

$$f: [0,1]^n \rightarrow [0,1]^n$$

$$x: [0,1]^n \quad M: [0,1]^{n,n}$$

$$a: [0,1]^n$$

$$f \cdot x = a + M \cdot x \quad \rightarrow \text{note row sums } \leq 1, \text{ because } a + M \cdot \underline{1} \leq \underline{1}$$

$$\mu.f = (\text{LIn. } f^n \cdot \underline{0})$$

$$\nu.f = (\text{rIn. } f^n \cdot \underline{1})$$

$$[r].f = \lim_{n \rightarrow \infty} f^n \cdot \underline{r} \quad \text{defined?}$$

$$\text{Yes: eg. } f^2 \cdot \underline{r} =$$

$$a + M \cdot (f \cdot \underline{r}) =$$

$$a + M \cdot a + M^2 \cdot \underline{r}$$

non-decreasing

non-increasing, because

$$M \cdot \underline{r} \leq \underline{r}$$

still continuous if only
finitely many \cap, \cup .

$$\text{More generally, } f \cdot x := f_1 \cdot x \cap f_2 \cdot x$$

$\uparrow \quad \quad \quad \uparrow$
both as above

\rightarrow $\mu.f, \nu.f$ as before, by monotonicity
(even if not continuous) eg. $\mu.f = (\cap x \mid f \cdot x \leq x)$

But $\lim_{n \rightarrow \infty} f^n \cdot \underline{r}$ now?