Process Theories and Graphical Language

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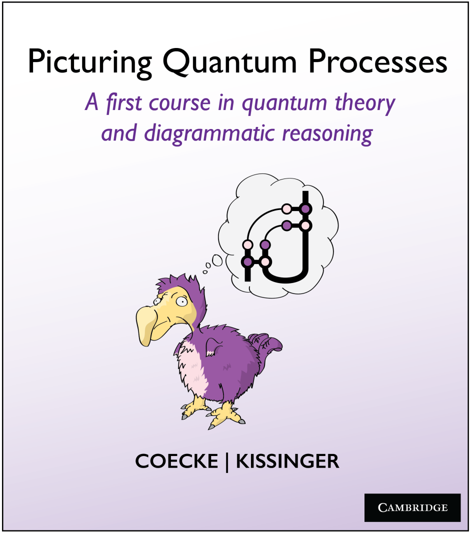
Picturing Quantum Processes
A first course in quantum theory
and diagrammatic reasoning
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Picturing Quantum Processes

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When two systems [...] enter into temporary physical interaction due to known forces between them, [...] then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. **I would not call that one but rather the characteristic trait of quantum mechanics**, the one that enforces its entire departure from classical lines of thought.

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In quantum theory, *interaction* of systems is everything. **Diagrams** are the language of interaction.

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A: Pretty much everything!

Outline

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Process theories and diagrams

Quantum processes

Classical and quantum interaction

Applications: a Hollywood-style trailer

Processes

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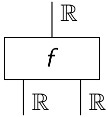

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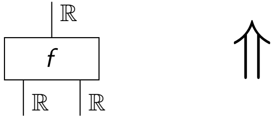

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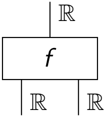

$\Uparrow$

- The labels on wires are called system-types or just types

More processes

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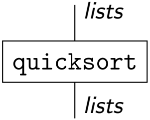

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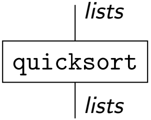
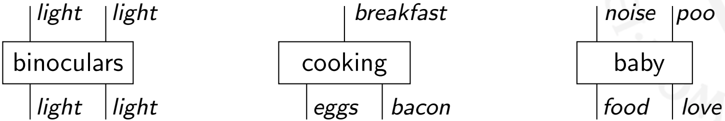
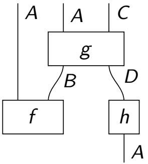


- These are also perfectly good processes:



Diagrams

- We can combine simple processes to make more complicted ones, described by diagrams:



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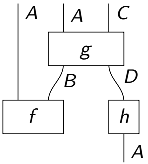
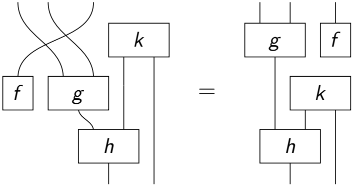


- The golden rule: only connectivity matters!



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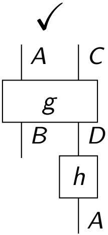

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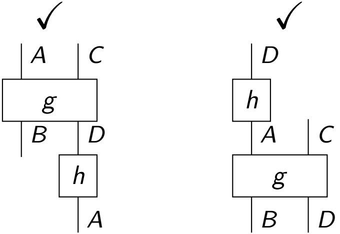

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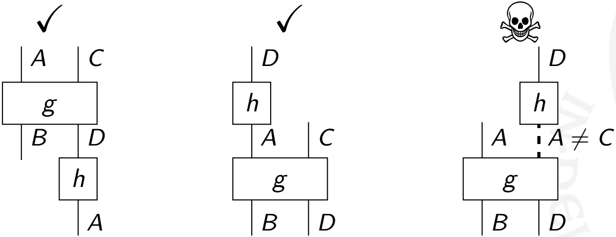

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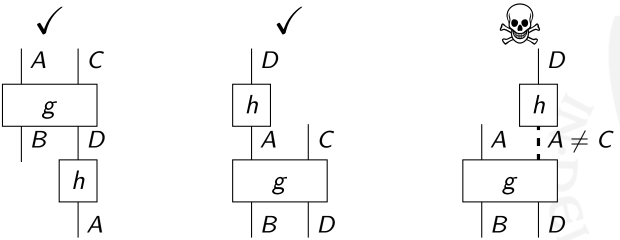
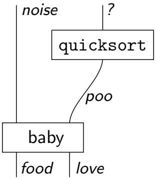


- Types tell us when it makes sense to plug processes together

Types and Process Theories

- Ill-typed diagrams are undefined:



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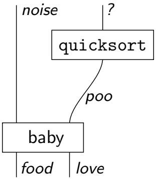

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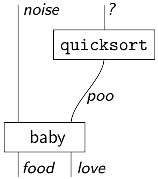

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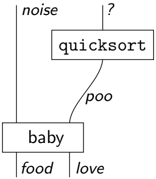
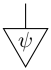


- In fact, these processes don't ever make sense to plug together
- A family of processes which do make sense together is called a process theory, e.g.
  - **functions**
  - **linear maps**
  - **optical devices**
  - **proofs**, ...

Special processes: states and effects

- Processes with no inputs are called states:



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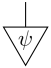

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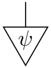

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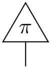

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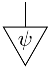

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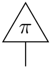

Interpret as: testing for a property π, where we don't care what happens after.

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or just: λ

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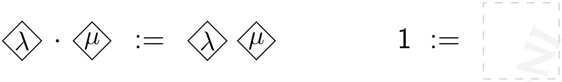

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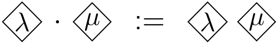

$1 :=$

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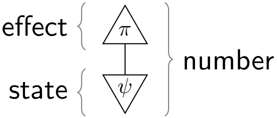

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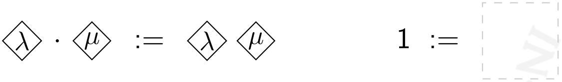

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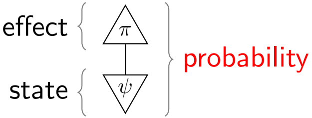

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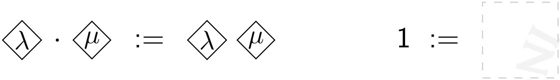

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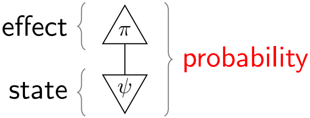

This is called the (generalised) Born rule

Process theories in general

Q: What kinds of behaviour can we study using just diagrams, and nothing else?

