Verification with Stochastic Games: Advances and Challenges

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Joint work with:

Gabriel Santos, Gethin Norman, Marta Kwiatkowska, …

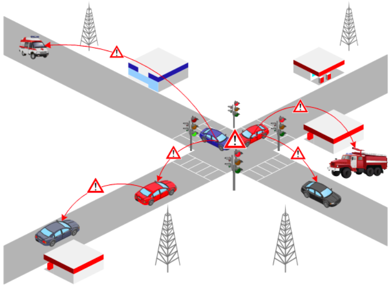
ERC Advanced Grant FUN2MODEL

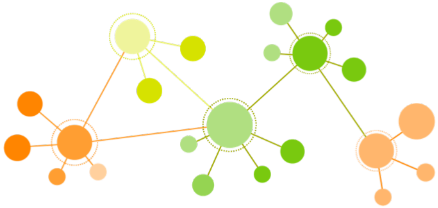
Verification of stochastic systems

- Formal verification needs stochastic modelling

faulty sensors/actuators

unpredictable/unknown environments

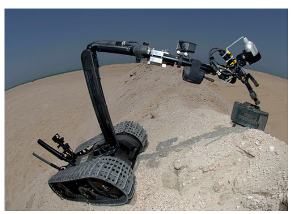
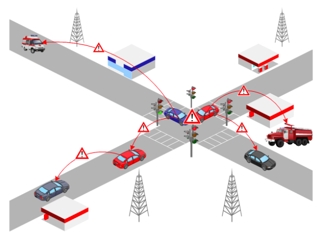
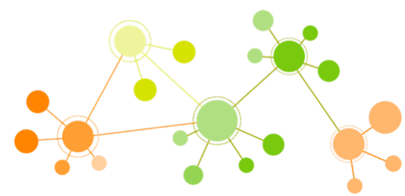




randomised protocols

Verification with stochastic games

- How do we verify stochastic systems with…
  - multiple autonomous agents acting concurrently
  - competitive or collaborative behaviour between agents, possibly with differing/opposing goals
  - e.g. security protocols, algorithms for distributed consensus, energy management, autonomous robotics, auctions

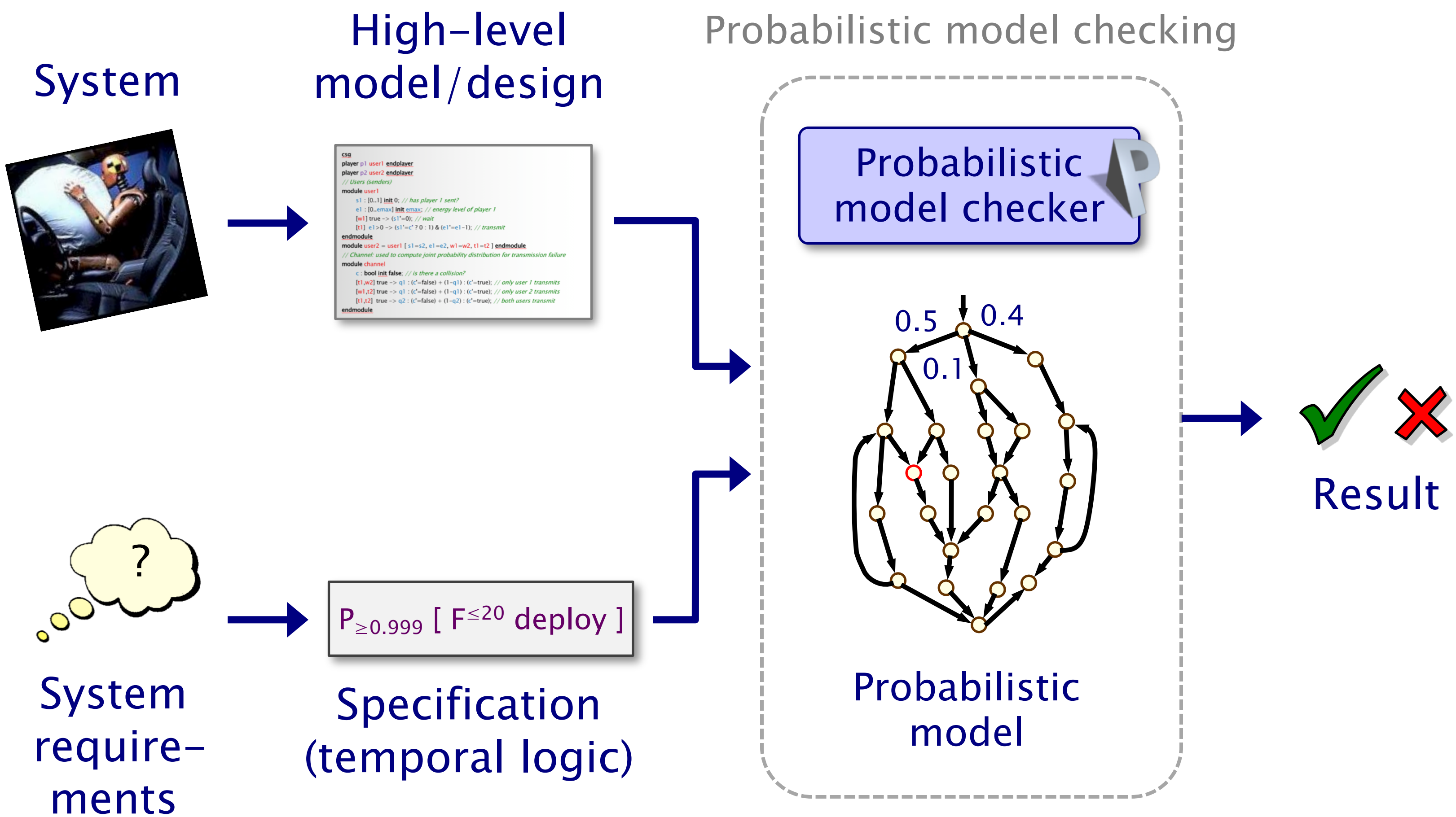
- This talk: verification with stochastic multi-player games
  - verification (and synthesis) of strategies that are robust in adversarial settings and stochastic environments
  - models, logics, algorithms, tools, examples

