Description Logic: A Formal Foundation for Ontology Languages and Tools

Part 1: Languages

Ian Horrocks
<ian.horrocks@comlab.ox.ac.uk>
Information Systems Group
Oxford University Computing Laboratory

Contents

- Motivation
- Brief review of (first order) logic
- Description Logics as fragments of FOL
- Description Logic syntax and semantics
- Brief review of relevant complexity notions
- Description Logics and OWL
- Ontology applications
- Ontologies –v- databases

DL Basics

What Are Description Logics?

What Are Description Logics?

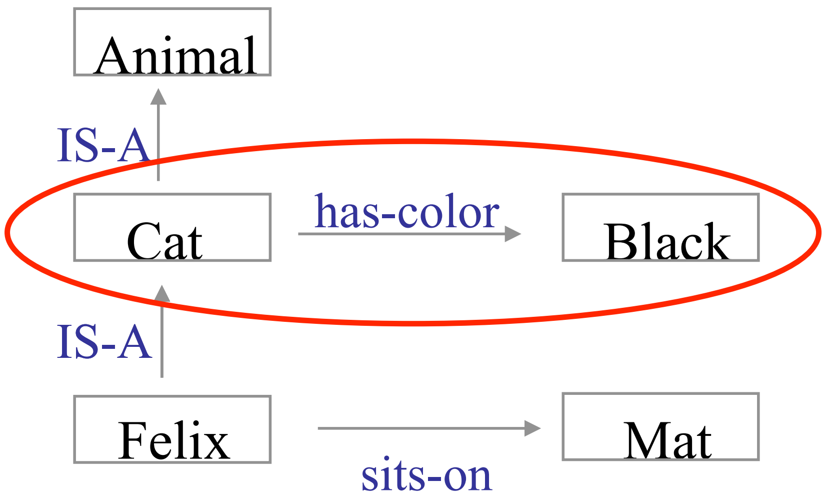
- Decidable fragments of First Order Logic

Thank you for listening

Any questions?

What Are Description Logics?

- A family of logic based Knowledge Representation formalisms
  - Originally descended from semantic networks and KL-ONE
  - Describe domain in terms of concepts (aka classes), roles (aka properties, relationships) and individuals



[Quillian, 1967]

What Are Description Logics?

- Modern DLs (after Baader et al) distinguished by:
  - Fully fledged logics with formal semantics
    - Decidable fragments of FOL (often contained in C_2)
    - Closely related to Propositional Modal/Dynamic Logics & Guarded Fragment
  - Computational properties well understood (worst case complexity)
  - Provision of inference services
    - Practical decision procedures (algorithms) for key problems (satisfiability, subsumption, query answering, etc)
    - Implemented systems (highly optimised)
- The basis for widely used ontology languages

Web Ontology Language OWL (2)

- **W3C recommendation(s)**
- Motivated by **Semantic Web** activity

  Add meaning to web content by annotating it with terms defined in ontologies

- Supported by **tools and infrastructure**
  - APIs (e.g., OWL API, Thea, OWLink)
  - Development environments (e.g., Protégé, Swoop, TopBraid Composer, Neon)
  - Reasoners & Information Systems (e.g., Pellet, Racer, HermiT, Quonto, …)
- Based on **Description Logics** ($\mathcal{SHOIN}$ / $\mathcal{SROIQ}$)

and now:

A Word from our Sponsors

Crash Course in (simplified) FOL

- Syntax
  - Non-logical symbols (signature)
    - Constants: Felix, MyMat
    - Predicates(arity): Animal(1), Cat(1), has-color(2), sits-on(2)
  - Logical symbols:
    - Variables: x, y
    - Operators: ∧, ∨, →, ¬, …
    - Quantifiers: ∃, ∀
    - Equality: =
  - Formulas:
    - $\mathrm{Cat}(\mathrm{Felix}),\quad \mathrm{Mat}(\mathrm{MyMat}),\quad \text{sits-on}(\mathrm{Felix}, \mathrm{MyMat})$
    - $\mathrm{Cat}(x),\quad \mathrm{Cat}(x) \vee \mathrm{Human}(x),\quad \exists y.\mathrm{Mat}(y) \wedge \text{sits-on}(x, y)$
    - $\forall x.\mathrm{Cat}(x) \rightarrow \mathrm{Animal}(x),\quad \forall x.\mathrm{Cat}(x) \rightarrow (\exists y.\mathrm{Mat}(y) \wedge \text{sits-on}(x, y))$

Crash Course in (simplified) FOL

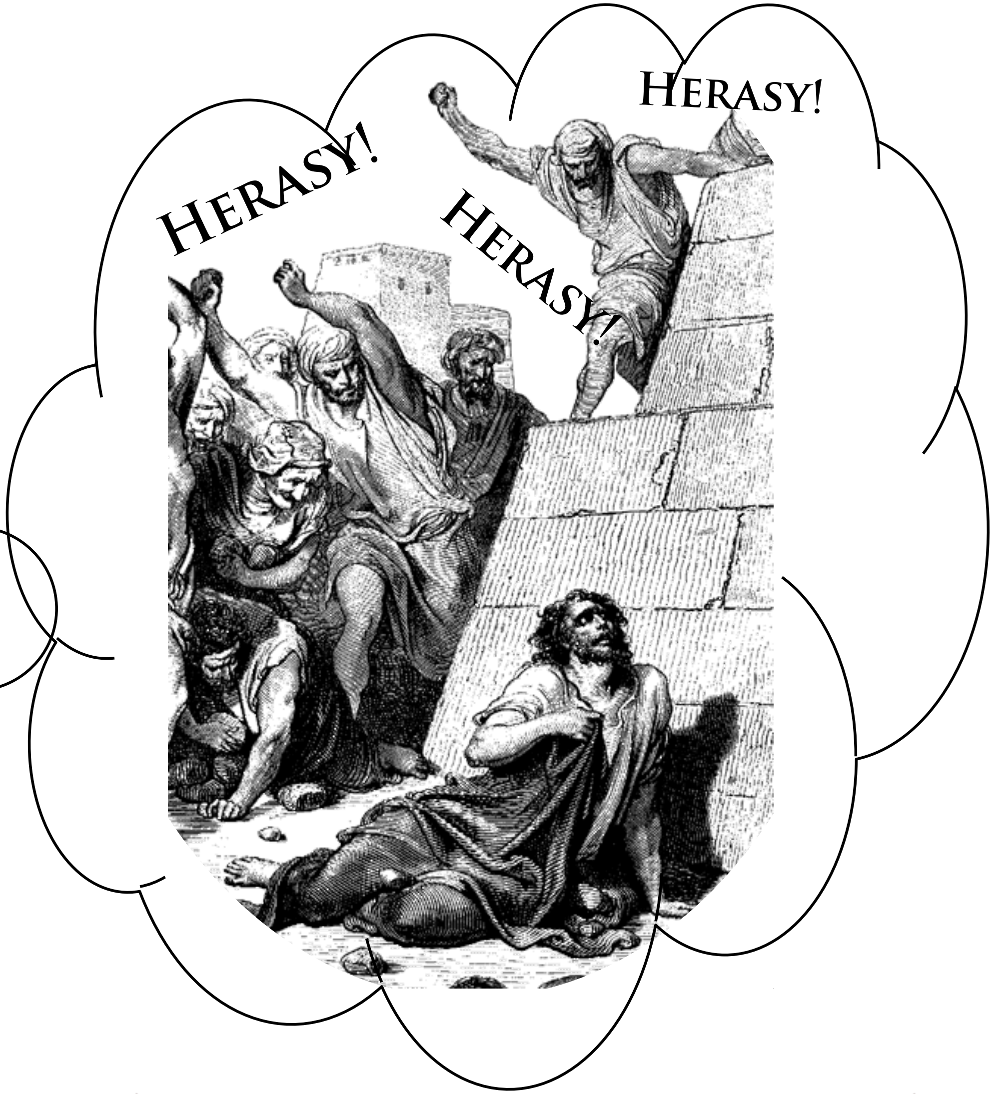
- Semantics

Crash Course in (simplified) FOL

- Semantics

Crash Course in (simplified) FOL

- Semantics



Crash Course in (simplified) FOL

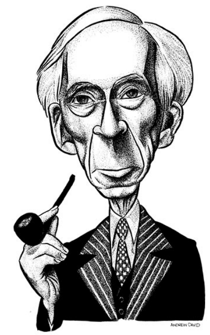
- Semantics

Why should I care about semantics? -- In fact I heard that a little goes a long way!

Crash Course in (simplified) FOL

- Semantics

Why should I care about semantics? -- In fact I heard that a little goes a long way!

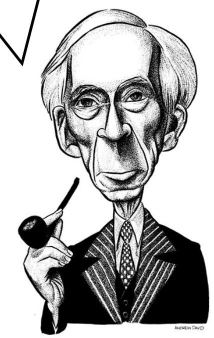


Crash Course in (simplified) FOL

- Semantics

Why should I care about semantics? -- In fact I heard that a little goes a long way!

Well, from a philosophical POV, we need to specify the relationship between statements in the logic and the existential phenomena they describe.



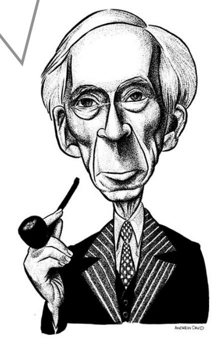
Crash Course in (simplified) FOL

- Semantics

Why should I care about semantics? -- In fact I heard that a little goes a long way!

Well, from a philosophical POV, we need to specify the relationship between statements in the logic and the existential phenomena they describe.

That's OK, but I don't get paid for philosophy.



Crash Course in (simplified) FOL

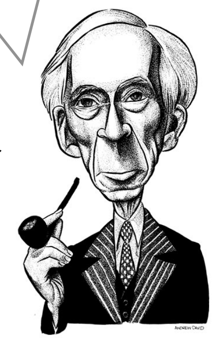
- Semantics

Why should I care about semantics? -- In fact I heard that a little goes a long way!

Well, from a philosophical POV, we need to specify the relationship between statements in the logic and the existential phenomena they describe.

That's OK, but I don't get paid for philosophy.

