Game Semantics and Block-Structured State

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GaLoP, York, March 2009

What this talk is about

Evaluation strategies vs scoping.

- Call-by-value and mobility: RML [AM98], Reduced ML [MT09], etc.
- Call-by-name and blocks: Idealized Algol [AM97].

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Call-by-value and (base-type) blocks?

- There is a gap.
- Name-mobility has been be described as a semantical intricacy of ML-like languages (e.g. [PS98]).

Reduced ML

Simply-typed λ-calculus with integers and references.

- Types:

$$\theta ::= \mathsf{unit} \mid \mathsf{int} \mid \mathsf{int\,ref} \mid \theta \to \theta$$

- Terms:

$$M ::= x \mid \lambda x.M \mid M\,M \mid () \mid \Omega$$
$$\mid n \mid M \odot M \mid \mathsf{if}\ M\ \mathsf{then}\ M\ \mathsf{else}\ M$$
$$\mid a \mid \mathsf{ref}\,M \mid\ !\,M \mid M := M$$

$$V ::= n \mid () \mid a \mid x \mid \lambda x.M$$

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- Moves carry carefully selective stores: to be included in the store a name has to be available to both participants.

- Names adhere to freshness conditions: an X-name which is fresh in the view of the other participant must be fresh.

These plays give full abstraction: see paper for details!

Go simpler: RML

RML = Reduced ML

$-$ a

$+$ mkvar

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- No names
- $\text{new}\,x\,\text{in}\,M \triangleq (\lambda x.\,M)(\text{ref}0)$

Games for RML [AM98]

- Games for PCF_v [HY99], without Innocence
- $[\![\mathsf{int\,ref}]\!] = ([\![\mathsf{unit}]\!] \Rightarrow [\![\mathsf{int}]\!]) \times ([\![\mathsf{int}]\!] \Rightarrow [\![\mathsf{unit}]\!])$
- $\mathsf{cell} : [\![\mathsf{unit}]\!] \longrightarrow [\![\mathsf{int\,ref}]\!]$

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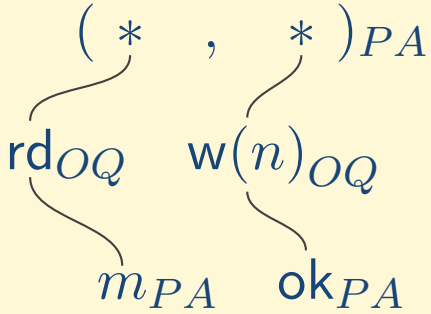

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Example

Taking $M \triangleq \lambda y.\,(x := !\,x{+}1\,;!\,x)$, $[\![\mathsf{new}\,x\,\mathsf{in}\,M : \mathsf{unit} \to \mathsf{int}]\!] =$

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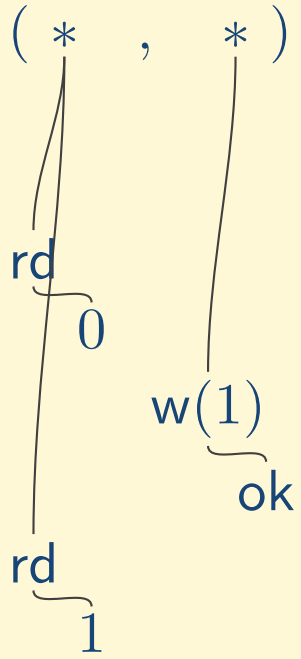
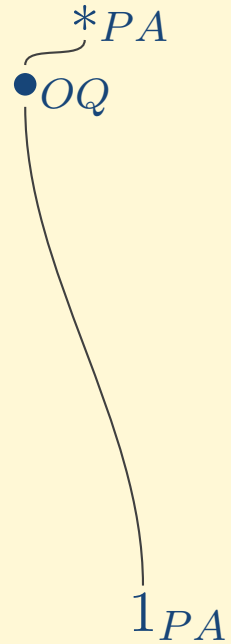
$\bullet_{OQ}$