Overview

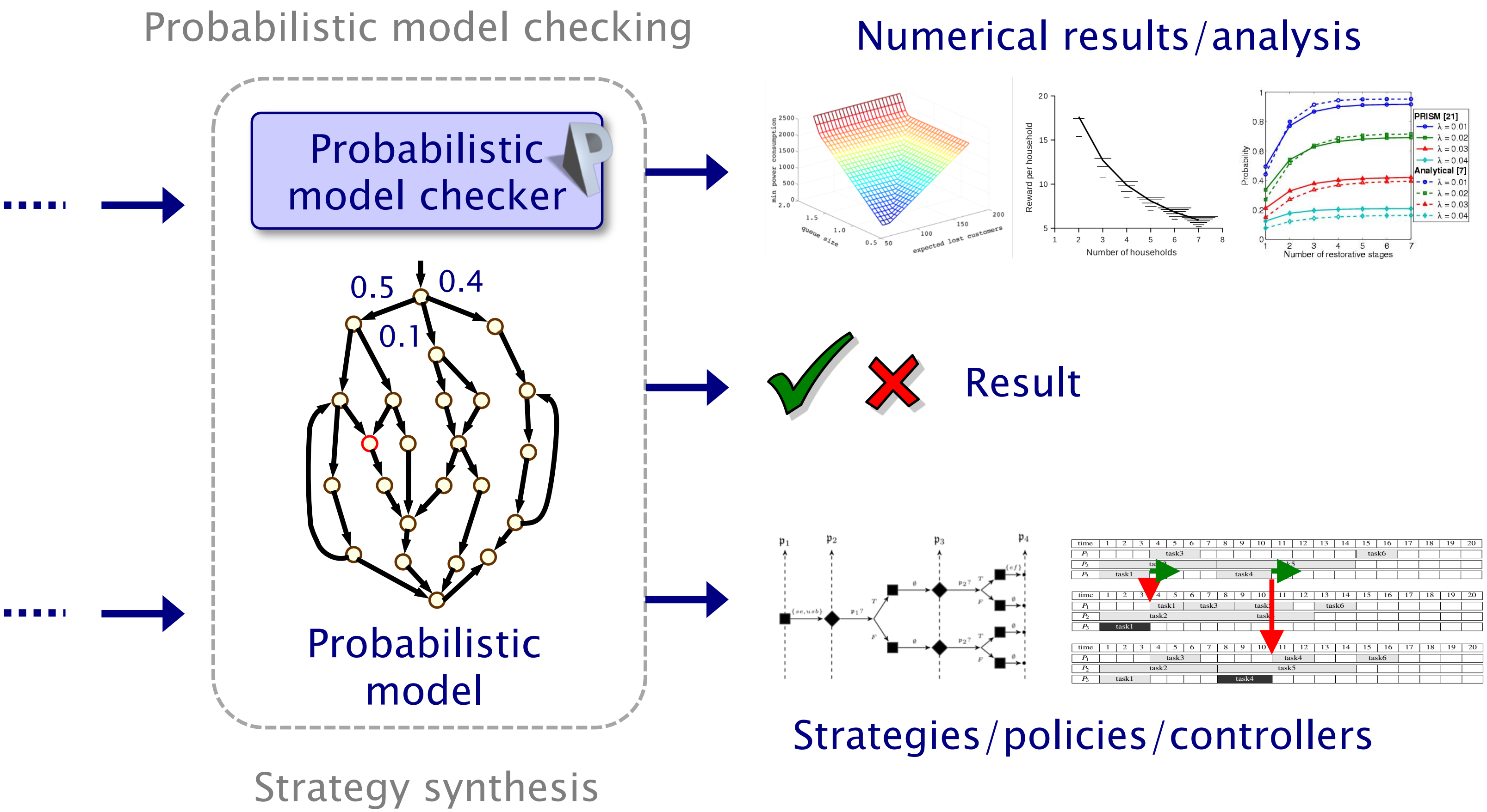
- Probabilistic model checking
  - Markov decision processes (MDPs)
  - example: robot navigation
- Stochastic multi-player games
  - example: energy management

– Concurrent stochastic games
  - example: investor models
- Equilibria-based properties
  - example: multi-robot coordination
- Future challenges