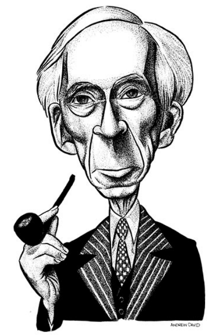
From a practical POV, in order to specify, build and test (ontology-based) tools/systems we need to precisely define relationships (like entailment) between logical statements – this defines the *intended* behaviour of tools/systems.



Crash Course in (simplified) FOL

- Semantics

In FOL we define the semantics in terms of models (a model theory). A model is supposed to be an analogue of (part of) the world being modeled. FOL uses a very simple kind of model, in which “objects” in the world (not necessarily physical objects) are modeled as elements of a set, and relationships between objects are modeled as sets of tuples.

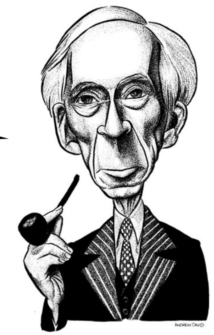


Crash Course in (simplified) FOL

- Semantics

In FOL we define the semantics in terms of models (a model theory). A model is supposed to be an analogue of (part of) the world being modeled. FOL uses a very simple kind of model, in which “objects” in the world (not necessarily physical objects) are modeled as elements of a set, and relationships between objects are modeled as sets of tuples.

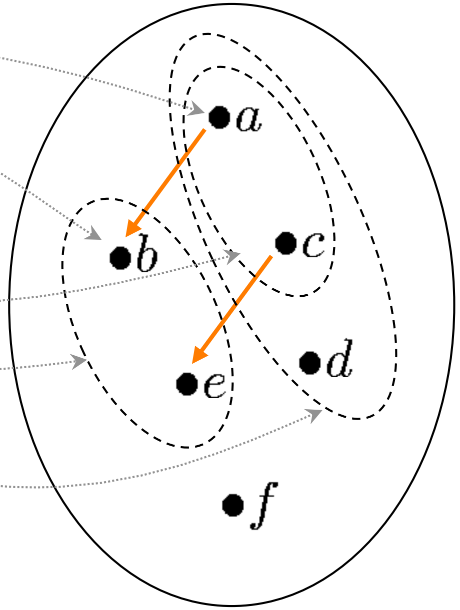
Note that this is exactly the same kind of model as used in a database: objects in the world are modeled as values (elements) and relationships as tables (sets of tuples).



Crash Course in (simplified) FOL

- Semantics
  - Model: a pair $\langle D, \cdot^I \rangle$ with D a non-empty set and $\cdot^I$ an interpretation
    - C^I is an element of D for C a constant
    - v^I is an element of D for v a variable
    - P^I is a subset of D^n for P a predicate of arity n
  - E.g., $D = \{a, b, c, d, e, f\}$, and

$$
\begin{aligned}
\text{Felix}^I &= a \\
\text{MyMat}^I &= b \\
\text{Cat}^I &= \{a, c\} \\
\text{Mat}^I &= \{b, e\} \\
\text{Animal}^I &= \{a, c, d\} \\
\text{sits-on}^I &= \{\langle a, b\rangle, \langle c, e\rangle\}
\end{aligned}
$$



Crash Course in (simplified) FOL

- Semantics
  - Evaluation: truth value in a given model M = $\langle D, \cdot^I \rangle$
    - $P(t_1, \ldots, t_n)$ is *true* iff $\langle t_1^I, \ldots, t_n^I \rangle \in P^I$
    - $A \wedge B$ is *true* iff A is *true* and B is *true*

      $\neg A$ is *true* iff A is not *true*
  - E.g.,

$\text{Cat}(\text{Felix})$	true
$\text{Cat}(\text{MyMat})$	false
$\neg\text{Mat}(\text{Felix})$	true
$\text{sits-on}(\text{Felix}, \text{MyMat})$	true
$\text{Mat}(\text{Felix}) \vee \text{Cat}(\text{Felix})$	true

$$
\boxed{
\begin{array}{l}
D = \{a, b, c, d, e, f\} \\
\text{Felix}^I = a \\
\text{MyMat}^I = b \\
\text{Cat}^I = \{a, c\} \\
\text{Mat}^I = \{b, e\} \\
\text{Animal}^I = \{a, c, d\} \\
\text{sits-on}^I = \{\langle a, b\rangle, \langle c, e\rangle\}
\end{array}
}
$$

Crash Course in (simplified) FOL

- Semantics
  - Evaluation: truth value in a given model M = $\langle D, \cdot^I \rangle$
    - $\exists x.A$ is *true* iff exists $\cdot^{I'}$ s.t. $\cdot^I$ and $\cdot^{I'}$ differ only w.r.t. x, and A is *true* w.r.t. $\langle D, \cdot^{I'} \rangle$
    - $\forall x.A$ is *true* iff for all $\cdot^{I'}$ s.t. $\cdot^I$ and $\cdot^{I'}$ differ only w.r.t. x, A is *true* w.r.t. $\langle D, \cdot^{I'} \rangle$

E.g.,

$\exists x.\text{Cat}(x)$	true
$\forall x.\text{Cat}(x)$	false
$\exists x.\text{Cat}(x) \wedge \text{Mat}(x)$	false
$\forall x.\text{Cat}(x) \rightarrow \text{Animal}(x)$	true
$\forall x.\text{Cat}(x) \rightarrow (\exists y.\text{Mat}(y) \wedge \text{sits-on}(x, y))$	true

$D = \{a, b, c, d, e, f\}$

$\text{Felix}^I = a$

$\text{MyMat}^I = b$

$\text{Cat}^I = \{a, c\}$

$\text{Mat}^I = \{b, e\}$

$\text{Animal}^I = \{a, c, d\}$

$\text{sits-on}^I = \{\langle a, b\rangle, \langle c, e\rangle\}$

Crash Course in (simplified) FOL

- Semantics
  - Given a model M and a formula F, M is a model of F (written $\mathrm{M} \models \mathrm{F}$) iff F evaluates to true in M
  - A formula F is **satisfiable** iff there exists a model M s.t. $\mathrm{M} \models \mathrm{F}$
  - A formula F **entails** another formula G (written $\mathrm{F} \models \mathrm{G}$) iff every model of F is also a model of G (i.e., $\mathrm{M} \models \mathrm{F}$ implies $\mathrm{M} \models \mathrm{G}$)

E.g.,

$$M \models \exists x.\mathrm{Cat}(x)$$
$$M \not\models \forall x.\mathrm{Cat}(x)$$
$$M \not\models \exists x.\mathrm{Cat}(x) \wedge \mathrm{Mat}(x)$$
$$M \models \forall x.\mathrm{Cat}(x) \rightarrow \mathrm{Animal}(x)$$
$$M \models \forall x.\mathrm{Cat}(x) \rightarrow (\exists y.\mathrm{Mat}(y) \wedge \text{sits-on}(x, y))$$

$$\boxed{\begin{aligned} & D = \{a, b, c, d, e, f\} \\ & \mathrm{Felix}^I = a \\ & \mathrm{MyMat}^I = b \\ & \mathrm{Cat}^I = \{a, c\} \\ & \mathrm{Mat}^I = \{b, e\} \\ & \mathrm{Animal}^I = \{a, c, d\} \\ & \text{sits-on}^I = \{\langle a, b\rangle, \langle c, e\rangle\} \end{aligned}}$$

Crash Course in (simplified) FOL

- Semantics
  - Given a model M and a formula F, M is a model of F (written $\mathrm{M} \vDash \mathrm{F}$) iff F evaluates to true in M
  - A formula F is **satisfiable** iff there exists a model M s.t. $\mathrm{M} \vDash \mathrm{F}$
  - A formula F **entails** another formula G (written $\mathrm{F} \vDash \mathrm{G}$) iff every model of F is also a model of G (i.e., $\mathrm{M} \vDash \mathrm{F}$ implies $\mathrm{M} \vDash \mathrm{G}$)

  E.g.,

✓ $\mathrm{Cat}(\mathrm{Felix}) \models \exists x.\mathrm{Cat}(x) \quad (\mathrm{Cat}(\mathrm{Felix}) \wedge \neg\exists x.\mathrm{Cat}(x)$ is not satisfiable$)$

✓ $(\forall x.\mathrm{Cat}(x) \rightarrow \mathrm{Animal}(x)) \wedge \mathrm{Cat}(\mathrm{Felix}) \models \mathrm{Animal}(\mathrm{Felix})$

✓ $(\forall x.\mathrm{Cat}(x) \rightarrow \mathrm{Animal}(x)) \wedge \neg\mathrm{Animal}(\mathrm{Felix}) \models \neg\mathrm{Cat}(\mathrm{Felix})$

✘ $\mathrm{Cat}(\mathrm{Felix}) \models \forall x.\mathrm{Cat}(x)$

✘ $\text{sits-on}(\mathrm{Felix}, \mathrm{Mat1}) \wedge \text{sits-on}(\mathrm{Tiddles}, \mathrm{Mat2}) \models \neg\text{sits-on}(\mathrm{Felix}, \mathrm{Mat2})$

✘ $\text{sits-on}(\mathrm{Felix}, \mathrm{Mat1}) \wedge \text{sits-on}(\mathrm{Tiddles}, \mathrm{Mat1}) \models \exists^{\geq 2} x.\text{sits-on}(x, \mathrm{Mat1})$

Decidable Fragments

- FOL (satisfiability) well known to be undecidable
  – A sound, complete and terminating algorithm is impossible
- Interesting decidable fragments include, e.g.,
  – C2: FOL with 2 variables and Counting quantifiers $(\exists^{\geq n}, \exists^{\leq n})$
    - Counting quantifiers abbreviate pairwise (in-) equalities, e.g.:

      $\exists^{\geq 3} x.\mathrm{Cat}(x)$ equivalent to

      $\exists x, y, z.\mathrm{Cat}(x) \wedge \mathrm{Cat}(y) \wedge \mathrm{Cat}(z) \wedge x \neq y \wedge x \neq z \wedge y \neq z$

      $\exists^{\leq 2} x.\mathrm{Cat}(x)$ equivalent to

      $\forall x, y, z.\mathrm{Cat}(x) \wedge \mathrm{Cat}(y) \wedge \mathrm{Cat}(z) \rightarrow x = y \vee x = z \vee y = z$