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A: (Non-)separability

Separable states

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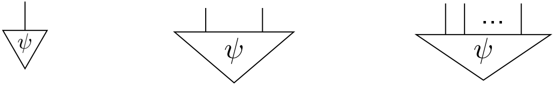

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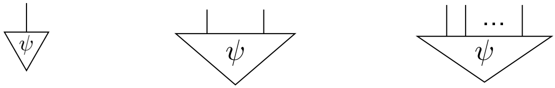
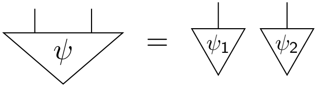


- A state ψ on two systems is $\otimes$-*separable* if there exist ψ_1, ψ_2 such that:



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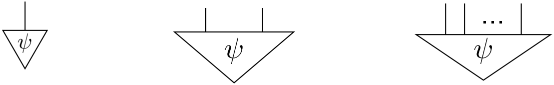

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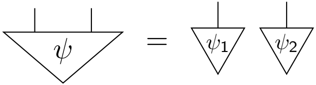

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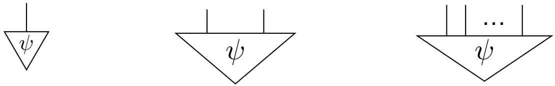

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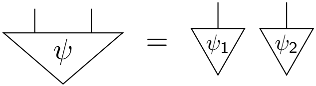

- **Intuitively:** the properties of the system on the left are *independent* from those on the right
- In classical (deterministic) world, we expect *all states* to $\otimes$-separate

Characterising non-separability

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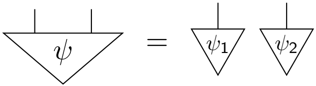

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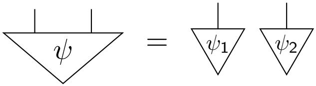

- **Problem:** Showing that something doesn't exist can be hard.

Characterising non-separability

Solution: Replace a negative property with a (stronger) postive one:

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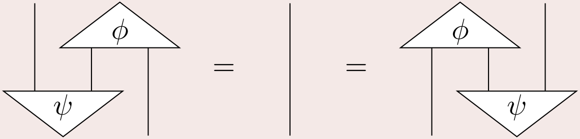
Definition

A state ψ is called *cup-state* if there exists an effect ϕ, called a *cap-effect*, such that:



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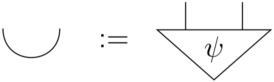

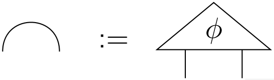

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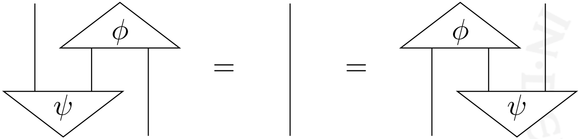

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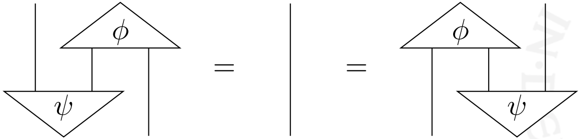

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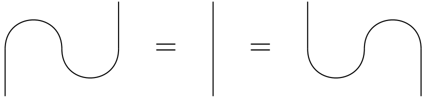

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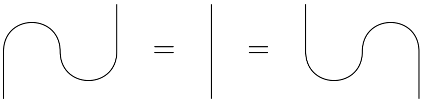

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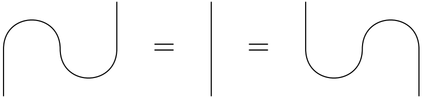

A no-go theorem for separability

Theorem

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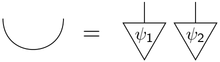

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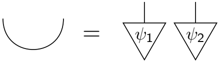

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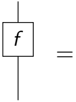

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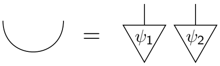

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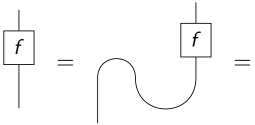

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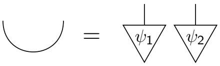
A no-go theorem for separability

Theorem

If a process theory (i) has cup-states for every type and (ii) every state separates, then it is trivial.

Proof. Suppose a cup-state separates:



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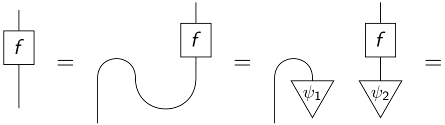

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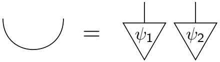

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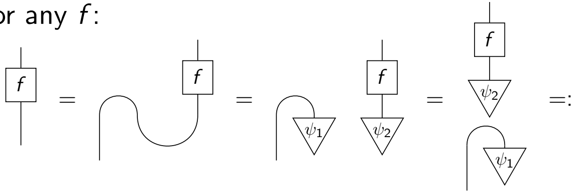

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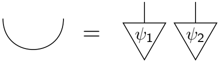

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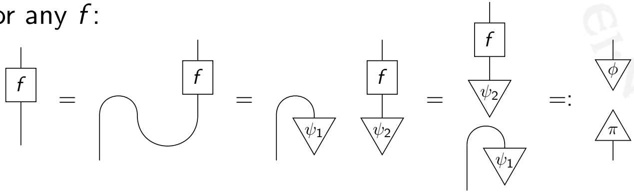
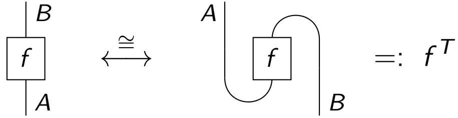


□

Transpose



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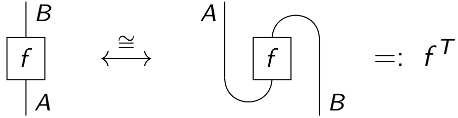
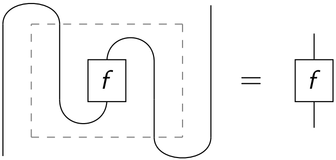


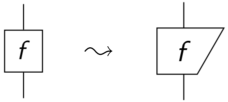


i.e.

$$(f^T)^T = f$$

Tranpose = rotation

A bit of a deformation:



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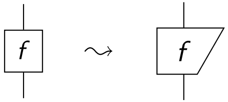
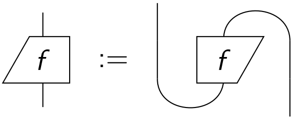


allows some clever notation:



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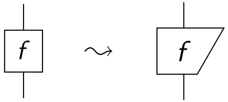

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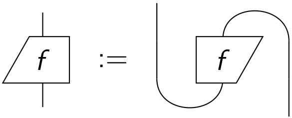