Probabilistic model checking

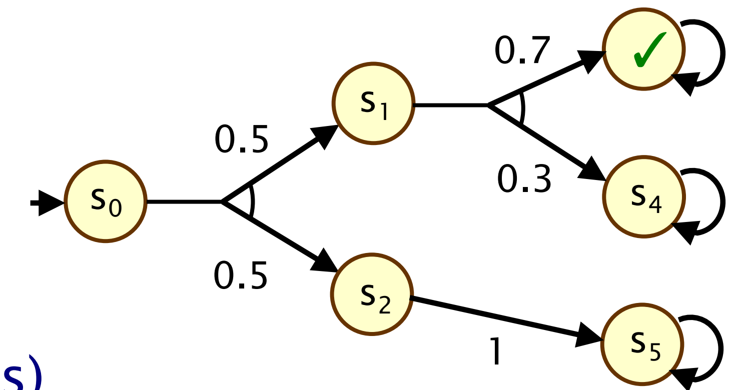


Probabilistic model checking

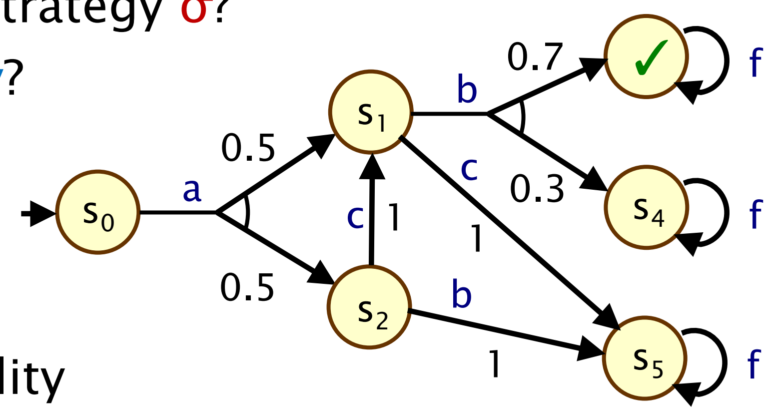


Probabilistic models

- Discrete-time Markov chains
  - e.g. what is the probability of reaching state ✓?



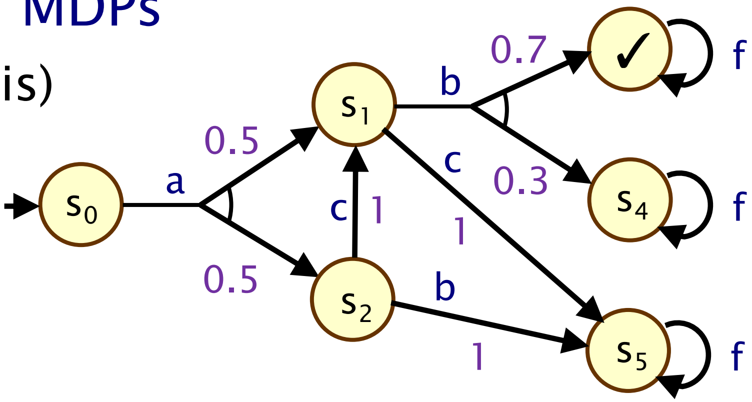
- Markov decision processes (MDPs)
  - strategies (or policies) resolve actions based on history
  - e.g. what is the <u>maximum</u> probability of reaching ✓ achievable by any strategy σ?
  - and what is an optimal strategy?



- Formally:
  - we write: $\sup_\sigma Pr_s^\sigma(F✓)$
  - where Pr_s^σ denotes the probability from state s under strategy σ

Solving MDPs

- Various techniques exist to solve MDPs
  - (and to perform strategy synthesis)



- Here, we focus on value iteration
  - dynamic programming approach
  - common for probabilistic model checking

let p(s) = $\sup_\sigma \mathrm{Pr}_s^{\sigma}(F\checkmark)$

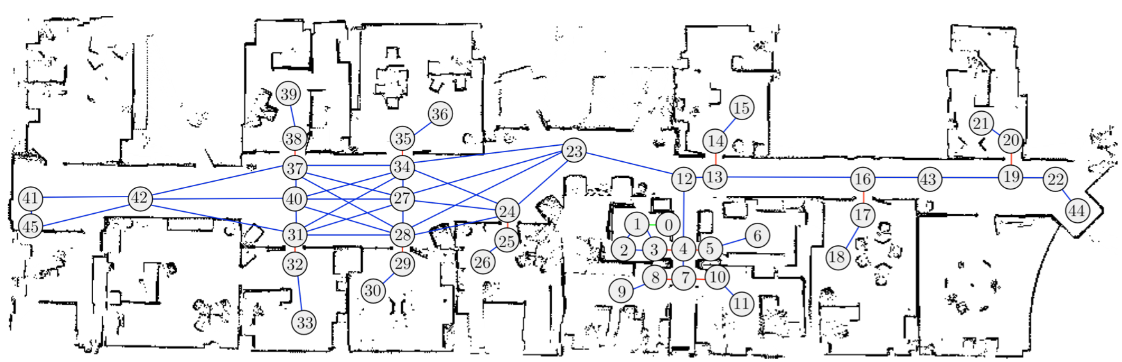
- For example:
  - maximum probability p(s) to reach ✓ from s
  - values p(s) are the least fixed point of:

$$p(s) = \begin{cases} 1 & \text{if } s \models \checkmark \\ \max_a \Sigma_{s'} \delta(s,a)(s') \cdot p(s') & \text{otherwise} \end{cases}$$

