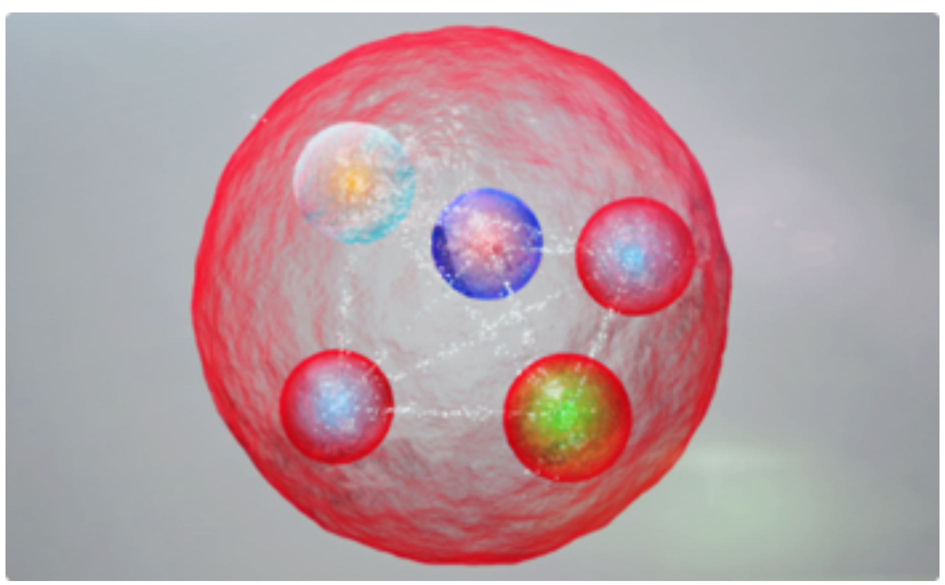
Global symmetry

$$\psi(x) \to e^{i\theta}\psi(x)$$

Local symmetry

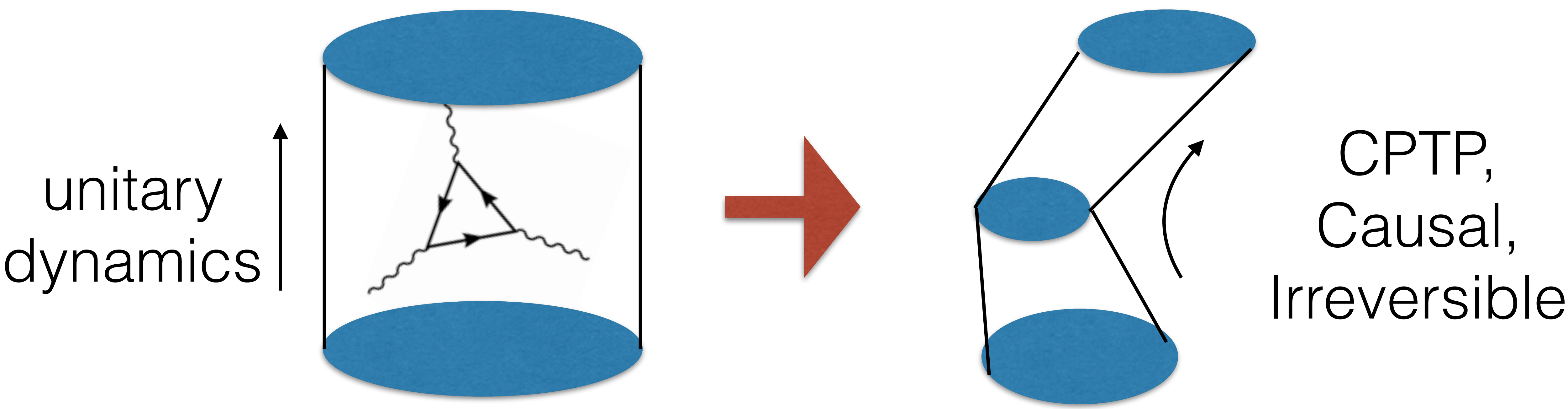
$$\psi(x) \to e^{i\theta(x)}\psi(x)$$

Gauge field $A(x)$



(QED, QCD, Standard Model)