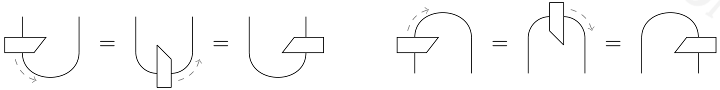

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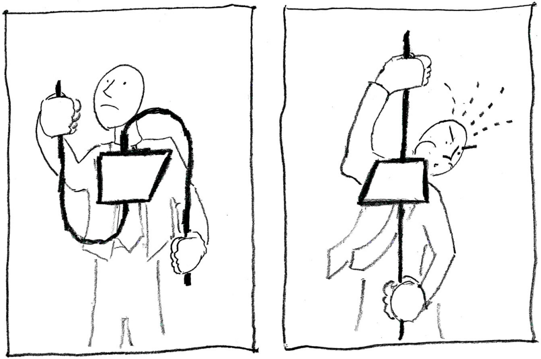
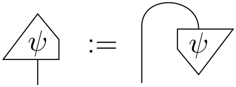


Tranpose = rotation

Specialised to states:



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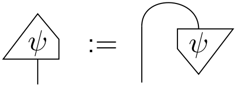

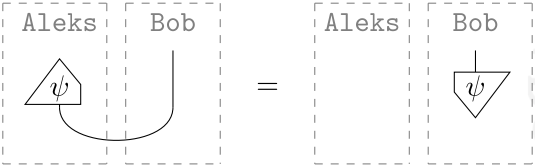

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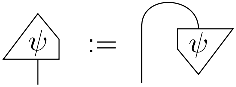
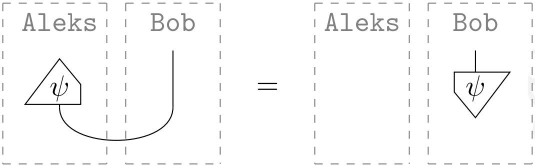
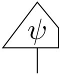


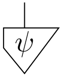


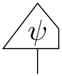
as soon as Aleks obtains 

Bob's system will be in state 

State/effect correspondence

states of system A $\cong$ *effects for correlated system B*





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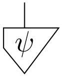

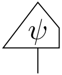

transpose

But what about...

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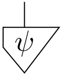

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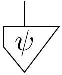

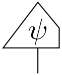

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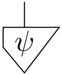
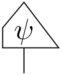
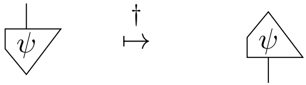




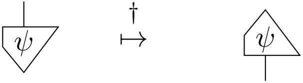
adjoint

Adjoints



state ψ **testing for** ψ

Adjoints



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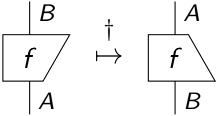
Extends from states/effects to all processes:



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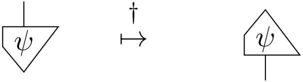

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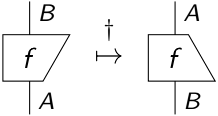

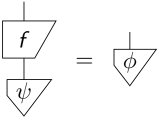

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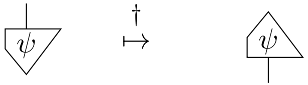

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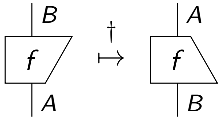
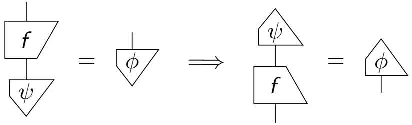
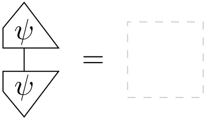




Normalised states and isometries

- Adjoints increase expressiveness, for instance can say when ψ is *normalised*:



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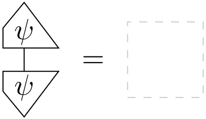
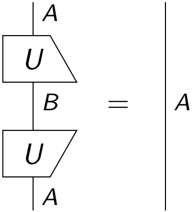


- U is an *isometry*:



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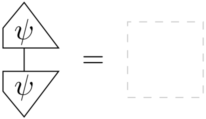

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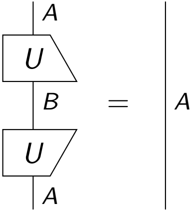
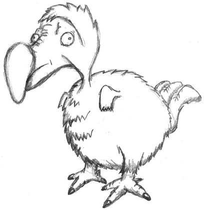


- ...and unitary, self-adjoint, positive, etc.

Conjugates

If we:



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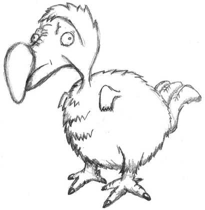
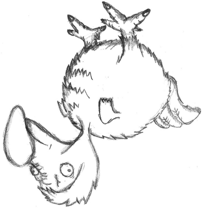


$\updownarrow$
$\mapsto$



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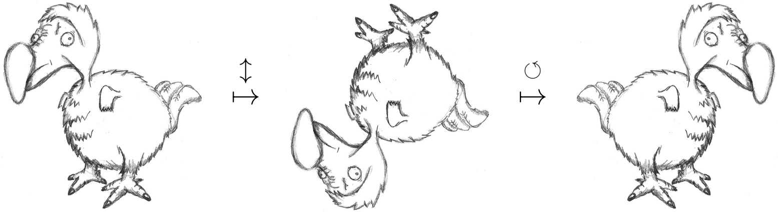

...we get horizontal reflection.

Conjugates

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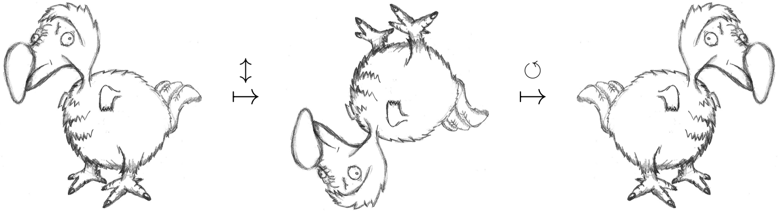
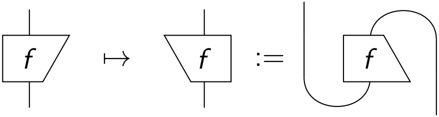
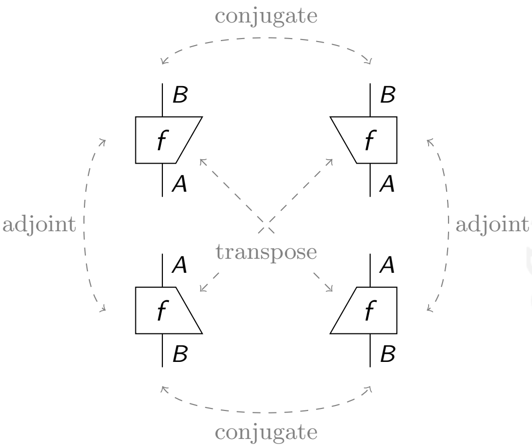