  - basis for iterative numerical computation

Example: Robot navigation

- Robot planning with probabilistic guarantees
  - MDP models navigation in (learnt) uncertain environment
  - temporal logic for formal robot task specification
    - $\neg zone_3 \; U \; (room_1 \wedge (F \; room_4 \wedge F \; room_5))$ (co-safe LTL)
  - strategy synthesis performed to generate controllers
    - also: costs & rewards, multi-objective, ..
  - PRISM built into a ROS module
    - 100s of hrs of autonomous robot deployment



[IROS'14,IJCAI'15,ICAPS'17,IJRR'19]

Stochastic games

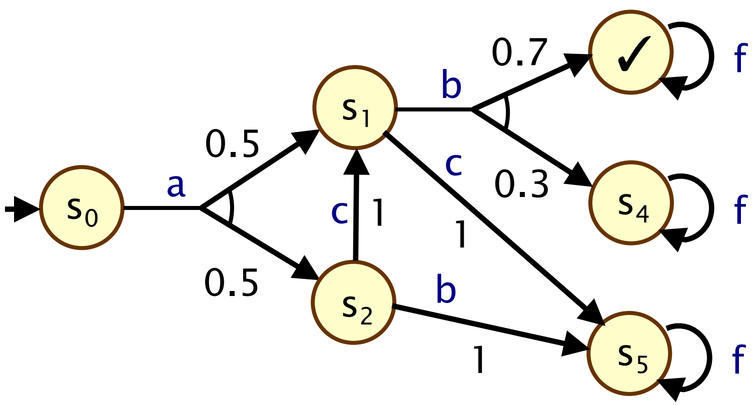
Stochastic multi-player games

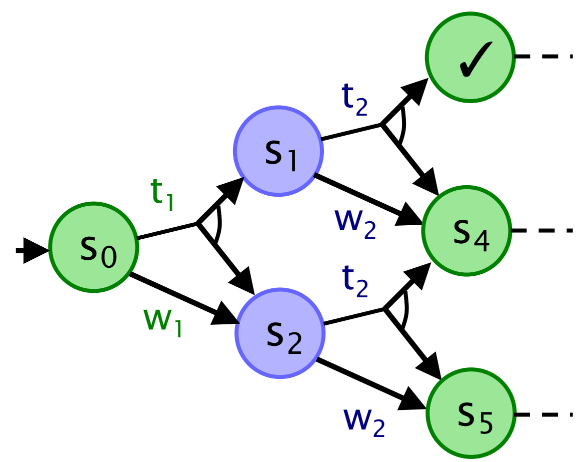
- Stochastic multi-player games
  - strategies + probability + multiple players
  - for now: turn-based (player i controls states S_i)

Markov decision processes (MDPs)

→

Turn-based stochastic games (TSGs)



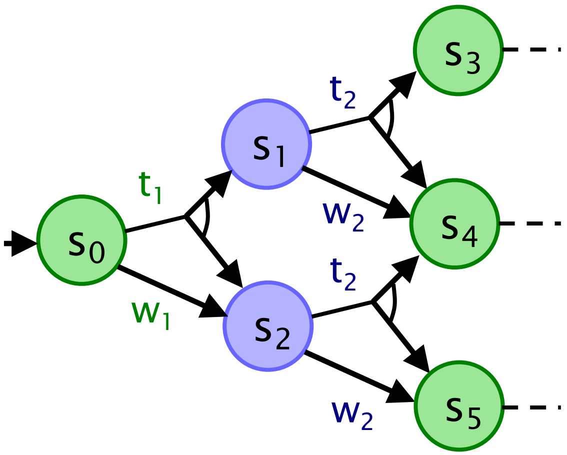


Property specification: rPATL

- rPATL (reward probabilistic alternating temporal logic)
  - branching-time temporal logic for stochastic games
- CTL, extended with:
  - coalition operator $\langle\langle C \rangle\rangle$ of ATL
  - probabilistic operator P of PCTL
  - generalised (expected) reward operator R from PRISM
- In short:
  - zero-sum, probabilistic reachability + expected total reward
- Example:
  - $\langle\langle \{robot_1, robot_3\} \rangle\rangle \; P_{>0.99} \; [\; F^{\leq 10} \; (goal_1 \vee goal_3) \;]$
  - "robots 1 and 3 have a strategy to ensure that the probability of reaching the goal location within 10 steps is >0.99, regardless of the strategies of other players"

Model checking rPATL

- Main task: checking individual P and R operators
  - reduces to solving a (zero-sum) stochastic 2-player game
  - e.g. max/min reachability probability: $\sup_{\sigma_1} \inf_{\sigma_2} Pr_s^{\sigma_1,\sigma_2}(F\checkmark)$
  - complexity: NP ∩ coNP (if we omit some reward operators)



- We again use value iteration
  - values p(s) are the least fixed point of:

$$p(s) = \begin{cases} 1 & \text{if } s \models \checkmark \\ \max_a \Sigma_{s'} \delta(s,a)(s')\cdot p(s') & \text{if } s \not\models \checkmark \text{ and } s \in S_1 \\ \min_a \Sigma_{s'} \delta(s,a)(s')\cdot p(s') & \text{if } s \not\models \checkmark \text{ and } s \in S_2 \end{cases}$$