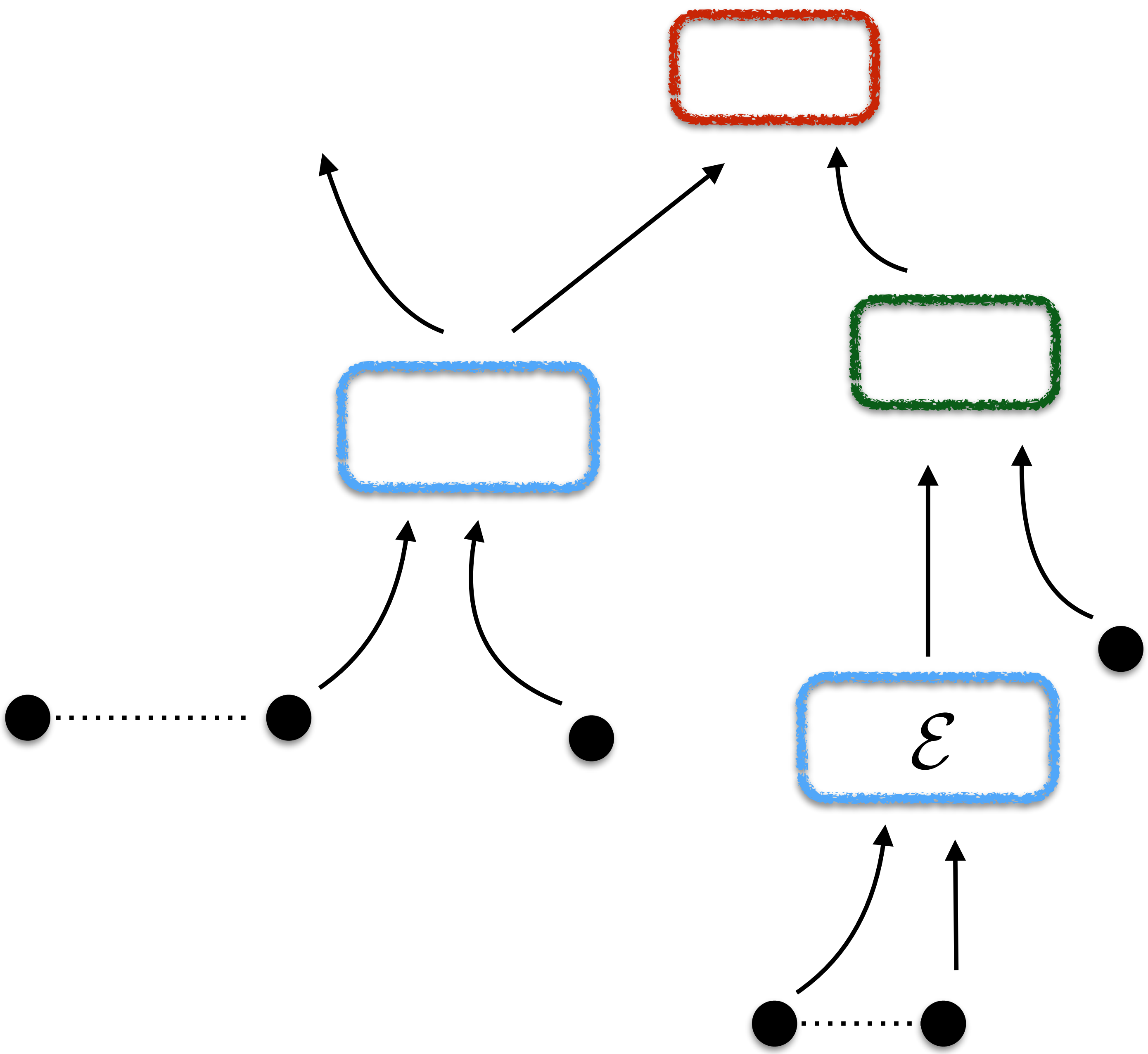
Irreversibility in gauge degrees of freedom



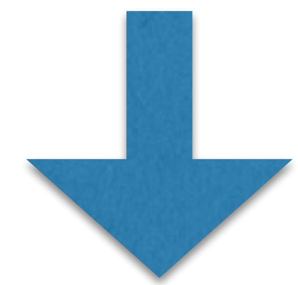
Global conservation law

Global monotonicity

Local quantum resources



Local group actions
Global covariance



How do local operations couple to obey Global covariance?