...we get horizontal reflection. The *conjugate*:



4 kinds of box



Quantum teleportation: take 1

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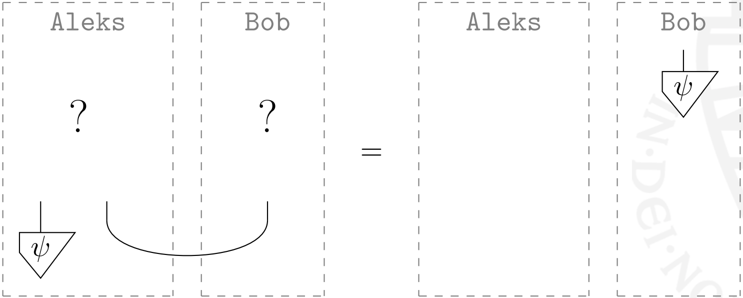
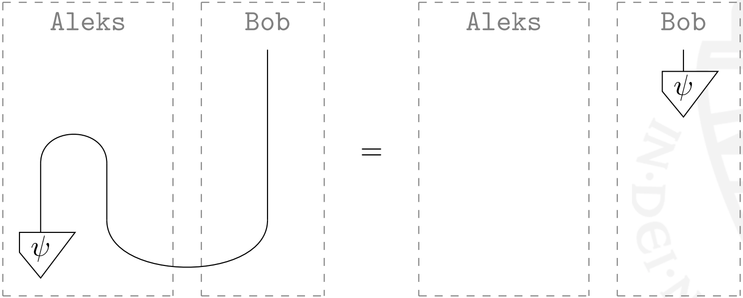
Can we fill in '?' to get this?



Quantum teleportation: take 1

Here's a simple solution:



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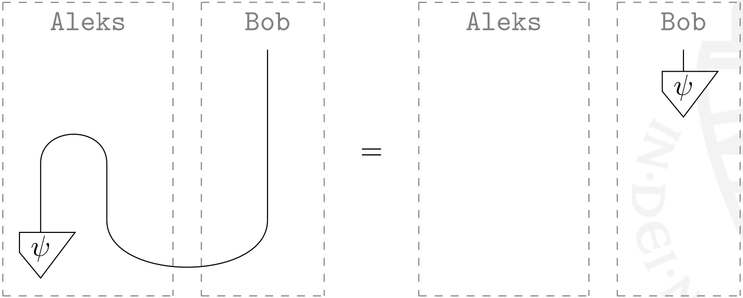
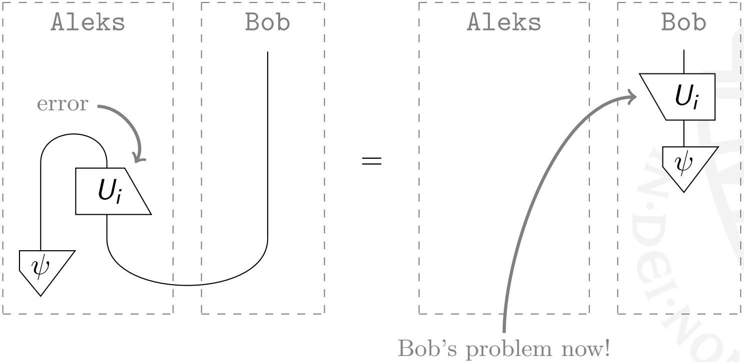


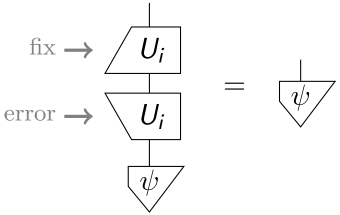
Problem: 'cap' can't be performed deterministically

Quantum teleportation: take 1



Quantum teleportation: take 1

Solution: Bob fixes the error.



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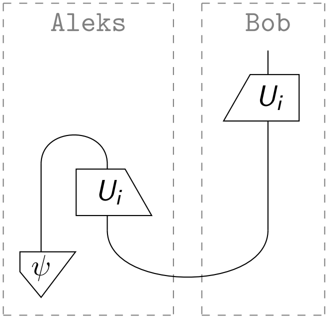

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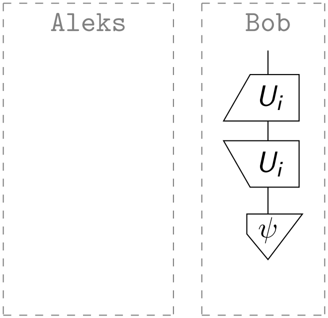
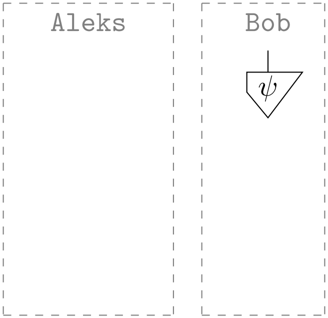


Quantum teleportation: take 1



Outline

Process theories and diagrams

Quantum processes

Classical and quantum interaction

Applications: a Hollywood-style trailer

Hilbert space

The starting point for quantum theory is the process theory of **linear maps**

Hilbert space

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1. **systems:** Hilbert spaces
2. **processes:** complex linear maps

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1. **systems:** Hilbert spaces
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...in particular, numbers are *complex numbers*.

Hilbert space

Looking at the ‘Born rule’ for **linear maps**, we have a problem:

Hilbert space

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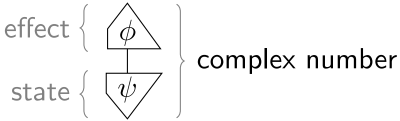

Hilbert space

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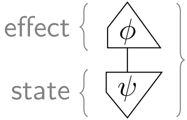
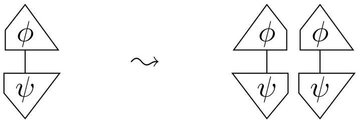


complex number $\neq$ probability!