  - and more: graph-algorithms, sequences of fixed points, …

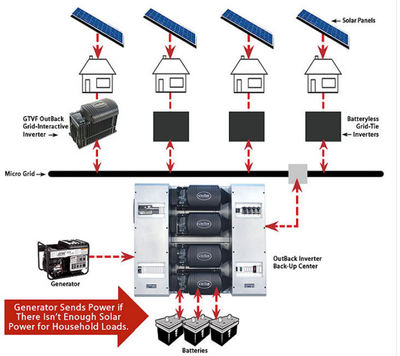
PRISM-games

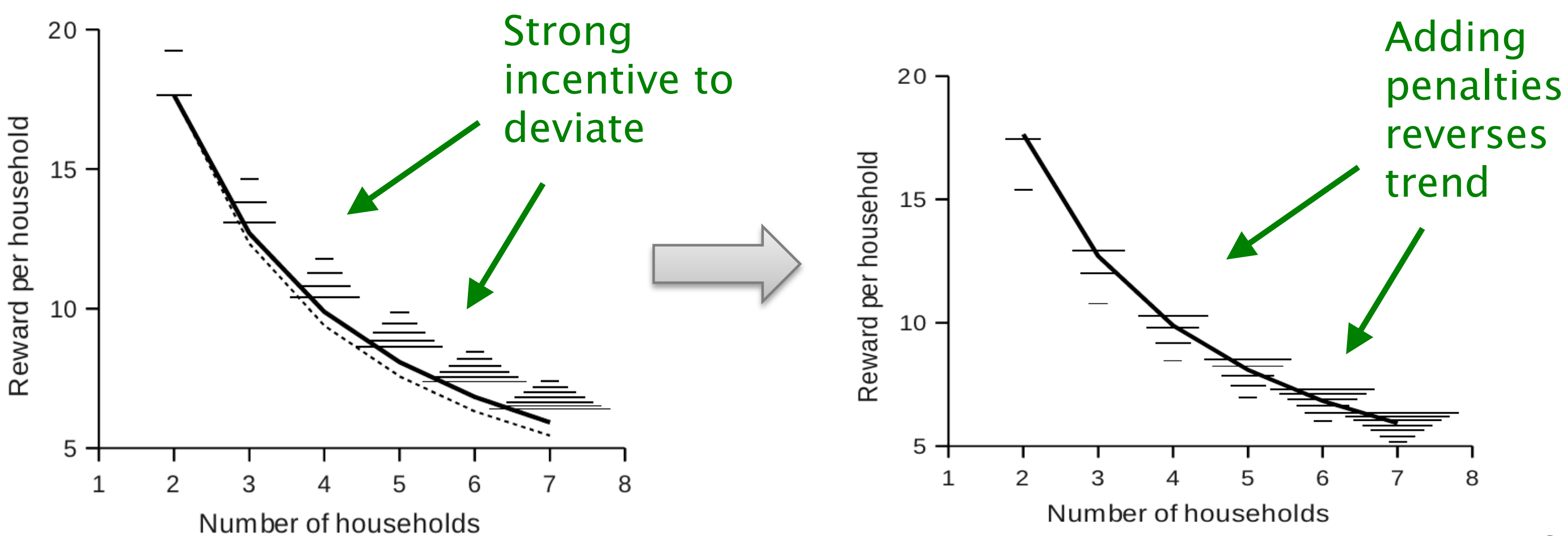
- PRISM-games: prismmodelchecker.org/games
  - extension of PRISM modelling language (see later)
  - implementation in explicit engine
  - prototype symbolic (MTBDD) version also available

- Example application domains
  - security: attack-defence trees; DNS bandwidth amplification
  - self-adaptive software architectures
  - autonomous urban driving
  - human-in-the-loop UAV mission planning
  - collective decision making and team formation protocols
  - energy management protocols

Example: Energy management

- Demand management protocol for microgrids
  - random back-off to minimise peaks
- Stochastic game model + rPATL
  - exposes protocol weakness (incentive for clients to act selfishly)
  - propose/verify simple fix using penalties



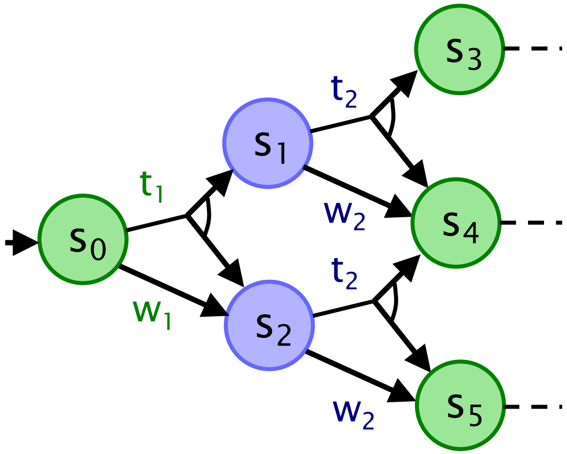


Concurrent stochastic games

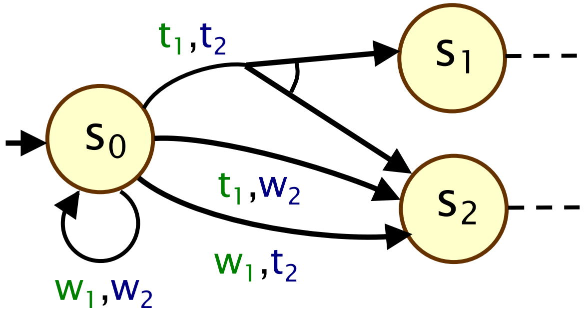
Concurrent stochastic games

- Motivation:
  - more realistic model of components operating concurrently, making action choices without knowledge of others

