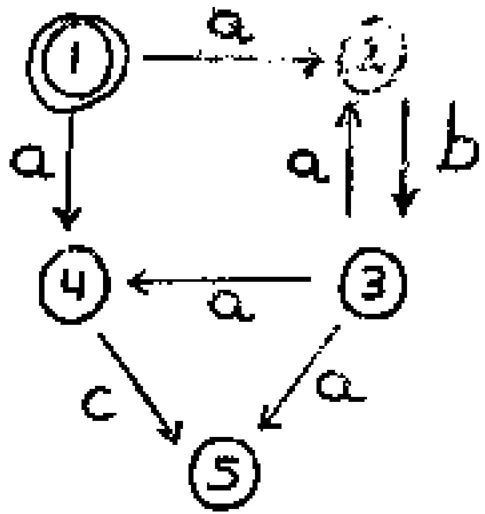
problem

given:
- edge-labelled directed graph
  vertex v
- regular expression P

compute: vertices w such that all paths $v \to w$ are in $L(P)$

graph G $v = 1$



pattern P

$(ab)^* a$

<u>division</u>

$$
\begin{tikzcd}
A & & C \arrow[ll, "R"'] \arrow[dl, "S"] \\
& B \arrow[ul, "R/S"] \arrow[u, phantom, "\rotatebox{90}{$\subseteq$}"] &
\end{tikzcd}
$$

$$X \subseteq R/S \quad \equiv \quad X \cdot S \subseteq R \;, \quad \text{all } X$$

$$a\,(R/S)\,b \quad \equiv \quad \forall c : bSc : aRc$$

automata & folds

$$\llparenthesis \text{init}, \text{step} \rrparenthesis \; [a_0, a_1, \ldots, a_{n-1}]$$

$$=$$

$$((\text{init} \;`\text{step}`\; a_0) \;`\text{step}`\; a_1) \;\ldots\; `\text{step}`\; a_{n-1}$$

$$\left.\begin{array}{l} \text{init} : S \leftarrow \mathbb{1} \\ \text{step} : S \leftarrow S \times A \end{array}\right\} \text{ "machine"}$$

specification

graph $(G_0 : V \leftarrow \mathbb{1},\ G_1 : V \leftarrow V \times A) = G$

pattern $(P_0 : S \leftarrow \mathbb{1},\ P_1 : S \leftarrow S \times A) = P$

$F : \mathbb{1} \leftarrow S$

compute

$$(F \cdot (\!|P|\!)) \,/\, (\!|G|\!) \quad : \quad \mathbb{1} \leftarrow V$$

derivation

$$(F \cdot [\![P]\!]) \,/\, [\![G]\!]$$

$$=$$

$$(F \cdot mem \cdot \Lambda[\![P]\!]) \,/\, [\![G]\!]$$

$$=$$

$$(F \cdot mem) \,/\, ([\![G]\!] \cdot (\Lambda[\![P]\!])^\circ)$$

$$=$$

$$(F \cdot mem) \,/\, (outl \cdot ran\langle [\![G]\!], \Lambda[\![P]\!] \rangle \cdot outr^\circ)$$

$$\begin{aligned}
& \mathrm{ran}\ \langle (\![G]\!),\ \Lambda (\![P]\!) \rangle \\
= \\
& \mathrm{ran}\ \langle (\![G]\!),\ (\![P']\!) \rangle \\
= \\
& \mathrm{ran}\ (\![GP']\!) \\
= \\
& \mathrm{ran}\ ((GP'_1 \cdot \mathit{all}^\circ)^* \cdot GP'_0)
\end{aligned}$$

what does it mean?!

- let P' be deterministic P
- take product automaton GP'
- compute reachable states in GP'

solution

$\{ v \mid \forall s : (v,s) \text{ reachable} : s \text{ final in } P' \}$