Doubling

Solution: multiply by the conjugate:



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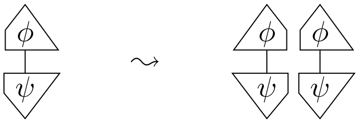
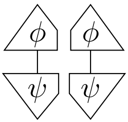


Then, for normalised ψ, ϕ:

$0 \leq$  ≤ 1

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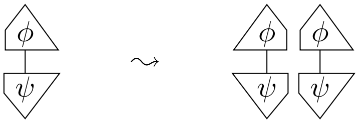

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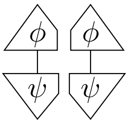
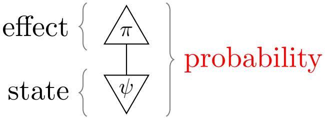
$0 \leq$  ≤ 1

(i.e. the 'usual' Born rule: $\overline{\langle\phi|\psi\rangle}\langle\phi|\psi\rangle = |\langle\phi|\psi\rangle|^2$)

Doubling

New problem: We lost this:



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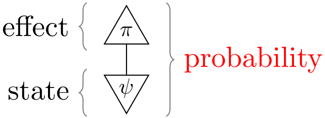

...which was the basis of our interpretation for states, effects, and numbers.

Doubling

Solution: Make a new process theory with doubling 'baked in':

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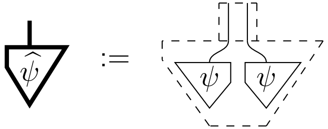

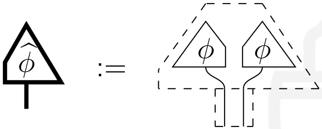

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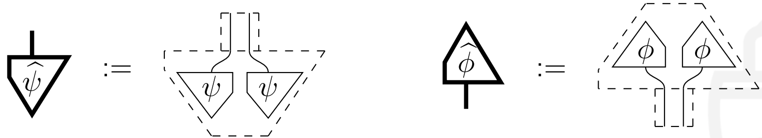
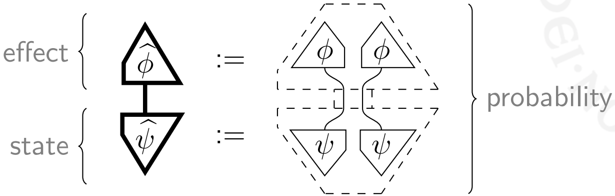
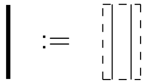


Then:



Doubling

The new process theory has doubled systems $\widehat{H} := H \otimes H$:



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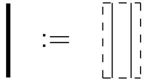
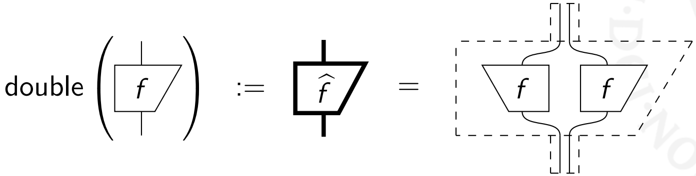
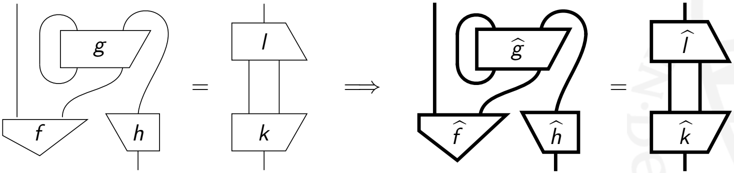


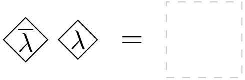
and processes:



Doubling preserves diagrams

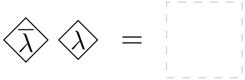


...but kills global phases



(i.e. $\lambda = e^{i\alpha}$)

...but kills global phases



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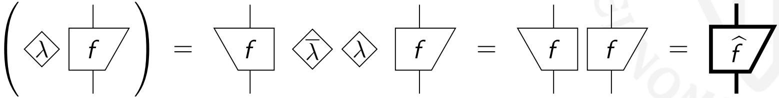
$$\Longrightarrow$$

double 

Discarding

Doubling also lets us do something we couldn't do before:

Discarding

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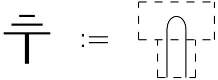
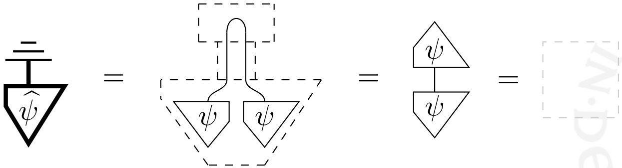
How? Like this:



Discarding

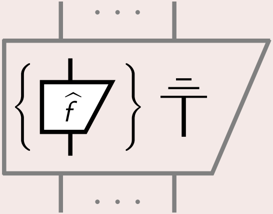
For normalised ψ, the two copies annihilate:



Quantum maps

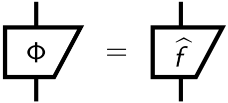
Definition

The process theory of **quantum maps** has as types (doubled) Hilbert spaces $\widehat{H}$ and as processes:



Two characterisations of ‘pure’

No discarding involved, i.e. for some f:



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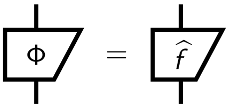
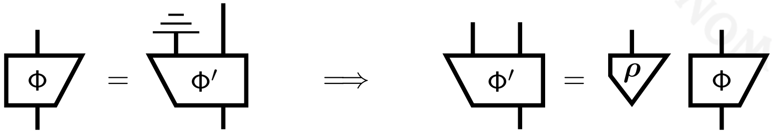


$$\Longleftrightarrow$$

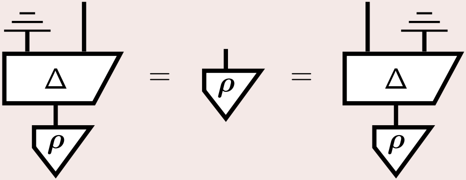
Any *extension* is trivial:



Consequence: no-broadcasting

Theorem (No universal broadcasting)

There exists no quantum map Δ *where:*



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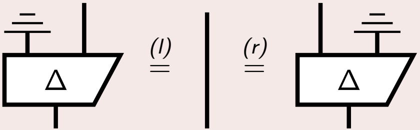

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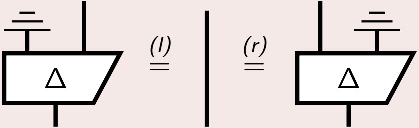
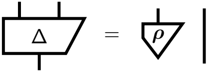


Proof. From (l):



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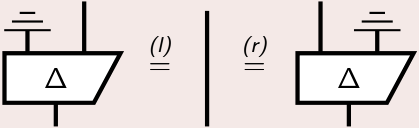