Turn-based stochastic games (TSGs)



Concurrent stochastic games (CSGs)



Concurrent stochastic games

- Concurrent stochastic games (CSGs)
  - players choose actions concurrently & independently
  - jointly determines (probabilistic) successor state
  - $\delta : S \times (A_1 \cup \{\bot\}) \times \ldots \times (A_n \cup \{\bot\}) \rightarrow Dist(S)$
  - generalises turn-based stochastic games

- We again use the logic rPATL for properties

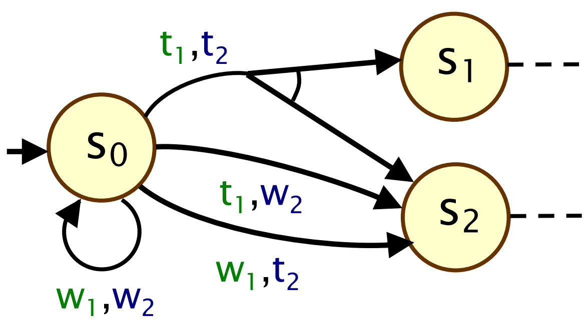
- Same overall rPATL model checking algorithm [QEST'18]
  - key ingredient is now solving (zero-sum) 2-player CSGs
  - this problem is in PSPACE
  - note that optimal strategies are now randomised

rPATL model checking for CSGs

- We again use a value iteration based approach
  - e.g. max/min reachability probabilities
  - $\sup_{\sigma_1} \inf_{\sigma_2} \Pr_s^{\sigma_1,\sigma_2}(F\ ✓)$ for all states s
  - values p(s) are the least fixed point of:

$$p(s) = \begin{cases} 1 & \text{if } s \models ✓ \\ val(Z) & \text{if } s \not\models ✓ \end{cases}$$

  - where Z is the matrix game with $z_{ij} = \Sigma_{s'}\ \delta(s,(a_i,b_j))(s') \cdot p(s')$
  - so each iteration requires solution of a matrix game for each state (LP problem of size |A|, where A = action set)



CSGs in PRISM-games

- CSG model checking implemented in PRISM-games 3.0
- Extension of PRISM modelling language
  - (see next slide)
- Explicit engine implementation
  - plus LP solvers for matrix game solution
  - this is the main bottleneck
  - experiments with CSGs up to ~3 million states
- Case studies:
  - future markets investor, trust models for user-centric networks, intrusion detection policies, jamming radio systems

CSGs in PRISM-games 3.0

Example model (medium access control)

```
csg
player p1 user1 endplayer
player p2 user2 endplayer
// Users (senders)
module user1
    s1 : [0..1] init 0; // has player 1 sent?
    e1 : [0..emax] init emax; // energy level of player 1
    [w1] true -> (s1'=0); // wait
    [t1]  e1>0 -> (s1'=c' ? 0 : 1) & (e1'=e1-1); // transmit
endmodule
module user2 = user1 [ s1=s2, e1=e2, w1=w2, t1=t2 ] endmodule
// Channel: used to compute joint probability distribution for transmission failure
module channel
    c : bool init false; // is there a collision?
    [t1,w2] true -> q1 : (c'=false) + (1-q1) : (c'=true); // only user 1 transmits
    [w1,t2] true -> q1 : (c'=false) + (1-q1) : (c'=true); // only user 2 transmits
    [t1,t2]  true -> q2 : (c'=false) + (1-q2) : (c'=true); // both users transmit
endmodule
```

CSGs in PRISM-games 3.0

```
csg
player p1 user1 endplayer
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	[t1,t2]  true -> q2 : (c'=false) + (1-q2) : (c'=true); // both users transmit
endmodule
```

Each player comprises one or more modules

Players have distinct actions, executed simultaneously

CSGs in PRISM-games 3.0

```
csg
player p1 user1 endplayer
player p2 user2 endplayer
// Users (senders)
module user1
	s1 : [0..1] init 0; // has player 1 sent?
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	[t1,t2]  true -> q2 : (c'=false) + (1-q2) : (c'=true); // both users transmit
endmodule
```
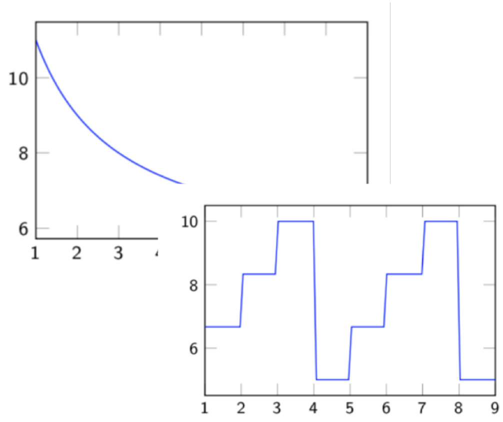