  – Propositional modal and description logics
  – Guarded fragment

Back to our Scheduled Program

DL Syntax

- Signature
  - **Concept** (aka class) names, e.g., Cat, Animal, Doctor
    - Equivalent to FOL unary predicates
  - **Role** (aka property) names, e.g., sits-on, hasParent, loves
    - Equivalent to FOL binary predicates
  - **Individual** names, e.g., Felix, John, Mary, Boston, Italy
    - Equivalent to FOL constants

DL Syntax

- Operators
  - Many kinds available, e.g.,
    - Standard FOL Boolean operators ($\sqcap$, $\sqcup$, $\neg$)
    - Restricted form of quantifiers ($\exists$, $\forall$)
    - Counting ($\geq$, $\leq$, $=$)
    - …

DL Syntax

- Concept expressions, e.g.,
  - Doctor $\sqcup$ Lawyer
  - Rich $\sqcap$ Happy
  - Cat $\sqcap$ $\exists$sits-on.Mat
- Equivalent to FOL formulae with one free variable
  - $\mathrm{Doctor}(x) \vee \mathrm{Lawyer}(x)$
  - $\mathrm{Rich}(x) \wedge \mathrm{Happy}(x)$
  - $\exists y.(\mathrm{Cat}(x) \wedge \text{sits-on}(x, y))$

DL Syntax

- Special concepts
  - $\top$ (aka top, Thing, most general concept)
  - $\bot$ (aka bottom, Nothing, inconsistent concept)

  used as abbreviations for
  - $(A \sqcup \neg A)$ for any concept A
  - $(A \sqcap \neg A)$ for any concept A

DL Syntax

- Role expressions, e.g.,
  - loves^-
  - $\text{hasParent} \circ \text{hasBrother}$
- Equivalent to FOL formulae with two free variables
  - $\text{loves}(y, x)$
  - $\exists z.(\text{hasParent}(x, z) \wedge \text{hasBrother}(z, y))$

DL Syntax

- "Schema" Axioms, e.g.,
  - $\text{Rich} \sqsubseteq \neg\text{Poor}$ (concept inclusion)
  - $\text{Cat} \sqcap \exists\text{sits-on.Mat} \sqsubseteq \text{Happy}$ (concept inclusion)
  - $\text{BlackCat} \equiv \text{Cat} \sqcap \exists\text{hasColour.Black}$ (concept equivalence)
  - $\text{sits-on} \sqsubseteq \text{touches}$ (role inclusion)
  - Trans(part-of) (transitivity)
- Equivalent to (particular form of) FOL sentence, e.g.,
  - $\forall x.(\text{Rich}(x) \rightarrow \neg\text{Poor}(x))$
  - $\forall x.(\text{Cat}(x) \wedge \exists y.(\text{sits-on}(x,y) \wedge \text{Mat}(y)) \rightarrow \text{Happy}(x))$
  - $\forall x.(\text{BlackCat}(x) \leftrightarrow (\text{Cat}(x) \wedge \exists y.(\text{hasColour}(x,y) \wedge \text{Black}(y)))$
  - $\forall x,y.(\text{sits-on}(x,y) \rightarrow \text{touches}(x,y))$
  - $\forall x,y,z.((\text{sits-on}(x,y) \wedge \text{sits-on}(y,z)) \rightarrow \text{sits-on}(x,z))$

DL Syntax

- "Data" Axioms (aka Assertions or Facts), e.g.,
  - BlackCat(Felix) (concept assertion)
  - Mat(Mat1) (concept assertion)
  - Sits-on(Felix,Mat1) (role assertion)
- Directly equivalent to FOL "ground facts"
  - Formulae with no variables

DL Syntax

- A set of axioms is called a TBox, e.g.:

$$\{\text{Doctor} \sqsubseteq \text{Person},$$
$$\text{Parent} \equiv \text{Person} \sqcap \exists\text{hasChild.Pers}$$
$$\text{HappyParent} \equiv \text{Parent} \sqcap \forall\text{hasChil}$$

- A set of facts is called an AE

$$\{\text{HappyParent(John)},$$
$$\text{hasChild(John,Mary)}\}$$

Note

Facts sometimes written
John:HappyParent,
John hasChild Mary,
$\langle$John,Mary$\rangle$:hasChild

- A Knowledge Base (KB) is just a TBox plus an Abox
  - Often written $\mathcal{K} = \langle\mathcal{T}, \mathcal{A}\rangle$

The DL Family

- Many different DLs, often with “strange” names
  - E.g., $\mathcal{EL}$, $\mathcal{ALC}$, $\mathcal{SHIQ}$
- Particular DL defined by:
  - Concept operators ($\sqcap$, $\sqcup$, $\neg$, $\exists$, $\forall$, etc.)
  - Role operators ($^-$, $\circ$, etc.)
  - Concept axioms ($\sqsubseteq$, $\equiv$, etc.)
  - Role axioms ($\sqsubseteq$, Trans, etc.)

The DL Family

- E.g., $\mathcal{EL}$ is a well known "sub-Boolean" DL
  - Concept operators: $\sqcap$, $\neg$, $\exists$
  - No role operators (only atomic roles)
  - Concept axioms: $\sqsubseteq$, $\equiv$
  - No role axioms
- E.g.:

$$\text{Parent} \equiv \text{Person} \sqcap \exists \text{hasChild.Person}$$

The DL Family

- $\mathcal{ALC}$ is the smallest propositionally closed DL
  - Concept operators: $\sqcap$, $\sqcup$, $\neg$, $\exists$, $\forall$
  - No role operators (only atomic roles)
  - Concept axioms: $\sqsubseteq$, $\equiv$
  - No role axioms
- E.g.:

$$\text{ProudParent} \equiv \text{Person} \sqcap \forall \text{hasChild}.(\text{Doctor} \sqcup \exists \text{hasChild}.\text{Doctor})$$

The DL Family

- $\mathcal{S}$ used for $\mathcal{ALC}$ extended with (role) transitivity axioms
- Additional letters indicate various extensions, e.g.:
  - $\mathcal{H}$ for role hierarchy (e.g., hasDaughter $\sqsubseteq$ hasChild)
  - $\mathcal{R}$ for role box (e.g., $\mathrm{hasParent} \circ \mathrm{hasBrother} \sqsubseteq$ hasUncle)
  - $\mathcal{O}$ for nominals/singleton classes (e.g., {Italy})
  - $\mathcal{I}$ for inverse roles (e.g., isChildOf $\equiv$ hasChild^{-})
  - $\mathcal{N}$ for number restrictions (e.g., $\geqslant 2$hasChild, $\leqslant 3$hasChild)
  - $\mathcal{Q}$ for qualified number restrictions (e.g., $\geqslant 2$hasChild.Doctor)
  - $\mathcal{F}$ for functional number restrictions (e.g., $\leqslant 1$hasMother)
- E.g., $\mathcal{SHIQ}$ = $\mathcal{S}$ + role hierarchy + inverse roles + QNRs

The DL Family

- Numerous other extensions have been investigated
  - Concrete domains (numbers, strings, etc)
  - DL-safe rules (Datalog-like rules)
  - Fixpoints
  - Role value maps
  - Additional role constructors ($\sqcap$, $\sqcup$, $\neg$, $\circ$, id, …)
  - Nary (i.e., predicates with arity >2)
  - Temporal
  - Fuzzy
  - Probabilistic
  - Non-monotonic
  - Higher-order
  - …

DL Semantics

Via translaton to FOL, or directly using FO model theory:

Interpretation function $\mathcal{I}$ Interpretation domain $\Delta^{\mathcal{I}}$

Individuals $i^{\mathcal{I}} \in \Delta^{\mathcal{I}}$

John

Mary

Concepts $C^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}}$

Lawyer

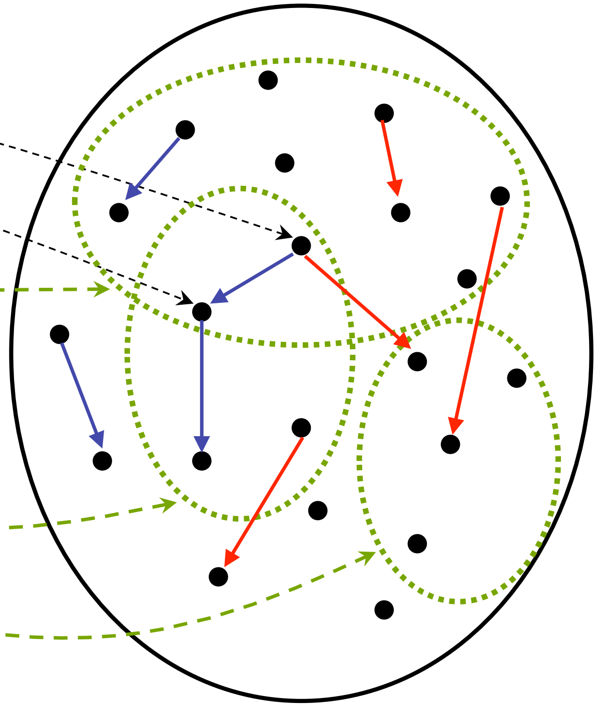
Doctor

Vehicle

Roles $r^{\mathcal{I}} \subseteq \Delta^{\mathcal{I}} \times \Delta^{\mathcal{I}}$

hasChild

owns



DL Semantics

- Interpretation function extends to **concept expressions** in the obvious(ish) way, e.g.:

$$
\begin{aligned}
(C \sqcap D)^{\mathcal{I}} &= C^{\mathcal{I}} \cap D^{\mathcal{I}} \\
(C \sqcup D)^{\mathcal{I}} &= C^{\mathcal{I}} \cup D^{\mathcal{I}} \\
(\neg C)^{\mathcal{I}} &= \Delta^{\mathcal{I}} \setminus C^{\mathcal{I}} \\
\{x\}^{\mathcal{I}} &= \{x^{\mathcal{I}}\} \\
(\exists R.C)^{\mathcal{I}} &= \{x \mid \exists y.\langle x, y\rangle \in R^{\mathcal{I}} \wedge y \in C^{\mathcal{I}}\} \\
(\forall R.C)^{\mathcal{I}} &= \{x \mid \forall y.(x, y) \in R^{\mathcal{I}} \Rightarrow y \in C^{\mathcal{I}}\} \\
(\leqslant n R)^{\mathcal{I}} &= \{x \mid \#\{y \mid \langle x, y\rangle \in R^{\mathcal{I}}\} \leqslant n\} \\
(\geqslant n R)^{\mathcal{I}} &= \{x \mid \#\{y \mid \langle x, y\rangle \in R^{\mathcal{I}}\} \geqslant n\}
\end{aligned}
$$