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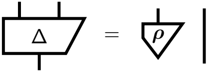
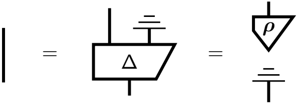


From (r):



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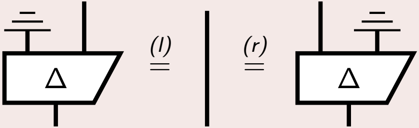

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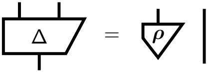

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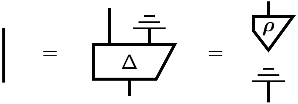

⇒ contradiction. □

Causality

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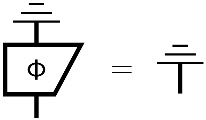
A quantum map is called *causal* if:



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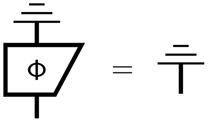

If we discard the output of a process,
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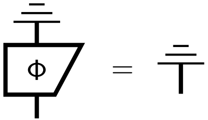

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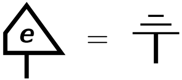
causal $\iff$ *deterministically physically realisable*

Consequence: no cap effect ☹

Consequence: there is a unique causal effect, discarding:



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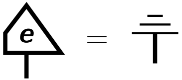
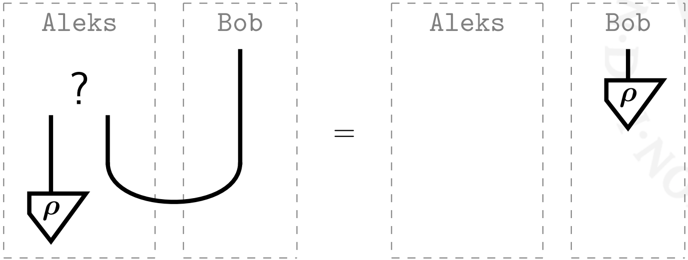


Hence 'deterministic quantum teleportation' must fail:



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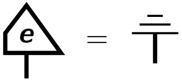

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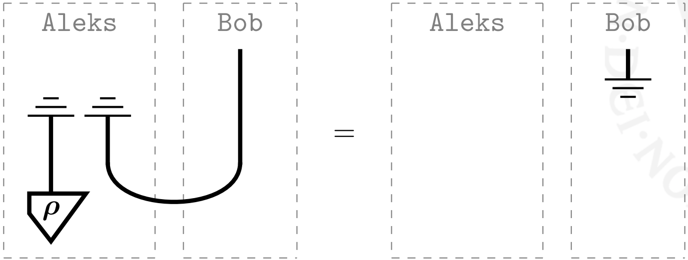
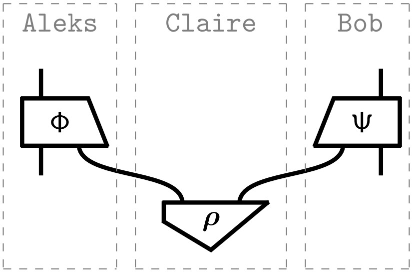


Consequence: no signalling ☺



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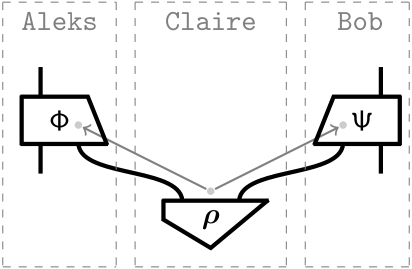

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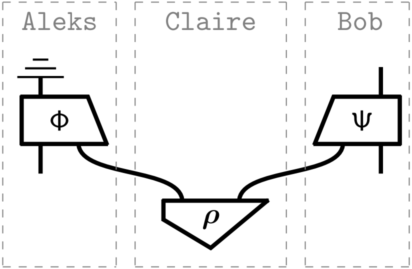

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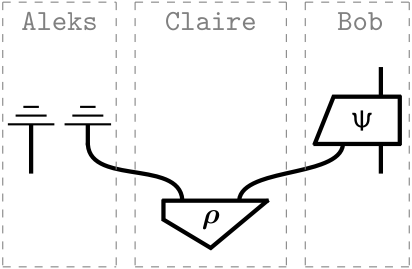

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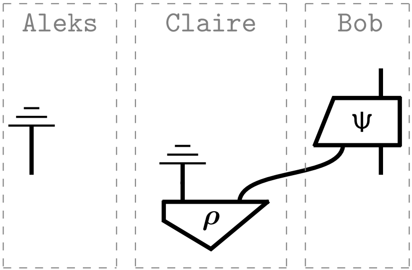


Outline

Process theories and diagrams

Quantum processes

Classical and quantum interaction

Applications: a Hollywood-style trailer

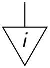
Double vs. single wires

$$\left(\text{quantum} \quad := \quad \Bigg| \right)$$

Double vs. single wires

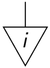
$$\left(\text{quantum} \quad := \quad \rule{3pt}{2em} \right) \quad \neq \quad \left(\text{classical} \quad := \quad \rule{0.5pt}{2em} \right)$$

Classical values



:= ‘providing classical value i’

Classical values



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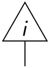
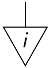


:= 'testing for classical value i'

Classical values



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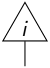

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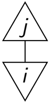
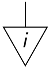


$$= \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

Classical values



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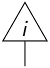

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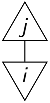

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($\Rightarrow$ ONB)

Classical states

General state of a classical system:

$$\bigtriangledown_{p} \;:=\; \sum_i p_i \, \bigtriangledown_{i} \qquad \leftarrow \quad \text{probability distributions}$$

Classical states

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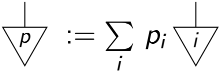
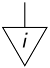


← probability distributions

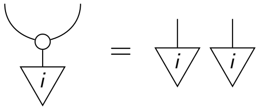
Hence:



← point distributions

Copy and delete

Unlike quantum states, classical values can be *copied*:



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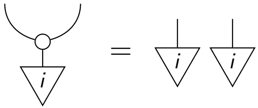
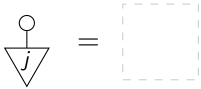
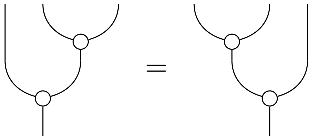


and *deleted*:



Copy and delete

These satisfy some equations you would expect:



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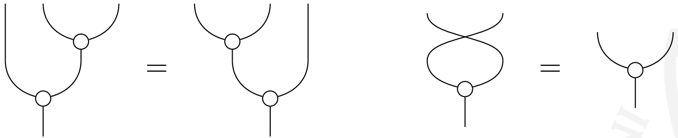

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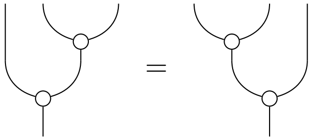

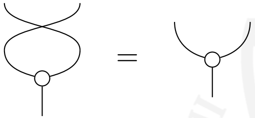

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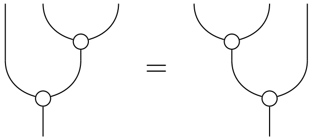
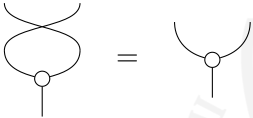
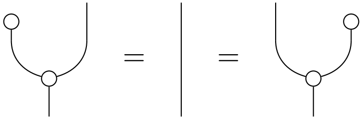
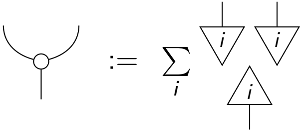
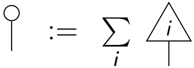






Other classical maps





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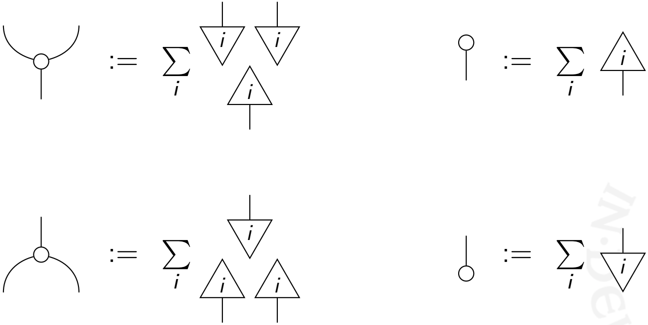

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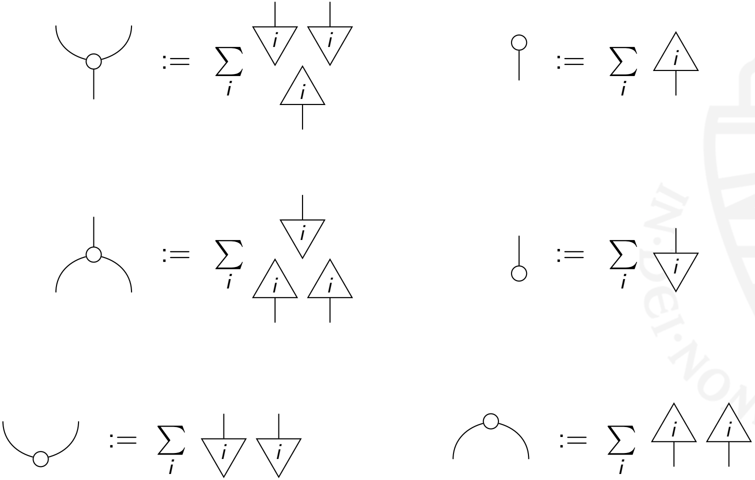

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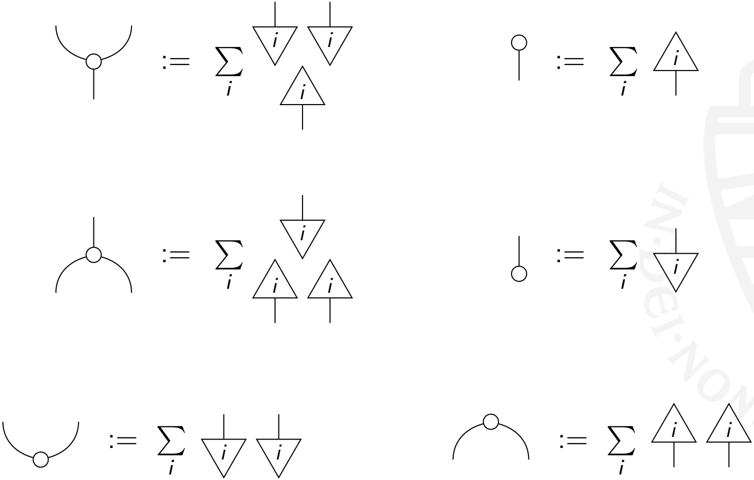
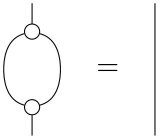
Other classical maps



...satisfying lots of equations



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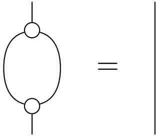

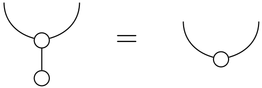

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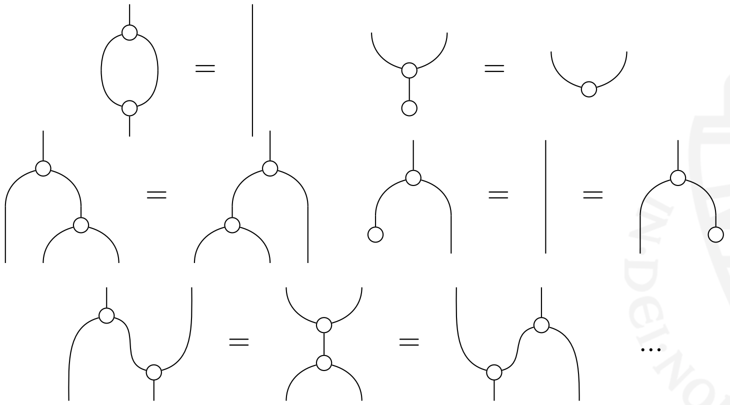

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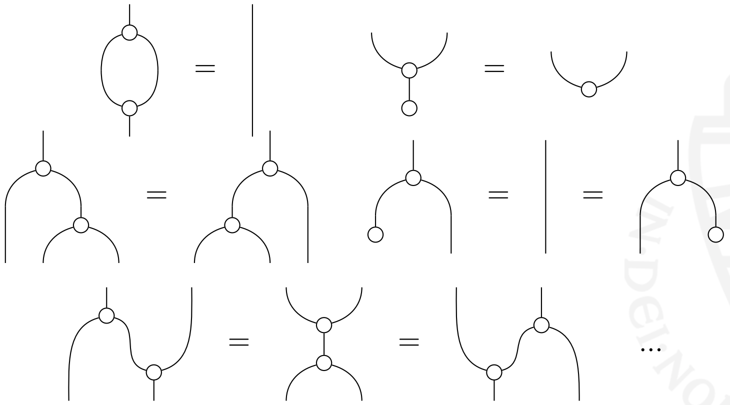
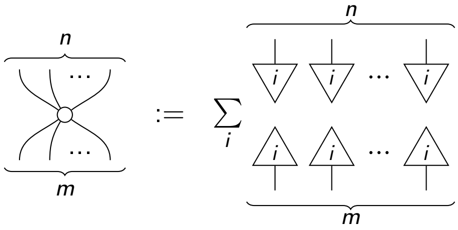


When does it end???