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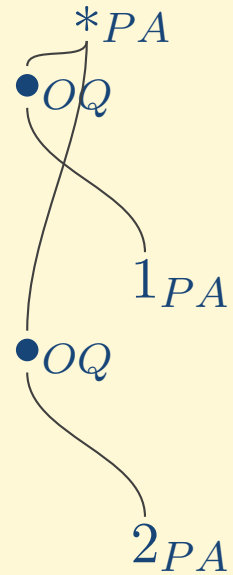

RML with blocks: bRML

- Non-innocence in this case crucially depends on $\mathsf{new}\_\,\mathsf{in}\,\_ : \mathsf{int}\,\mathsf{ref} \Rightarrow (\mathsf{unit} \Rightarrow \mathsf{int}) \Rightarrow (\mathsf{unit} \Rightarrow \mathsf{int})$
- What if we use blocks? Take $\beta = \{\mathsf{unit}, \mathsf{int}\}$ and:

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Explicitly, bRML terms are:

$$\begin{aligned} M ::= \; & x \mid \lambda x.M \mid M\,M \mid () \mid \Omega \\ & \mid n \mid M \odot M \mid \mathsf{if}\ M\ \mathsf{then}\ M\ \mathsf{else}\ M \\ & \mid \mathsf{mkvar}\ M\ M \mid \mathsf{new}\ x\ \mathsf{in}\ M \mid\ !\,M \mid M := M \end{aligned}$$

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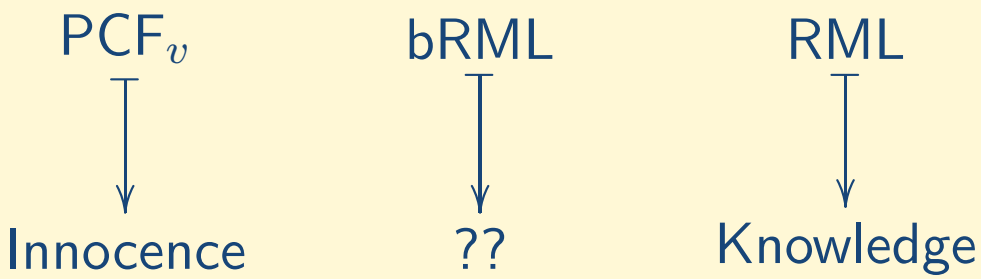
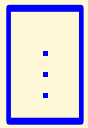


Binnocence

bRML strategies exhibit a particular kind of uniformity:

$$A \longrightarrow B_1 \to B_2 \to \cdots \to B_n \to \beta$$

$\bullet_{OQ}$



$*_{PA}$

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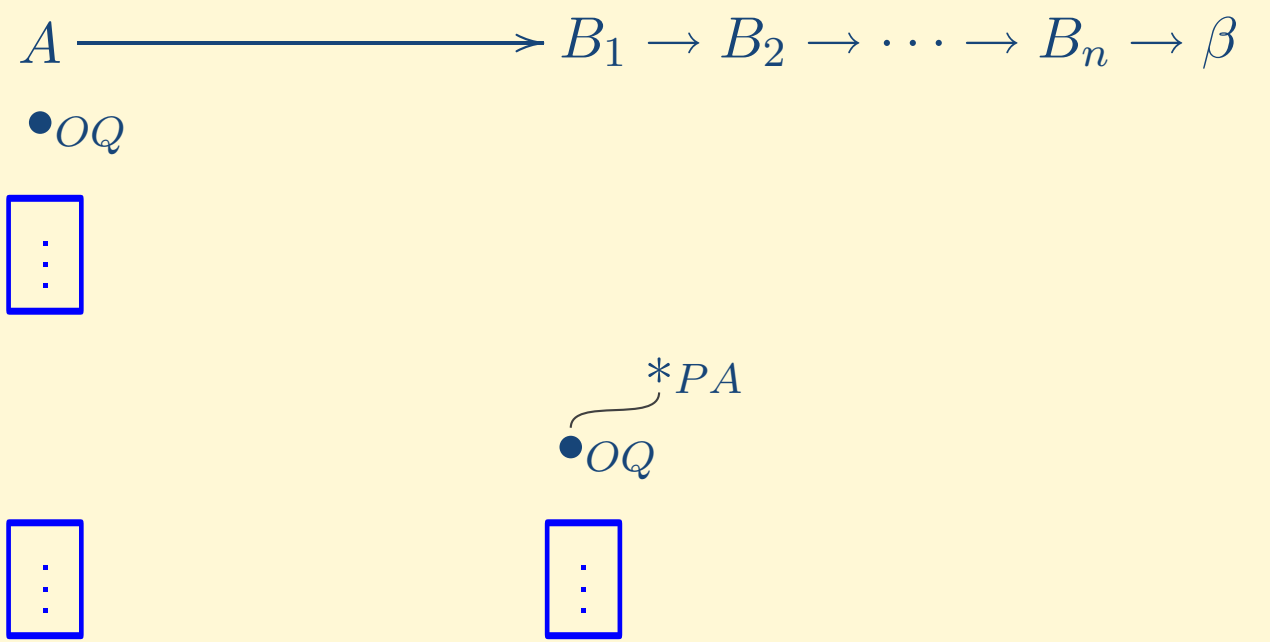