DL Semantics

- Given a model $\mathrm{M} = \langle D, \cdot^I \rangle$
  - $M \models C \sqsubseteq D$ iff $C^I \subseteq D^I$
  - $M \models C \equiv D$ iff $C^I = D^I$
  - $M \models C(a)$ iff $a^I \in C^I$
  - $M \models R(a, b)$ iff $\langle a^I, b^I \rangle \in R^I$
  - $M \models \langle \mathcal{T}, \mathcal{A} \rangle$ iff for every axiom $ax \in \mathcal{T} \cup \mathcal{A}$, $M \models ax$

DL Semantics

- Satisfiability and entailment
  - A KB $\mathcal{K}$ is satisfiable iff there exists a model M s.t. $\mathrm{M} \models \mathcal{K}$
  - A concept C is satisfiable w.r.t. a KB $\mathcal{K}$ iff there exists a model $\mathrm{M} = \langle \mathrm{D}, \cdot^{\mathrm{I}} \rangle$ s.t. $\mathrm{M} \models \mathcal{K}$ and $\mathrm{C}^{\mathrm{I}} \neq \emptyset$
  - A KB $\mathcal{K}$ entails an axiom ax (written $\mathcal{K} \models \mathrm{ax}$) iff for every model M of $\mathcal{K}$, $\mathrm{M} \models \mathrm{ax}$ (i.e., $\mathrm{M} \models \mathcal{K}$ implies $\mathrm{M} \models \mathrm{ax}$)

DL Semantics

E.g.,

$\mathcal{T}$ = {Doctor $\sqsubseteq$ Person, Parent $\equiv$ Person $\sqcap$ $\exists$hasChild.Person,
HappyParent $\equiv$ Parent $\sqcap$ $\forall$hasChild.(Doctor $\sqcup$ $\exists$hasChild.Doctor)}

$\mathcal{A}$ = {John:HappyParent, John hasChild Mary, John hasChild Sally,
Mary:$\neg$Doctor, Mary hasChild Peter, Mary:($\leq$ 1 hasChild)}

- ✓ – $\mathcal{K} \models$ John:Person ?
- ✓ – $\mathcal{K} \models$ Peter:Doctor ?
- ✓ – $\mathcal{K} \models$ Mary:HappyParent ?
- – What if we add "Mary hasChild Jane" ?

  $\mathcal{K} \models$ Peter = Jane
- – What if we add "HappyPerson $\equiv$ Person $\sqcap$ $\exists$hasChild.Doctor" ?

  $\mathcal{K} \models$ HappyPerson $\sqsubseteq$ Parent

DL and FOL

- Most DLs are subsets of C2
  - But reduction to C2 may be (highly) non-trivial
    - Trans(R) naively reduces to $\forall x, y, z . R(x, y) \wedge R(y, z) \rightarrow R(x, z)$
- Why use DL instead of C2?
  - Syntax is succinct and convenient for KR applications
  - Syntactic conformance guarantees being inside C2
    - Even if reduction to C2 is non-obvious
  - Different combinations of constructors can be selected
    - To guarantee decidability
    - To reduce complexity
  - DL research has mapped out the decidability/complexity landscape in great detail
    - See Evgeny Zolin's DL Complexity Analyzer http://www.cs.man.ac.uk/~ezolin/dl/

Complexity of reasoning in Description Logics

Note: the information here is (always) incomplete and updated often

Base description logic: $\mathcal{A}$ttributive $\mathcal{L}$anguage with $\mathcal{C}$omplements

$\mathcal{ALC}$ ::= $\bot$ | A | $\neg C$ | $C \wedge D$ | $C \vee D$ | $\exists R.C$ | $\forall R.C$

Concept constructors:	**Role constructors:** trans reg
☐ $\mathcal{F}$ – functionality[2]: $(\leq 1\ R)$ ☑ $\mathcal{N}$ – (unqualified) number restrictions: $(\geq n\ R)$, $(\leq n\ R)$ ☐ $\mathcal{Q}$ – qualified number restrictions: $(\geq n\ R.C)$, $(\leq n\ R.C)$ ☑ $\mathcal{O}$ – nominals: $\{a\}$ or $\{a_1,\ldots,a_n\}$ ("one-of" constructor) ☐ μ – least fixpoint operator: $\mu X.C$ ☐ $R \subseteq S$ – role-value-maps ☐ $f = g$ – agreement of functional role chains ("same-as")	☑ $\mathcal{I}$ – role inverses: R^- ☐ $\cap$ – role intersection[3]: $R \cap S$ ☐ $\cup$ – role union: $R \cup S$ ☐ $\neg$ – role complement: full ☐ $\circ$ – role chain (composition): $R \circ S$ ☐ $*$ – reflexive-transitive closure[4]: R^* ☐ id – concept identity: $id(C)$ Forbid complex roles[5] in number restrictions[6]
TBox is *internalized* in extensions of $\mathcal{ALCIO}$, see [76, Lemma 4.12], [54, p.3] ◉ Empty TBox ○ Acyclic TBox ($A \equiv C$, A is a concept name; no cycles) ○ General TBox ($C \sqsubseteq D$ for arbitrary concepts C and D)	**Role axioms (RBox):** OWL-Lite OWL-DL OWL 1.1 ☑ $\mathcal{S}$ – Role transitivity: Trans(R) ☑ $\mathcal{H}$ – Role hierarchy: $R \sqsubseteq S$ ☐ $\mathcal{R}$ – Complex role inclusions: $R \circ S \sqsubseteq R$, $R \circ S \sqsubseteq S$ ☐ s – some additional features

Reset You have selected the Description Logic: $\mathcal{SHOIN}$

Complexity of reasoning problems[7]

Reasoning problem	**Complexity**[8]	**Comments and references**
Concept satisfiability	**NExpTime-complete**	• Hardness of even $\mathcal{ALCFIO}$ is proved in [76, Corollary 4.13]. In that paper, the result is formulated for $\mathcal{ALCQIO}$, but only number restrictions of the form $(\leq 1 R)$ are used in the proof. • A different proof of the NExpTime-hardness for $\mathcal{ALCFIO}$ is given in [54] (even with 1 nominal, and role inverses not used in number restrictions). • Upper bound for $\mathcal{SHOIQ}$ is proved in [77, Corollary 6.31] with numbers coded in unary (for binary coding, the upper bound remains an open problem for all logics in between $\mathcal{ALCNIO}$ and $\mathcal{SHOIQ}$). • **Important:** in number restrictions, only *simple* roles (i.e. which are neither transitive nor have a transitive subroles) are allowed; otherwise we gain undecidability even in $\mathcal{SHN}$; see [46]. • **Remark:** recently [47] it was observed that, in many cases, one can use transitive roles in number restrictions – and still have a decidable logic! So the above notion of a *simple* role could be substantially extended.
ABox consistency	**NExpTime-complete**	By reduction to concept satisfiability problem in presence of nominals shown in [69, Theorem 3.7].

Complexity Measures

- Taxonomic complexity

  Measured w.r.t. total size of “schema” axioms

- Data complexity

  Measured w.r.t. total size of “data” facts

- Query complexity

  Measured w.r.t. size of query

- Combined complexity

  Measured w.r.t. total size of KB (plus query if appropriate)

Complexity Classes

- LogSpace, PTime, NP, PSpace, ExpTime, etc
  - worst case for a given problem w.r.t. a given parameter
  - X-hard means at-least this hard (could be harder); in X means no harder than this (could be easier); X-complete means both hard and in, i.e., exactly this hard
    - e.g., $\mathcal{SROIQ}$ KB satisfiability is 2NExpTime-complete w.r.t. combined complexity and NP-hard w.r.t. data complexity
- Note that:
  - this is for the worst case, not a typical case
  - complexity of problem means we can never devise a more efficient (in the worst case) algorithm
  - complexity of algorithm may, however, be even higher (in the worst case)

DLs and Ontology Languages

DLs and Ontology Languages

- W3C's OWL 2 (like OWL, DAML+OIL & OIL) based on DL
  - OWL 2 based on $\mathcal{SROIQ}$, i.e., $\mathcal{ALC}$ extended with transitive roles, a role box nominals, inverse roles and qualified number restrictions
    - OWL 2 EL based on $\mathcal{EL}$
    - OWL 2 QL based on DL-Lite
    - OWL 2 EL based on $\mathcal{DLP}$
  - OWL was based on $\mathcal{SHOIN}$
    - only simple role hierarchy, and unqualified NRs

Class/Concept Constructors

OWL Constructor	DL Syntax	Example
intersectionOf	$C_1 \sqcap \ldots \sqcap C_n$	Human $\sqcap$ Male
unionOf	$C_1 \sqcup \ldots \sqcup C_n$	Doctor $\sqcup$ Lawyer
complementOf	$\neg C$	$\neg$Male
oneOf	$\{x_1\} \sqcup \ldots \sqcup \{x_n\}$	{john} $\sqcup$ {mary}
allValuesFrom	$\forall P.C$	$\forall$hasChild.Doctor
someValuesFrom	$\exists P.C$	$\exists$hasChild.Lawyer
maxCardinality	$\leqslant nP$	$\leqslant$1hasChild
minCardinality	$\geqslant nP$	$\geqslant$2hasChild

Ontology Axioms

OWL Syntax	DL Syntax	Example
subClassOf	$C_1 \sqsubseteq C_2$	Human $\sqsubseteq$ Animal $\sqcap$ Biped
equivalentClass	$C_1 \equiv C_2$	Man $\equiv$ Human $\sqcap$ Male
subPropertyOf	$P_1 \sqsubseteq P_2$	hasDaughter $\sqsubseteq$ hasChild
equivalentProperty	$P_1 \equiv P_2$	cost $\equiv$ price
transitiveProperty	$P^+ \sqsubseteq P$	ancestor$^+$ $\sqsubseteq$ ancestor

OWL Syntax	DL Syntax	Example
type	$a : C$	John : Happy-Father
property	$\langle a, b\rangle : R$	$\langle$John, Mary$\rangle$: has-child

- An Ontology is *usually* considered to be a TBox
  – but an OWL ontology is a mixed set of TBox and ABox axioms