Traditional Physics

Lagrangian ~ Kinetic energy - Potential energy

$$L = \frac{1}{2}\dot{x}^2 - \frac{1}{2}x^2$$

Dynamics: $\ddot{x} + x = 0$

Encode symmetries in L, e.g. $L = \bar{\psi}(x)(i\gamma^\mu\partial_\mu - m)\psi(x)$

Rigidity

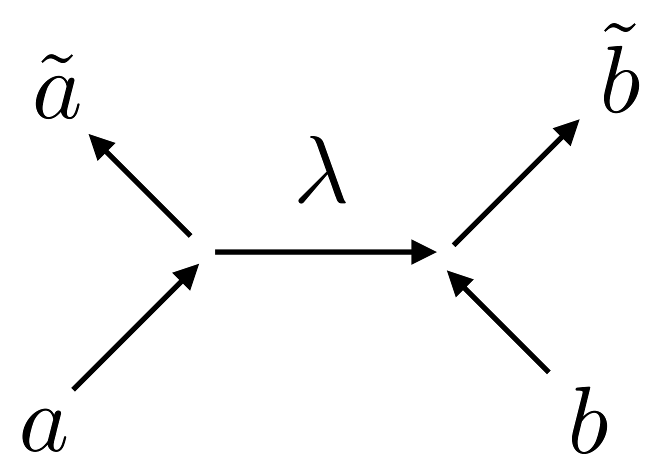
CPTP maps?

Core structure

Theorem: The space of bipartite covariant maps is spanned by

$$\Phi_{\Theta} : \mathcal{B}(\mathcal{H}_A \otimes \mathcal{H}_B) \to \mathcal{B}(\mathcal{H}_{\tilde{A}} \otimes \mathcal{H}_{\tilde{B}})$$

$$\Theta \equiv (a, \tilde{a}) \xrightarrow{\lambda} (b, \tilde{b})$$

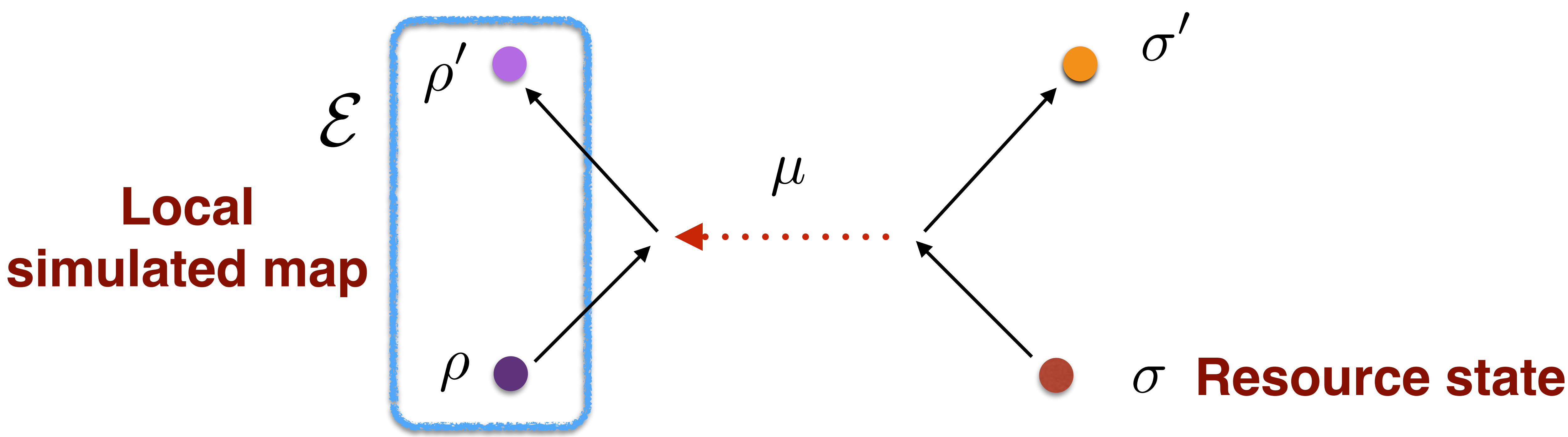


(irrep labels)

$$\Phi_{\Theta} = \sum_k \Phi^k_{A,(-\lambda)} \otimes \Phi^k_{B,(+\lambda)}$$

** Cristina Cirstoiu, DJ arxiv:015.xx (2015)*

Asymmetry charges



Temporal/casual aspect: some irreps ruled out.