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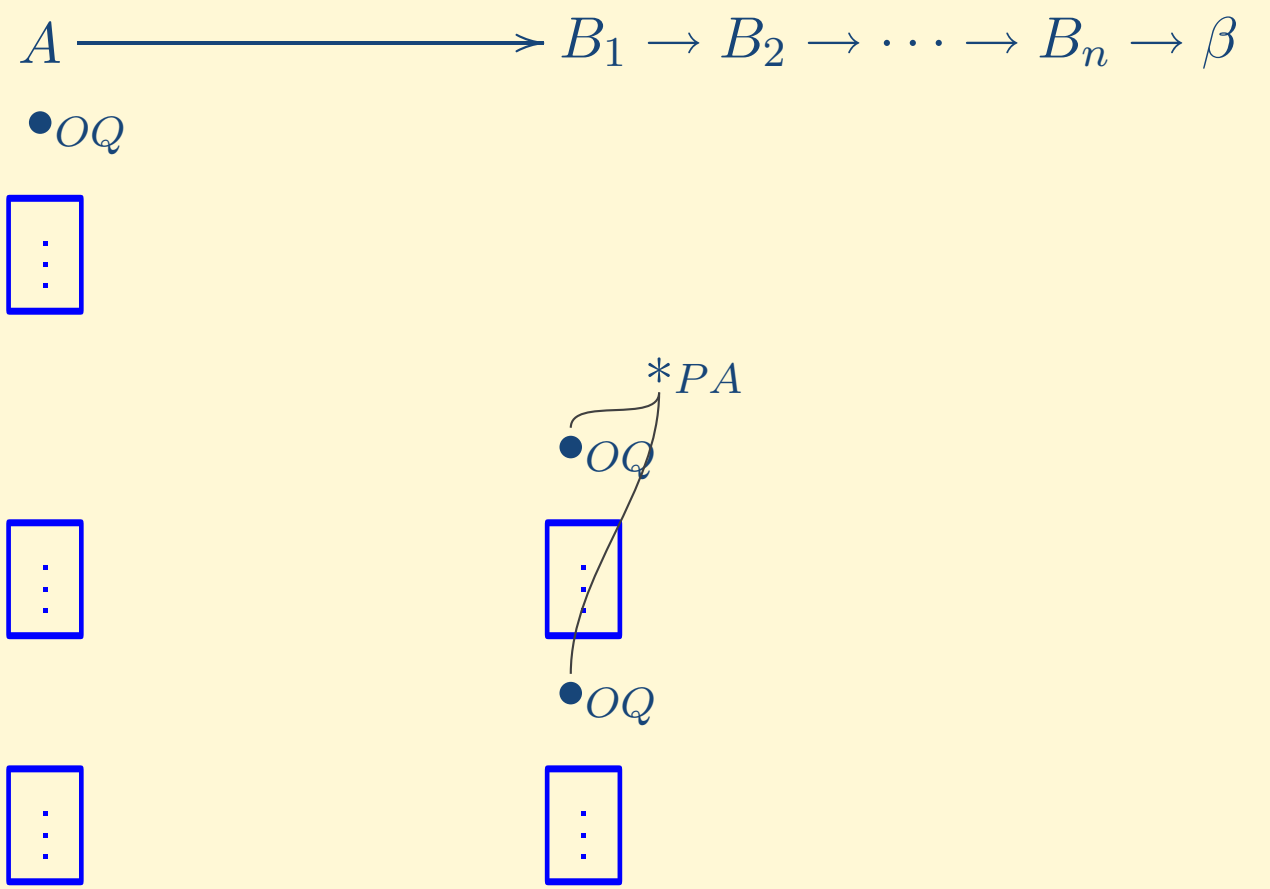