Other OWL Features

- XSD datatypes and (in OWL 2) facets, e.g.,
  - integer, string and (in OWL 2) real, float, decimal, datetime, …
  - minExclusive, maxExclusive, length, …
  - PropertyAssertion(hasAge Meg "17"^^xsd:integer)
  - DatatypeRestriction(xsd:integer xsd:minInclusive "5"^^xsd:integer xsd:maxExclusive "10"^^xsd:integer)

  These are equivalent to (a limited form of) **DL concrete domains**

- Keys
  - E.g., HasKey(Vehicle Country LicensePlate)
    - Country + License Plate is a unique identifier for vehicles

  This is equivalent to (a limited form of) **DL safe rules**

OWL RDF/XML Exchange Syntax

E.g., Person $\sqcap$ $\forall$hasChild.(Doctor $\sqcup$ $\exists$hasChild.Doctor):

```
<owl:Class>
  <owl:intersectionOf rdf:parseType=" collection">
    <owl:Class rdf:about="#Person"/>
    <owl:Restriction>
      <owl:onProperty rdf:resource="#hasChild"/>
      <owl:allValuesFrom>
        <owl:unionOf rdf:parseType=" collection">
          <owl:Class rdf:about="#Doctor"/>
          <owl:Restriction>
            <owl:onProperty rdf:resource="#hasChild"/>
            <owl:someValuesFrom rdf:resource="#Doctor"/>
          </owl:Restriction>
        </owl:unionOf>
      </owl:allValuesFrom>
    </owl:Restriction>
  </owl:intersectionOf>
</owl:Class>
```

Complexity/Scalability

- From the complexity navigator we can see that:
  - OWL (aka $\mathcal{SHOIN}$) is NExpTime-complete
  - OWL Lite (aka $\mathcal{SHIF}$) is ExpTime-complete (oops!)
  - OWL 2 (aka $\mathcal{SROIQ}$) is 2NExpTime-complete
  - OWL 2 EL (aka $\mathcal{EL}$) is PTIME-complete (robustly scalable)
  - OWL 2 RL (aka $\mathcal{DLP}$) is PTIME-complete (robustly scalable)
    - And implementable using rule based technologies e.g., rule-extended DBs
  - OWL 2 QL (aka DL-Lite) is in AC^0 w.r.t. size of data
    - same as DB query answering -- nice!

Why (Description) Logic?

- OWL exploits results of 20+ years of DL research
  - Well defined (model theoretic) **semantics**

Constructor	DL Syntax	Example	FOL Syntax
intersectionOf	$C_1 \sqcap \ldots \sqcap C_n$	Human $\sqcap$ Male	$C_1(x) \wedge \ldots \wedge C_n(x)$
unionOf	$C_1 \sqcup \ldots \sqcup C_n$	Doctor $\sqcup$ Lawyer	$C_1(x) \vee \ldots \vee C_n(x)$
complementOf	$\neg C$	$\neg$Male	$\neg C(x)$
oneOf	$\{x_1\} \sqcup \ldots \sqcup \{x_n\}$	{john} $\sqcup$ {mary}	$x = x_1 \vee \ldots \vee x = x_n$
allValuesFrom	$\forall P.C$	$\forall$hasChild.Doctor	$\forall y.P(x,y) \rightarrow C(y)$
someValuesFrom	$\exists P.C$	$\exists$hasChild.Lawyer	$\exists y.P(x,y) \wedge C(y)$
maxCardinality	$\leqslant nP$	$\leqslant$1hasChild	$\exists^{\leqslant n} y.P(x,y)$
minCardinality	$\geqslant nP$	$\geqslant$2hasChild	$\exists^{\geqslant n} y.P(x,y)$

Why (Description) Logic?

- OWL exploits results of 20+ years of DL research
  - Well defined (model theoretic) **semantics**
  - **Formal properties** well understood (complexity, decidability)

I can't find an efficient algorithm, but neither can all these famous people.

[Garey & Johnson. Computers and Intractability: A Guide to the Theory of NP-Completeness. Freeman, 1979.]

Why (Description) Logic?

- OWL exploits results of 20+ years of DL research
  - Well defined (model theoretic) **semantics**
  - **Formal properties** well understood (complexity, decidability)
  - Known **reasoning algorithms**

$\sqcap$-rule	if 1. $(C_1 \sqcap C_2) \in \mathcal{L}(v)$, v is not indirectly blocked, and 2. $\{C_1, C_2\} \not\subseteq \mathcal{L}(v)$ then $\mathcal{L}(v) \longrightarrow \mathcal{L}(v) \cup \{C_1, C_2\}$.
$\sqcup$-rule	if 1. $(C_1 \sqcup C_2) \in \mathcal{L}(v)$, v is not indirectly blocked, and 2. $\{C_1, C_2\} \cap \mathcal{L}(v) = \emptyset$ then $\mathcal{L}(v) \longrightarrow \mathcal{L}(v) \cup \{E\}$ for some $E \in \{C_1, C_2\}$
$\exists$-rule	if 1. $\exists r.C \in \mathcal{L}(v_1)$, v_1 is not blocked, and 2. v_1 has no safe r-neighbour v_2 with $C \in \mathcal{L}(v_1)$, then create a new node v_2 and an edge $\langle v_1, v_2 \rangle$ with $\mathcal{L}(v_2) = \{C\}$ and $\mathcal{L}(\langle v_1, v_2 \rangle) = \{r\}$.
$\forall$-rule	if 1. $\forall r.C \in \mathcal{L}(v_1)$, v_1 is not indirectly blocked, and 2. there is an r-neighbour v_2 of v_1 with $C \notin \mathcal{L}(v_2)$ then $\mathcal{L}(v_2) \longrightarrow \mathcal{L}(v_2) \cup \{C\}$.
$\forall_+$-rule	if 1. $\forall r.C \in \mathcal{L}(v_1)$, v_1 is not indirectly blocked, and 2. there is some role r' with $\mathsf{Trans}(r')$ and $r' \sqsubseteq^* r$ 3. there is an r'-neighbour v_2 of v_1 with $\forall r'.C \notin \mathcal{L}(v_2)$ then $\mathcal{L}(v_2) \longrightarrow \mathcal{L}(v_2) \cup \{\forall r'.C\}$.
choose-rule	if 1. $\leqslant n\, r.C \in \mathcal{L}(v_1)$, v_1 is not indirectly blocked, and 2. there is an r-neighbour v_2 of v_1 with $\{C, \dot{\neg} C\} \cap \mathcal{L}(v_2) = \emptyset$ then $\mathcal{L}(v_2) \longrightarrow \mathcal{L}(v_2) \cup \{E\}$ for some $E \in \{C, \dot{\neg} C\}$.
$\geqslant$-rule	if 1. $\geqslant n\, r.C \in \mathcal{L}(v)$, v is not blocked, and 2. there are not n safe r-neighbours $v_1, \ldots, v_n$ of v with $C \in \mathcal{L}(v_i)$ and $v_i \neq v_j$ for $1 \leqslant i < j \leqslant n$

Why (Description) Logic?

- OWL exploits results of 20+ years of DL research
  - Well defined (model theoretic) **semantics**
  - **Formal properties** well understood (complexity, decidability)
  - Known **reasoning algorithms**
  - **Scalability** demonstrated by **implemented systems**

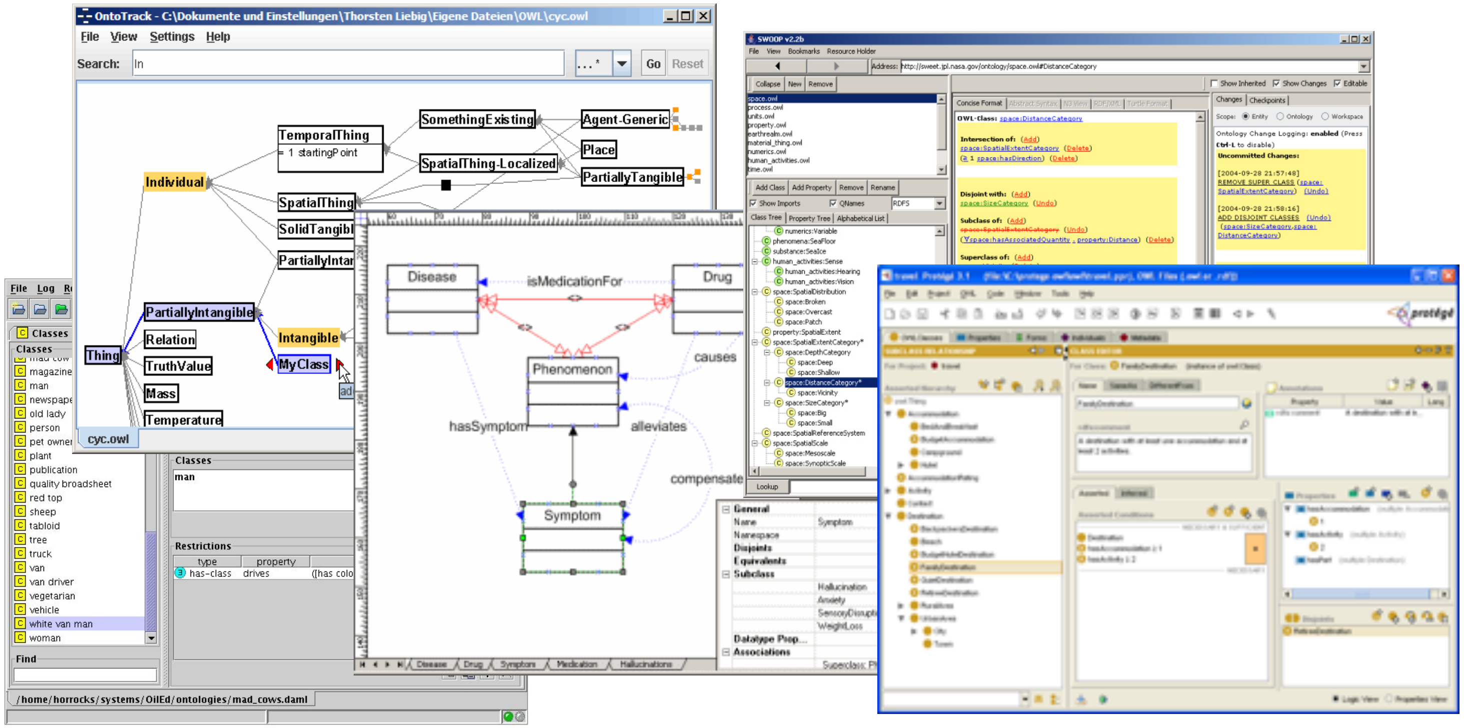
Tools, Tools, Tools

Major benefit of OWL has been huge increase in range and sophistication of tools and infrastructure:

Tools, Tools, Tools

Major benefit of OWL has been huge increase in range and sophistication of tools and infrastructure:

- Editors/development environments



Tools, Tools, Tools

Major benefit of OWL has been huge increase in range and sophistication of tools and infrastructure:

- Editors/development environments
- Reasoners

Hermit

FaCT++

Racer

MINDSWAP Pellet

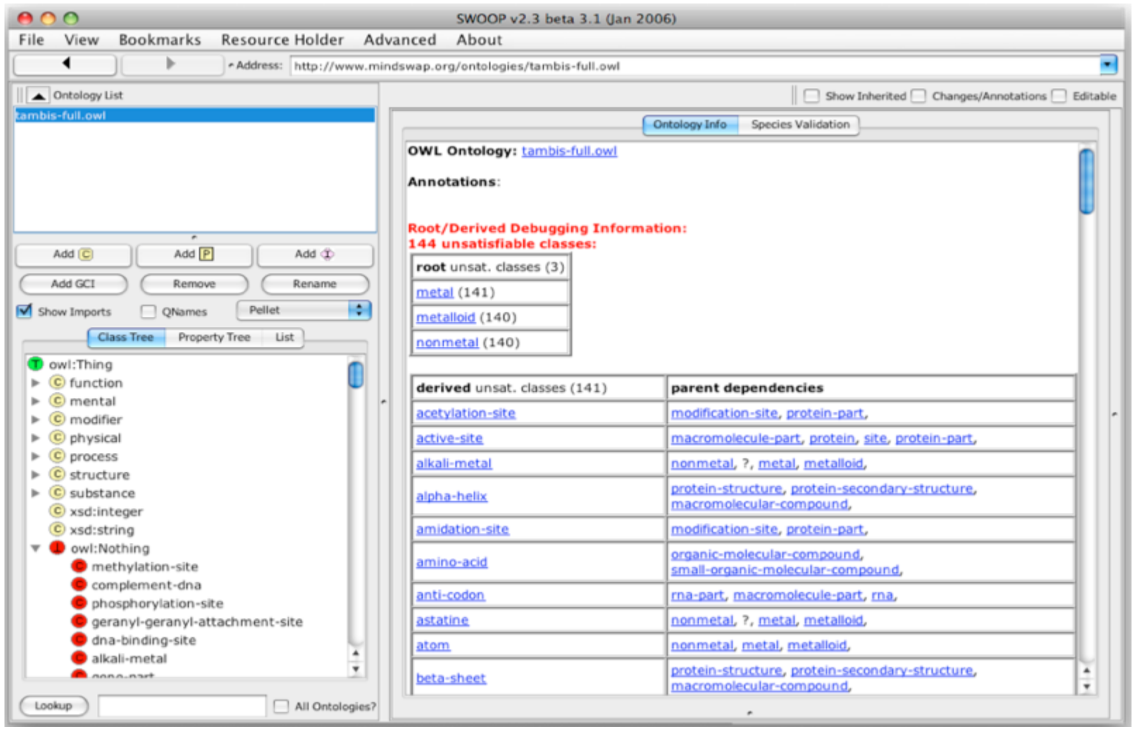
KAON2

CEL

Tools, Tools, Tools

Major benefit of OWL has been huge increase in range and sophistication of tools and infrastructure:

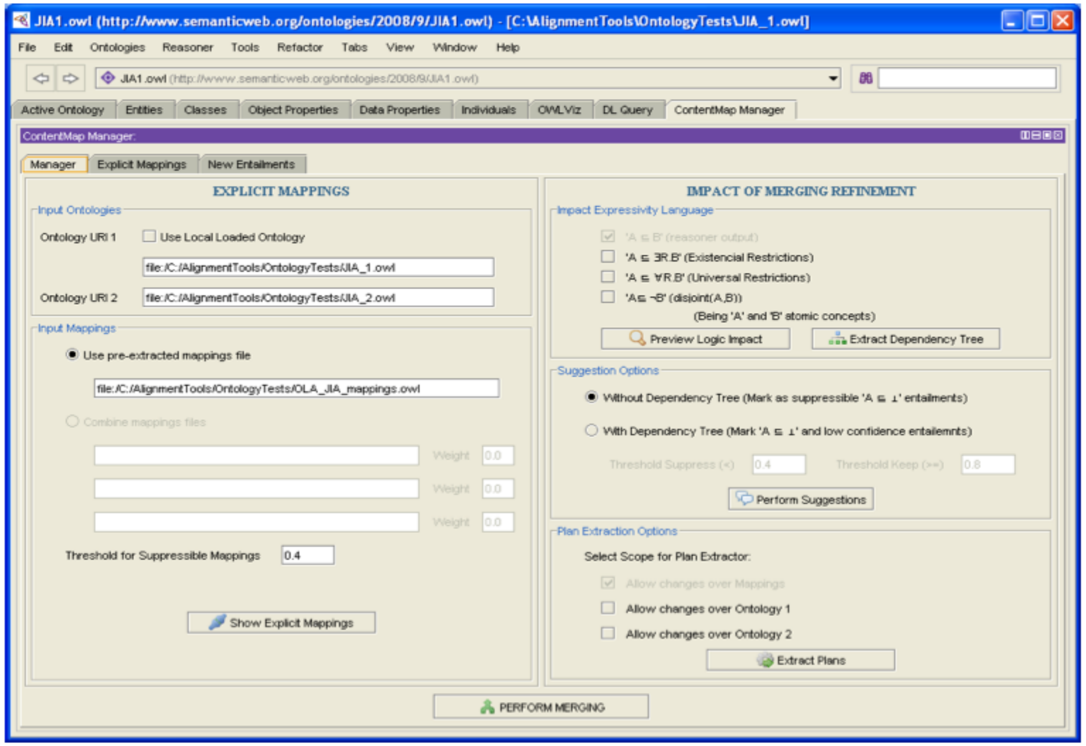
- Editors/development environments
- Reasoners
- Explanation, justification and pinpointing



Tools, Tools, Tools

Major benefit of OWL has been huge increase in range and sophistication of tools and infrastructure:

- Editors/development environments
- Reasoners
- Explanation, justification and pinpointing
- Integration and modularisation



Tools, Tools, Tools

Major benefit of OWL has been huge increase in range and sophistication of tools and infrastructure:

- Editors/development environments
- Reasoners
- Explanation, justification and pinpointing
- Integration and modularisation

Revision 1403 - (download) (annotate)
Fri Dec 18 17:14:37 2009 UTC (4 months, 2 weeks ago) by matthewhorridge
File size: 4711 byte(s)

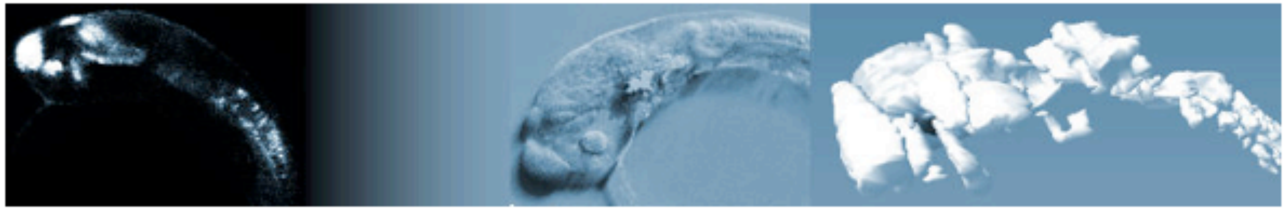
```
package org.coode.owlapi.examples;

import org.semanticweb.owlapi.apibinding.OWLManager;
import org.semanticweb.owlapi.model.*;
import org.semanticweb.owlapi.util.DefaultPrefixManager;
/*
 * Copyright (C) 2009, University of Manchester
 *
 * Modifications to the initial code base are copyright of their
 * respective authors, or their employers as appropriate.  Authorship
 * of the modifications may be determined from the ChangeLog placed at
 * the end of this file.
 *
 * This library is free software; you can redistribute it and/or
 * modify it under the terms of the GNU Lesser General Public
 * License as published by the Free Software Foundation; either
 * version 2.1 of the License, or (at your option) any later version.

 * This library is distributed in the hope that it will be useful,
 * but WITHOUT ANY WARRANTY; without even the implied warranty of
 * MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the GNU
 * Lesser General Public License for more details.
```

- APIs, in particular the [OWL API]

Motivating Applications

- OWL playing **key role** in increasing number & range of applications
  - eScience

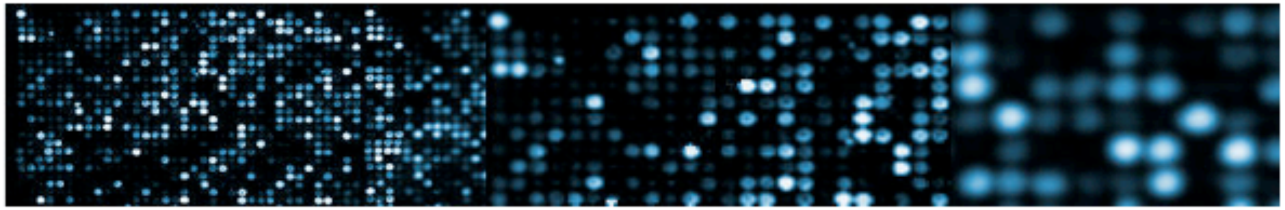
3D Analysis of Patterns of Gene Expression



