Research Project

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This article provides an overview of the initial problem that motivated my D.Phil. research.

1 Project Description

The official title of my research project is 'Numerical Analysis for the Multiple Extrusion of Pastes'.

This project is being carried out in the Oxford University Computing Laboratory, in conjunction with Unilever Research, Port Sunlight. It is supported by an EPSRC industrial CASE award, (EPSRC/Smith Institute CASE Studentship) made available through the Faraday Partnership for Industrial Mathematics.

The aim of this Faraday Partnership project is to develop 2d and 3d finite element simulations of incompressible, slow, viscous multiple-extrusion flows, for shear-thinning power-law fluids such as toothpaste. The results will help to identify novel multiple extrusion opportunities.

Further examples of shear-thinning power-law fluids in the food and cosmetics industries are margarine, molten chocolate, jam, jelly, and cosmetic cold creams. The nonlinearities that are present in such materials mean that the simulation and prediction of flow phenomena in industrially relevant geometries rely on computational tools. This project will advance the quality and accuracy of existing numerical algorithms through the use of state-of-the-art adaptive techniques.

2 Technical Background

During the last five years, tremendous advances have been made in the area of adaptive computational algorithms. Through a feedback loop, an adaptive algorithm is capable of automatically adjusting the granularity of the computational grid to deliver an approximate solution to the problem under consideration to within a user-specified tolerance. The use of adaptive algorithms guarantees that the computational grid is refined only where necessary, so ensuring that computations are performed in an economical manner, leading to a reduction in computational cost while accurately capturing all of the relevant flow features.

The basic principle behind an adaptive algorithm that is driven by an a posteriori error bound is to insert the computed solution into the differential equation under consideration, in order to quantify the extent to which it fails to satisfy the equation. Decisions about grid-refinement are then taken by monitoring the local size of the associated residual error. The use of a posteriori error bounds makes it possible to validate the computational results against the mathematical model and then also against experimental data.

For shear-thinning power-law fluids with a low power-law exponent, such as toothpaste (for which the exponent is up to approximately 0.2), thin boundary layers appear in the flow, which may only be accurately captured by using adaptively refined computational meshes. A further computational challenge is to predict the interface between two or more pastes of different rheologies, in extrusions through tubes or nozzles.

Of particular interest in this project is the computational identification of multiple-extrusion flow regimes where these interfaces are stable and visually sharp.

Summary

Specifically, this work centres around the finite element analysis of non-Newtonian flows. Non-Newtonian flows in physical geometries are modeled by a system of nonlinear partial differential equations. These equations are numerically approximated by using the finite element method. We improve the overall accuracy and efficiency of the approximation procedure by using adaptivity. A useful adaptive routine requires a sharp and reliable error estimator, and so a posteriori error analysis of mixed finite element methods for non-Newtonian flow problems has been and will continue to be central to my work.