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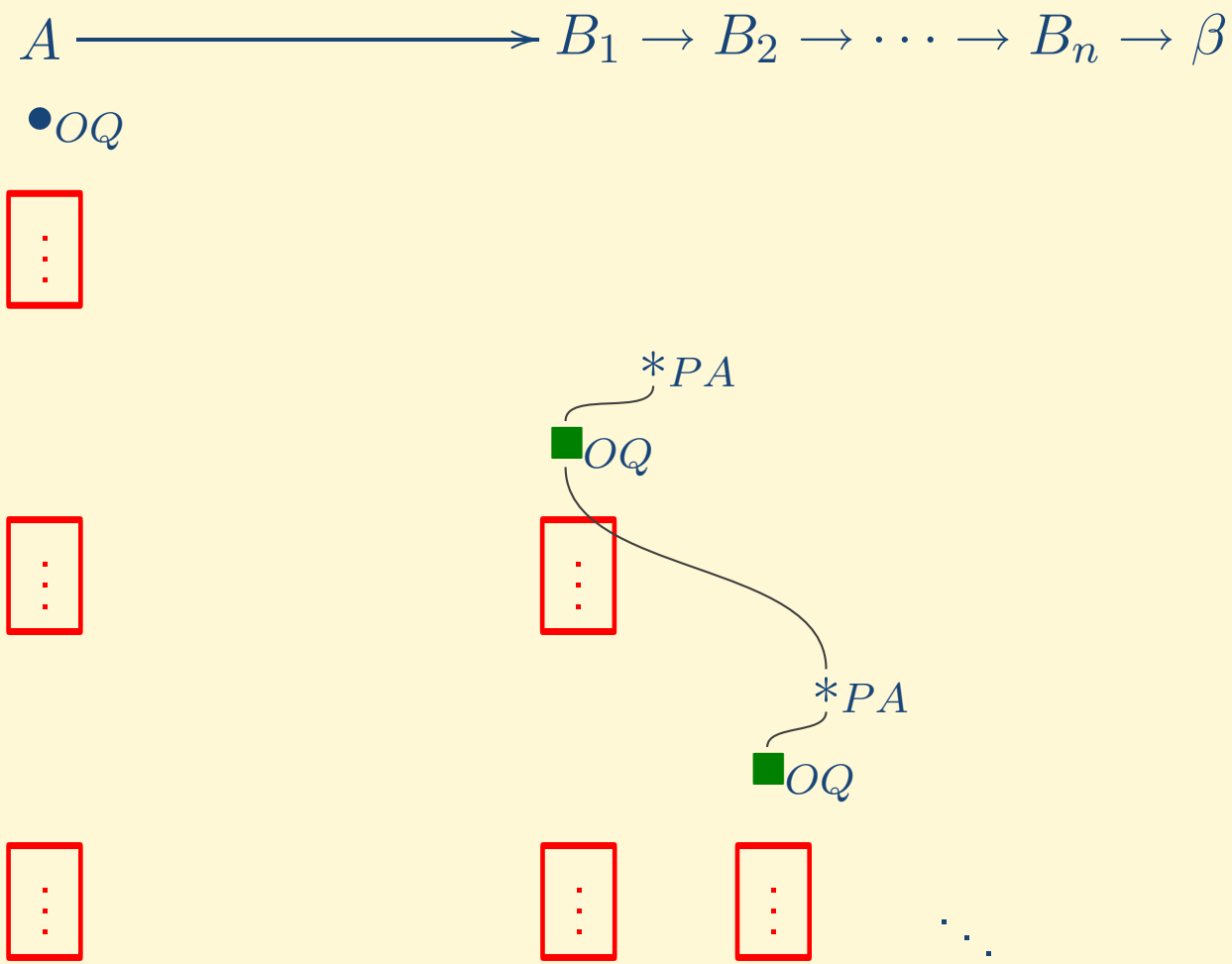