Ontology of Zebrafish Developmental Anatomy

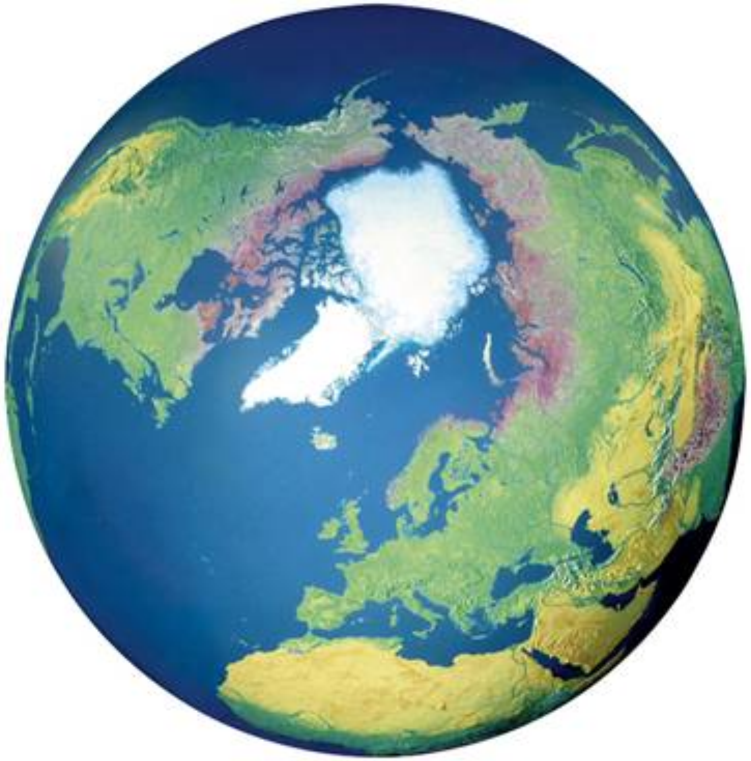
trigeminal (V) ganglion	20 somite	...Head	Periphiral Nervous Systen
Rohon-Beard neurons	20 somite	...Head	Central Nervous System
primary motorneurons	20 somite	...Head	Central Nervous System
primary neurons	20 somite	...Head	Central Nervous System
brain	14 somite	...Head	Central Nervous System
hindbrain	14 somite	...Head	Central Nervous System
midbrain	14 somite	...Head	Central Nervous System
forebrain	14 somite	...Head	Central Nervous System
ear	20 somite	...Head	Auditory
eye	14 somite	...Head	Visual

Integration of Heterogeneous gene expression data



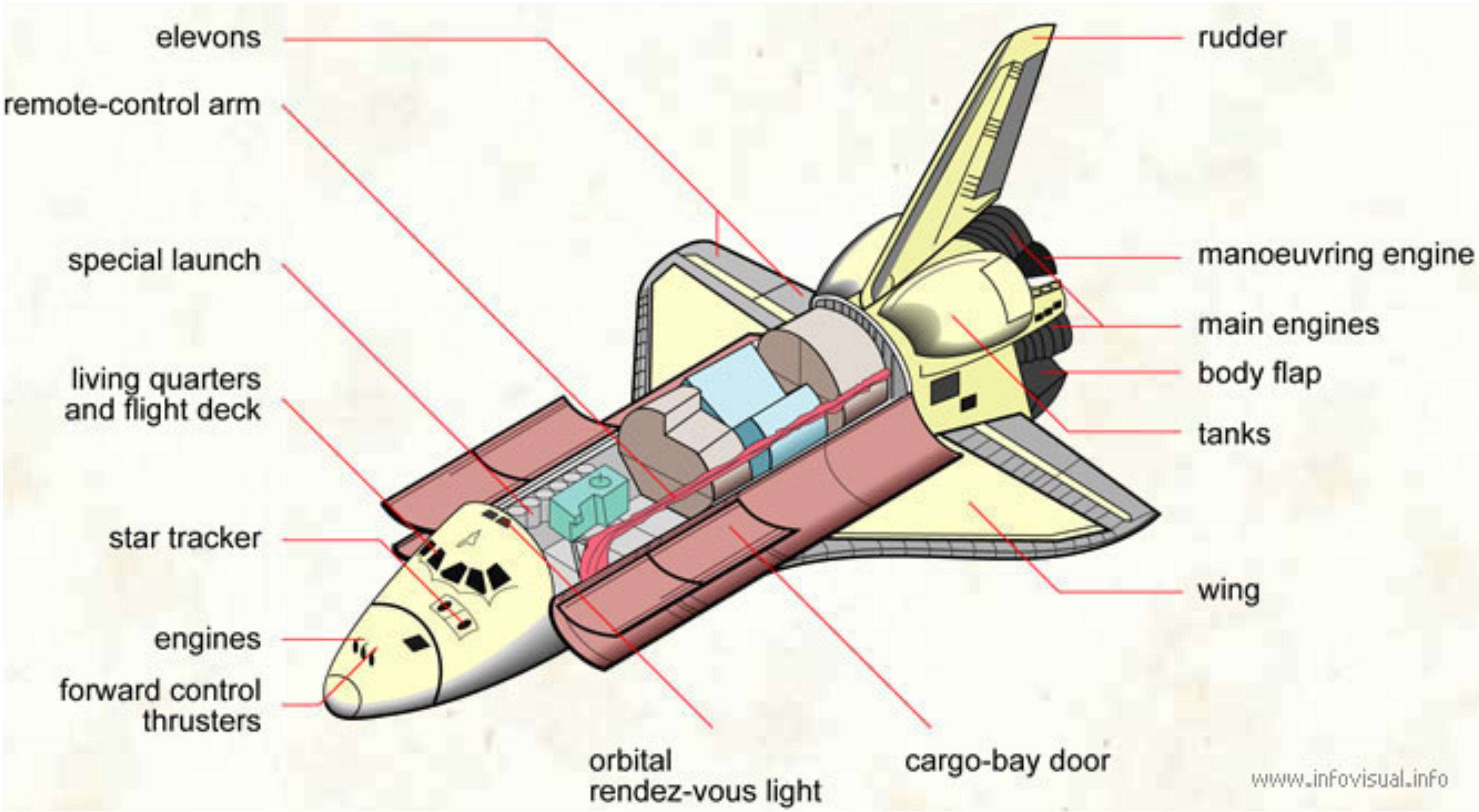
Motivating Applications

- OWL playing **key role** in increasing number & range of applications
  - eScience, geography



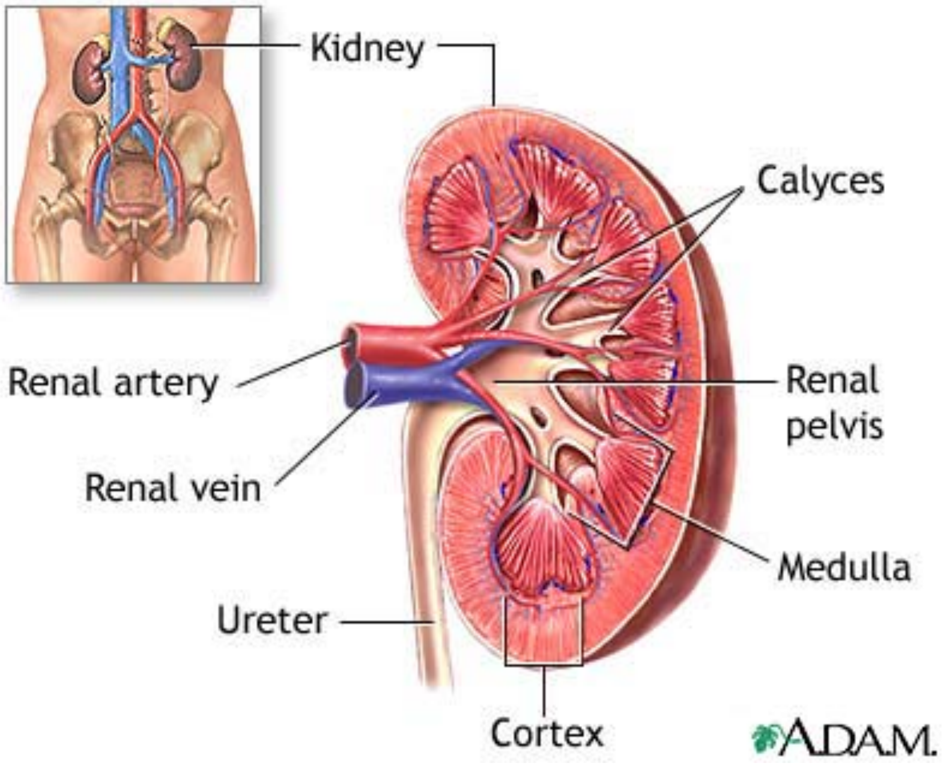
Motivating Applications

- OWL playing **key role** in increasing number & range of applications
  - eScience, geography, engineering,



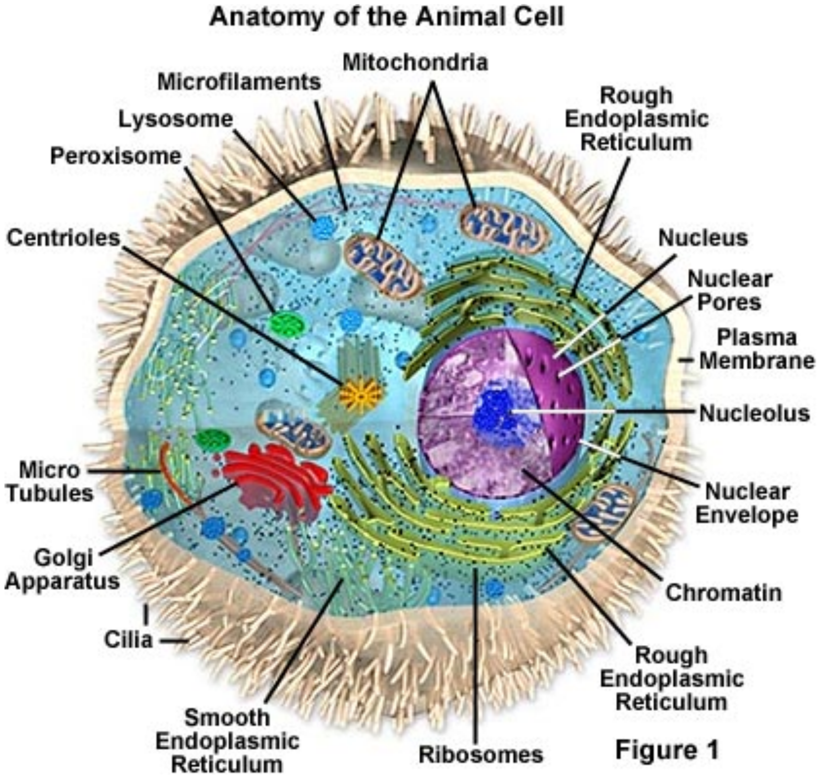
Motivating Applications

- OWL playing **key role** in increasing number & range of applications
  - eScience, geography, engineering, medicine