Variable updates can refer to other variables updated simultaneously

Action lists used to specify synchronisation

Example: Future markets investor

- Model of interactions between:
  - stock market, evolves stochastically
  - two investors i_1, i_2 decide when to invest
  - market decides whether to bar investors

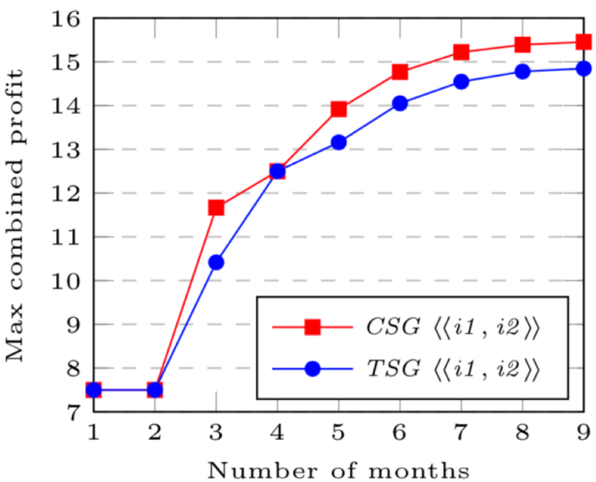


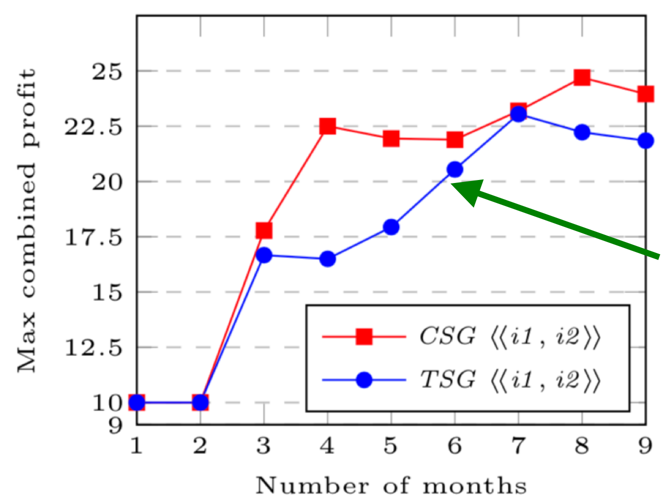
- Modelled as a 3-player CSG
  - extends simpler model originally from [McIver/Morgan'07]
  - investing/barring decisions are simultaneous
  - profit reduced for simultaneous investments
  - market cannot observe investors' decisions

- Analysed with rPATL model checking & strategy synthesis
  - distinct profit models considered: 'normal market', 'later cash-ins' and 'later cash-ins with fluctuation'
  - comparison between TSG and CSG models

Example: Future markets investor

- Example rPATL query:
  - $\langle\langle \text{investor}_1,\text{investor}_2 \rangle\rangle\ \text{R}^{\text{profit}_{1,2}}_{\text{max}=?}\ [\ \text{F finished}_{1,2}\ ]$
  - i.e. maximising joint profit
- Results: with (left) and without (right) fluctuations
  - optimal (randomised) investment strategies synthesised
  - CSG yields more realistic results (market has less power due to limited observation of investor strategies)





Too pessimistic: unrealistic strategy for adversary

Equilibria-based properties

Equilibria-based properties

- Motivation:
  - players/components may have distinct objectives but which are not directly opposing (zero-sum)

Zero-sum properties → Equilibria-based properties

$\langle\langle robot_1 \rangle\rangle_{max=?}\ P\ [\ F^{\leq k}\ goal_1\ ]$

$\langle\langle robot_1{:}robot_2 \rangle\rangle_{max=?}$
$(P\ [\ F^{\leq k}\ goal_1\ ]+P\ [F^{\leq k}\ goal_2])$

- We use Nash equilibria (NE)
  - no incentive for any player to unilaterally change strategy
  - actually, we use ϵ-NE, which always exist for CSGs
  - a strategy profile $\sigma=(\sigma_{1,\dots,}\sigma_n)$ for a CSG is an ϵ-NE for state s and objectives $X_1,\dots,X_n$ iff:
  - $Pr_s^{\sigma}(X_i) \geq \sup \{ Pr_s^{\sigma'}(X_i) \mid \sigma'=\sigma_{-i}[\sigma_i'] \text{ and } \sigma_i' \in \Sigma_i \} - \epsilon$ for all i

Social-welfare Nash equilibria

- Key idea: formulate model checking (strategy synthesis) in terms of social-welfare Nash equilibria (SWNE)
  - these are NE which maximise the sum $E_s^\sigma(X_1) + \ldots E_s^\sigma(X_n)$
  - i.e., optimise the players combined goal

- We extend rPATL accordingly

Zero-sum properties → **Equilibria-based properties**

$\langle\langle robot_1 \rangle\rangle_{max=?}\ P\ [\ F^{\leq k}\ goal_1\ ]$

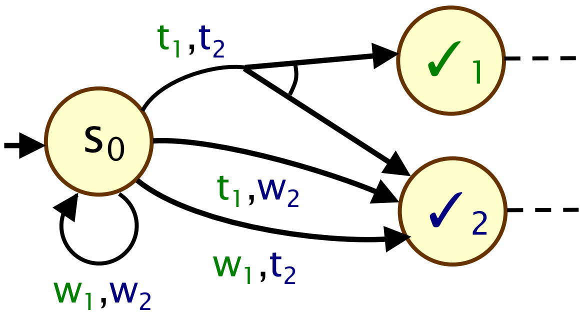
find a robot 1 strategy which maximises the probability of it reaching its goal, regardless of robot 2