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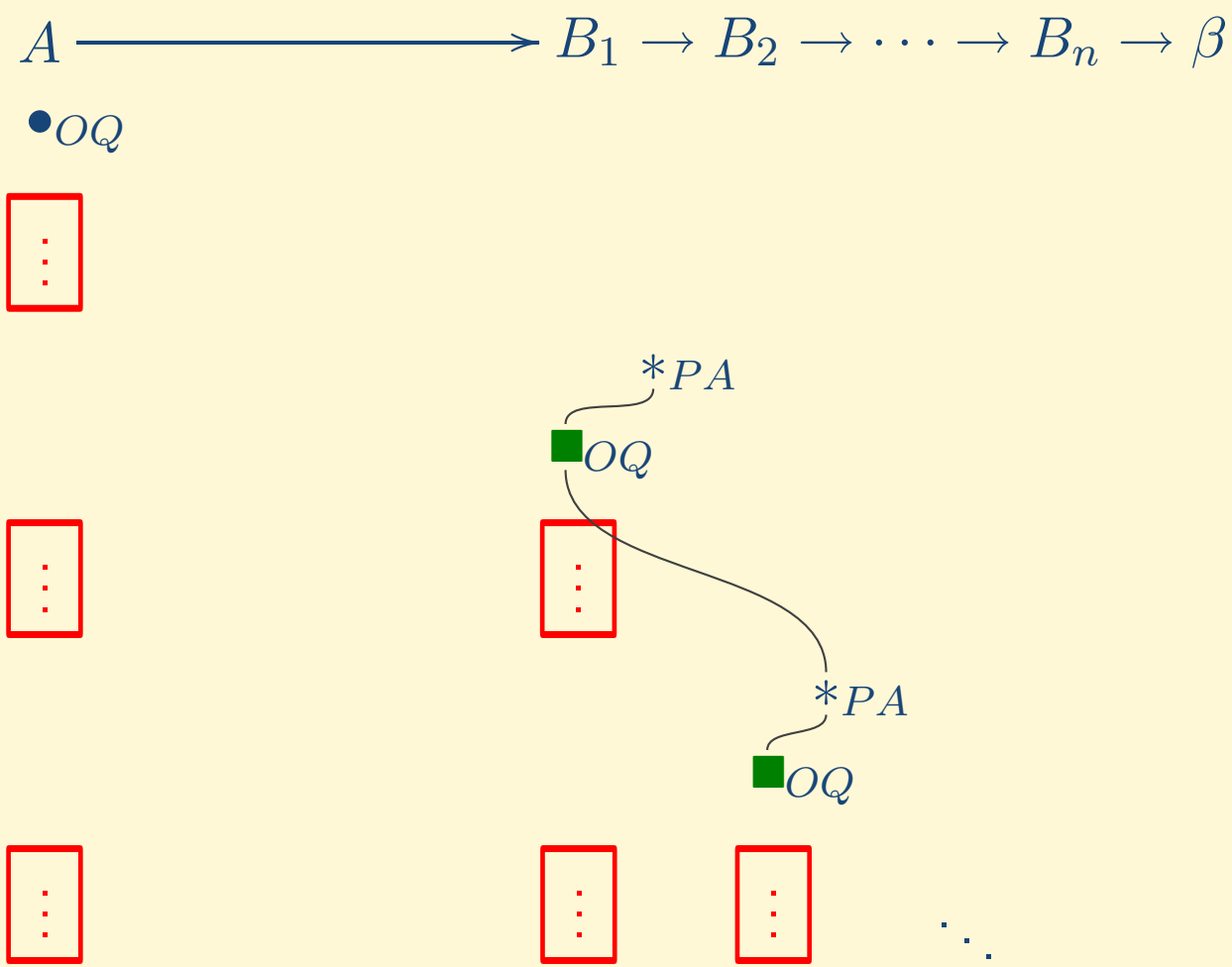
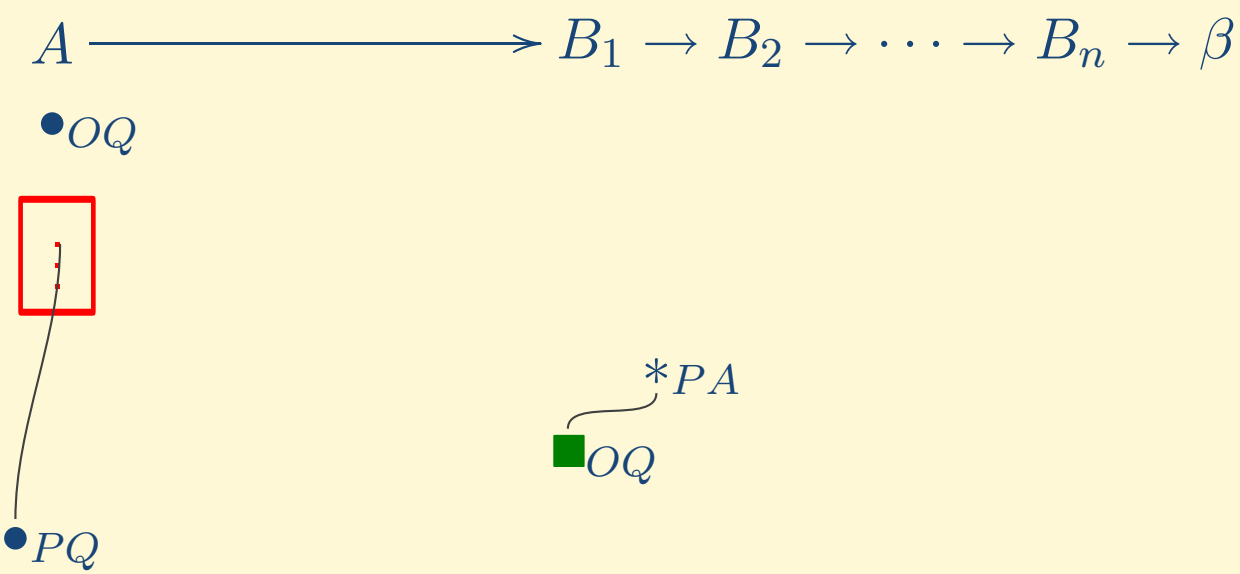