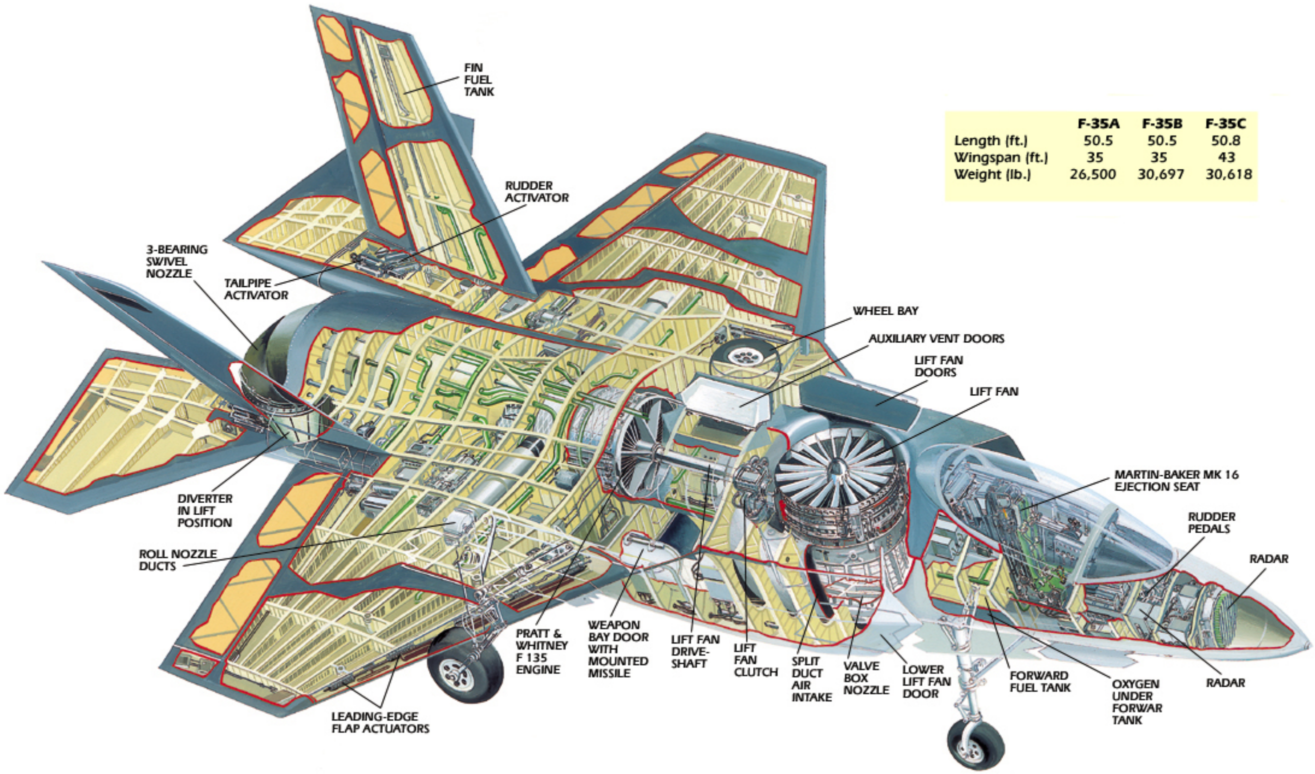
Motivating Applications

- OWL playing **key role** in increasing number & range of applications
  - eScience, geography, engineering, medicine, biology



Motivating Applications

- OWL playing **key role** in increasing number & range of applications
  - eScience, geography, engineering, medicine, biology, defence, …



	F-35A	F-35B	F-35C
Length (ft.)	50.5	50.5	50.8
Wingspan (ft.)	35	35	43
Weight (lb.)	26,500	30,697	30,618

NHS ~~£6.2~~ £12 Billion IT Programme

Key component is "Care Records Service"

- "Live, interactive patient record service accessible 24/7"
- Patient data **distributed** across local and national DBs
  - Diverse applications support radiology, pharmacy, etc
  - Applications exchange "semantically rich clinical information"
  - Summaries sent to national database
- SNOMED-CT ontology provides clinical **vocabulary**
  - Data uses terms drawn from ontology
  - New terms with well defined meaning can be added "on the fly"

Ontology -v- Database

Obvious Database Analogy

- Ontology axioms analogous to DB **schema**
  - Schema describes structure of and constraints on data
- Ontology facts analogous to DB **data**
  - Instantiates schema
  - Consistent with schema constraints
- But there are also important differences…

Obvious Database Analogy

Database:

- Closed world assumption (**CWA**)
  - Missing information treated as false
- Unique name assumption (**UNA**)
  - Each individual has a single, unique name
- Schema behaves as **constraints** on structure of data
  - Define legal database states

Ontology:

- Open world assumption (**OWA**)
  - Missing information treated as unknown
- **No UNA**
  - Individuals may have more than one name
- Ontology axioms behave like **implications** (inference rules)
  - Entail implicit information

Database -v- Ontology

E.g., given the following **ontology/schema**:

$\text{HogwartsStudent} \equiv \text{Student} \sqcap \exists\,\text{attendsSchool.Hogwarts}$

$\text{HogwartsStudent} \sqsubseteq \forall\text{hasPet.(Owl or Cat or Toad)}$

$\text{hasPet} \equiv \text{isPetOf}^{-}$ (i.e., hasPet inverse of isPetOf)

$\exists\text{hasPet}.\top \sqsubseteq \text{Human}$ (i.e., domain of hasPet is Human)

$\text{Phoenix} \sqsubseteq \forall\text{isPetOf.Wizard}$ (i.e., only Wizards have Phoenix pets)

$\text{Muggle} \sqsubseteq \neg\text{Wizard}$ (i.e., Muggles and Wizards are disjoint)

Database -v- Ontology

And the following **facts/data**:

HarryPotter: Wizard
DracoMalfoy: Wizard
HarryPotter hasFriend RonWeasley
HarryPotter hasFriend HermioneGranger
HarryPotter hasPet Hedwig

Query: Is Draco Malfoy a friend of HarryPotter?

- DB: No
- Ontology: Don't Know

  OWA (didn't say Draco was not Harry's friend)

Database -v- Ontology

And the following **facts/data**:

HarryPotter: Wizard
DracoMalfoy: Wizard
HarryPotter hasFriend RonWeasley
HarryPotter hasFriend HermioneGranger
HarryPotter hasPet Hedwig

Query: How many friends does Harry Potter have?

- DB: 2
- Ontology: at least 1

  No UNA (Ron and Hermione may be 2 names for same person)

Database -v- Ontology

And the following **facts/data**:

HarryPotter: Wizard
DracoMalfoy: Wizard
HarryPotter hasFriend RonWeasley
HarryPotter hasFriend HermioneGranger
HarryPotter hasPet Hedwig

➔ **RonWeasley ≠ HermioneGranger**

Query: How many friends does Harry Potter have?

- DB: 2
- Ontology: at least 2

  OWA (Harry may have more friends we didn't mention yet)

Database -v- Ontology

And the following **facts/data**:

HarryPotter: Wizard
DracoMalfoy: Wizard
HarryPotter hasFriend RonWeasley
HarryPotter hasFriend HermioneGranger
HarryPotter hasPet Hedwig

RonWeasley $\neq$ HermioneGranger

➔ **HarryPotter: $\forall$hasFriend.{RonWeasley} $\sqcup$ {HermioneGranger}**

Query: How many friends does Harry Potter have?

– DB: 2

– Ontology: 2!

Database -v- Ontology

Inserting new facts/data:

Dumbledore: Wizard
Fawkes: Phoenix
Fawkes isPetOf Dumbledore

$$\exists \text{hasPet}.\top \sqsubseteq \text{Human}$$
$$\text{Phoenix} \sqsubseteq \forall \text{isPetOf}.\text{Wizard}$$

What is the response from DBMS?

– Update rejected: **constraint violation**

Domain of hasPet is Human; Dumbledore is not Human (CWA)

What is the response from Ontology reasoner?

– **Infer** that Dumbledore is Human (domain restriction)

– Also infer that Dumbledore is a Wizard (only a Wizard can have a pheonix as a pet)

DB Query Answering

- Schema plays **no role**
  - Data must explicitly satisfy schema constraints
- Query answering amounts to **model checking**
  - I.e., a “look-up” against the data
- Can be very **efficiently implemented**
  - Worst case complexity is low (logspace) w.r.t. size of data

Ontology Query Answering

- Ontology axioms play a powerful and **crucial role**
  - Answer may include implicitly derived facts
  - Can answer conceptual as well as extensional queries
    - E.g., Can a Muggle have a Phoenix for a pet?
- Query answering amounts to **theorem proving**
  - I.e., logical entailment
- May have very **high worst case complexity**
  - E.g., for OWL, NP-hard w.r.t. size of data
    (upper bound is an open problem)
  - Implementations may still behave well in typical cases
  - Fragments/profiles may have much better complexity

Ontology Based Information Systems

- Analogous to **relational database management systems**
  - – Ontology ≈ schema; instances ≈ data
- Some important (**dis**)**advantages**
  - \+ (Relatively) easy to maintain and update schema
    - Schema plus data are integrated in a logical theory
  - \+ Query answers reflect both schema and data
  - \+ Can deal with incomplete information
  - \+ Able to answer both intensional and extensional queries
  - – Semantics can seem counter-intuitive, particularly w.r.t. data
    - Open -v- closed world; axioms -v- constraints
  - – Query answering (logical entailment) may be much more difficult
    - Can lead to scalability problems with expressive logics

Ontology Based Information Systems

- Analogous to **relational d** **anagement systems**
  - Ontology ≈ schem
- Some important
  - \+ (Relatively)
    - Schema
  - \+ Query ans
  - \+ Can deal
  - \+ Able to an eries
  - – Semantics w.r.t. data
    - Open -v- c
  - – Query answering much more difficult
    - Can lead to scalab ssive logics