$\langle\langle robot_1{:}robot_2 \rangle\rangle_{max=?}\ (P\ [\ F^{\leq k}\ goal_1\ ] + P\ [F^{\leq k}\ goal_2])$

find (SWNE) strategies for robots 1 and 2 where there is no incentive to change actions and which maximise joint goal probability

Model checking for extended rPATL

- Model checking for CSGs with equilibria
  - first: 2-coalition case [FM'19]
  - needs solution of bimatrix games
  - (basic problem is EXPTIME)
  - we adapt a known approach using labelled polytopes, and implement with an SMT encoding



- We further extend the value iteration approach:

$$p(s) = \begin{cases} (1,1) & \text{if } s \models \checkmark_1 \wedge \checkmark_2 \\ (p_{max}(s,\checkmark_2),1) & \text{if } s \models \checkmark_1 \wedge \neg\checkmark_2 \\ (1,p_{max}(s,\checkmark_1)) & \text{if } s \models \neg\checkmark_1 \wedge \checkmark_2 \\ val(Z_1,Z_2) & \text{if } s \models \neg\checkmark_1 \wedge \neg\checkmark_2 \end{cases}$$

standard MDP analysis

bimatrix game

  - where Z_1 and Z_2 encode matrix games similar to before

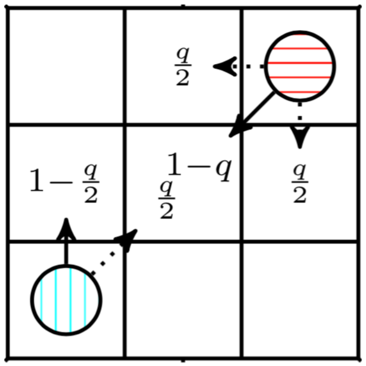
PRISM-games support

- Implementation in PRISM-games 3.0
  - bimatrix games solved using Z3/Yices encoding
  - optimised filtering of dominated strategies
  - scales up to CSGs with ~2 million states
  - extended to n-coalition case in [QEST'20]

- Applications & results
  - robot navigation in a grid, medium access control, Aloha communication protocol, power control
  - SWNE strategies outperform those found with rPATL
  - ϵ-Nash equilibria found typically have $\epsilon=0$

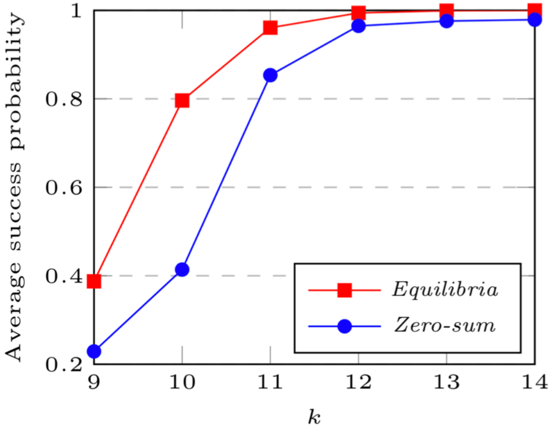
Example: multi-robot coordination

- 2 robots navigating an l x l grid
  - start at opposite corners, goals are to navigate to opposite corners
  - obstacles modelled stochastically: navigation in chosen direction fails with probability q



- We synthesise SWNEs to maximise the average probability of robots reaching their goals within time k
  - $\langle\langle robot1{:}robot2 \rangle\rangle_{max=?}$ (P [$F^{\leq k}$ $goal_1$]+P [$F^{\leq k}$ $goal_2$])

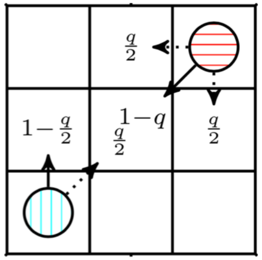
- Results (10 x 10 grid)
  - better performance obtained than using zero-sum methods, i.e., optimising for robot 1, then robot 2



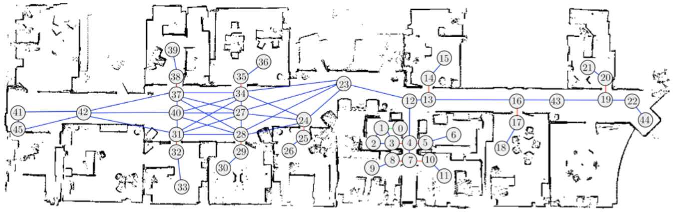
Future challenges

Challenges

- Partial information/observability
  - we need realisable strategies
  - leverage progress on POMDPs?



- Managing model uncertainty
  - integration with learning
  - robust verification



- Accuracy of model checking results
  - value iteration improvements; exact methods

- Scalability & efficiency
  - e.g. symbolic methods, abstraction, symmetry reduction
  - sampling-based strategy synthesis methods

PRISM-games

- See the PRISM-games website for more info
  - prismmodelchecker.org/games/
  - documentation, examples, case studies, papers
  - downloads: + CAV'20 artefact VM
  - open source (GPLV2):