Binnocence can be described as a "recursive version" of 3rd-move-binnocence (*thread-independence*).

A problem



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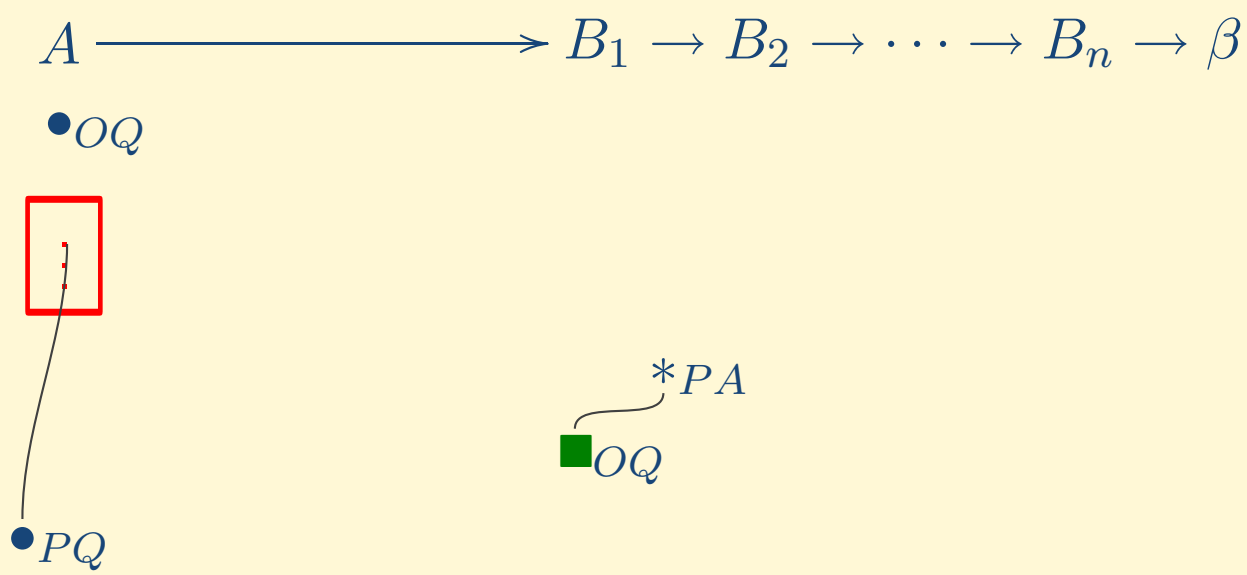

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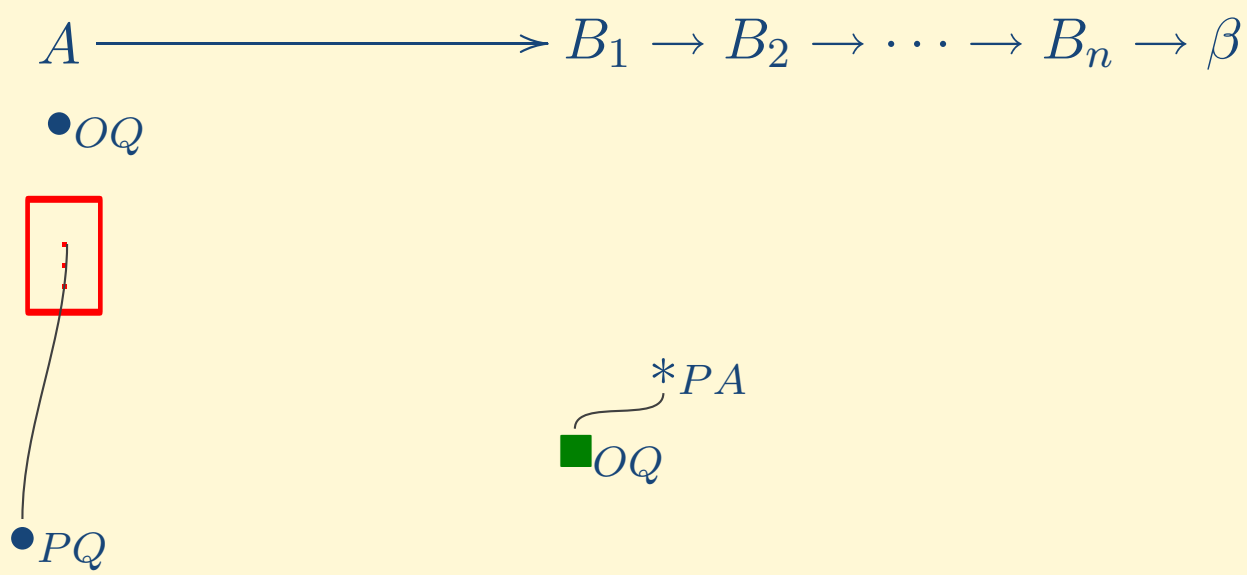

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- In fact, P cannot play $\bullet_{PQ}$ if it is justified by a move in an open block

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- Annotate explicitly blocks in plays.
- Annotations give us a means to express blocks, but binnocence has become very complicated.
- Use stores as annotations, and go innocent.