Traditional observables (energy,charge, density…) insufficient.

Asymmetry modes: **gauge degrees of freedom**.

* *Cristina Cirstoiu, DJ arxiv:015.xx (2015)*

High level Picture

Multipartite irreversible asymmetry

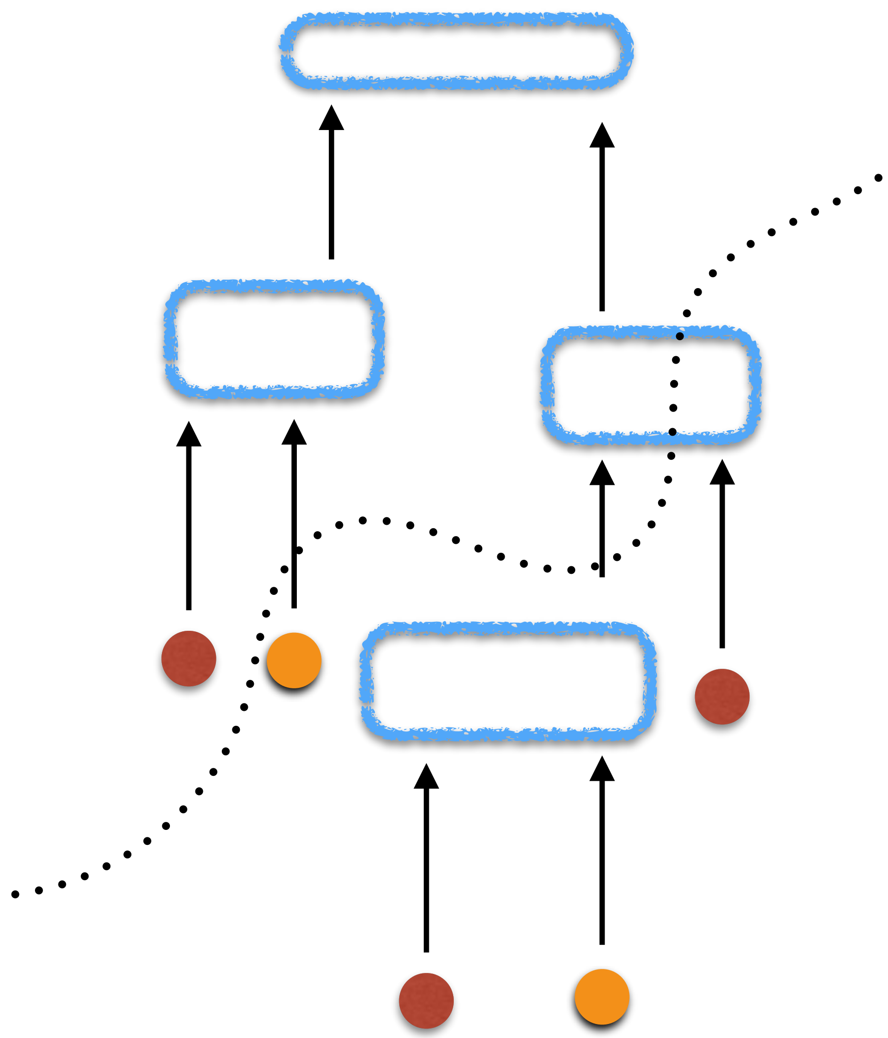
$G = U(1)$

quantum thermodynamics

$\mathrm{CPTP} \to \mathrm{unitary}$

reversible gauge dynamics

Outlook



Analysis of general processes
(causal, quantum switches…)

Tool-kit for quantum operations

QI techniques to traditional
gauge theory topics

Interplay of energetic +
quantum properties.

For more see…

1. Lostaglio, DJ, Rudolph,
   Nature Communications (2015)
2. Lostaglio, Korzekwa, DJ, Rudolph,
   Physical Review X (2015)
3. Korzekwa, Lostaglio, Oppenheim, DJ,
   New Journal of Physics (2015)
4. Cirstoiu, DJ (soon!)

(insufficiency of free energy)

(general coherence bounds)

(coherence and work)

(non-abelian resources)