Spiders

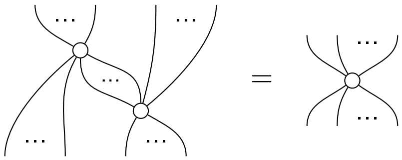
Spiders

All of these are special cases of *spiders*:



Spiders

The only equation you need to remember is this one:



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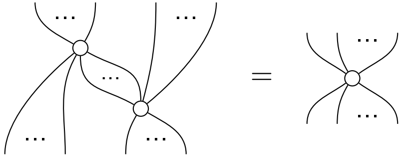
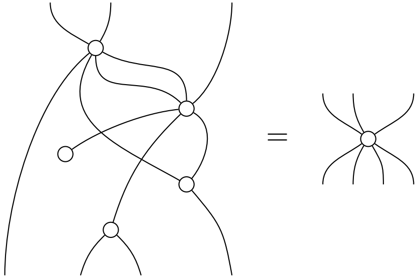


When spiders meet, they fuse together.

Spider reasoning



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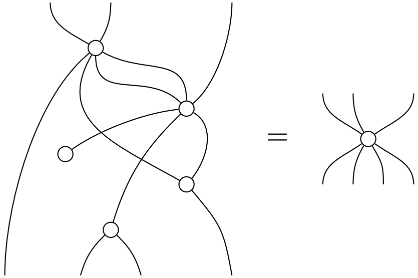
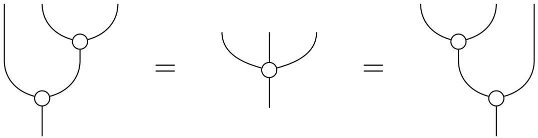
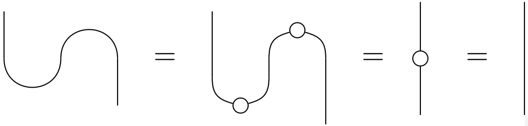


For example:



Spider reasoning ⇒ string diagram reasoning



How do we recognise spiders?

Suppose we have something that ‘behaves like’ a spider:

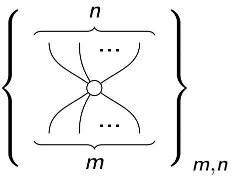
How do we recognise spiders?

Suppose we have something that 'behaves like' a spider:

Do we know it is one?

Spiders = 'diagrammatic ONBs'

Yes!



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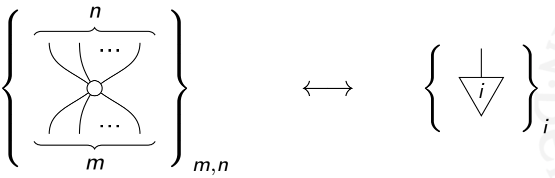
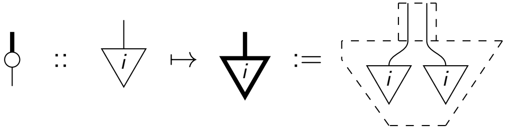


Classical and quantum interaction

Classical and quantum interaction

Classical values can be encoded as quantum states, via doubling:



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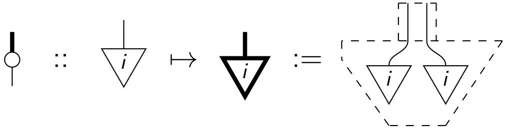

This is our first classical-quantum map, *encode*.

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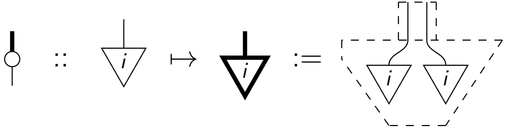
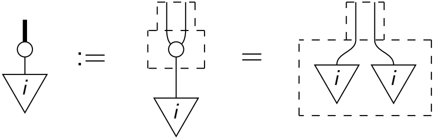


This is our first classical-quantum map, *encode*. It's a copy-spider in disguise:



Measuring quantum states

The adjoint of *encode* is *measure*:

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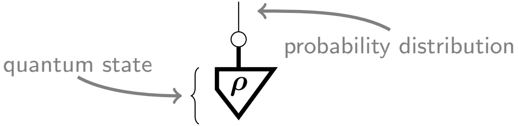
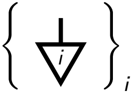


This represents measuring w.r.t.



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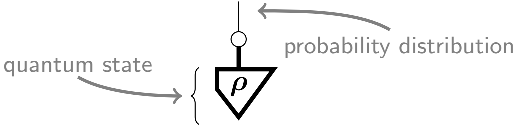

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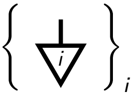

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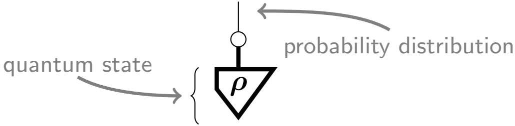

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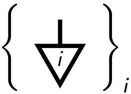

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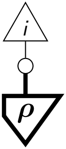
$P(i|\rho) :=$ 

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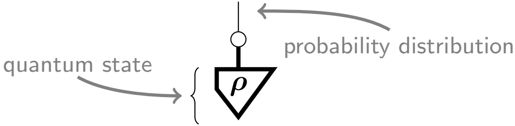

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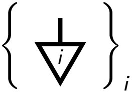

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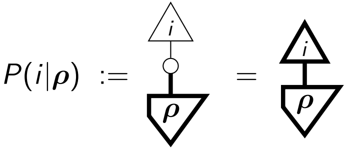
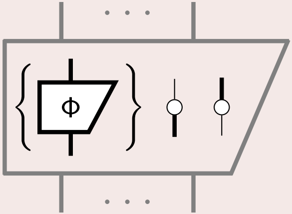


Classical-quantum maps

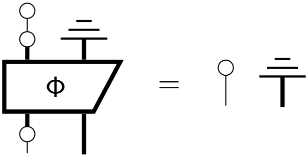
Definition

The process theory of **cq-maps** has as processes diagrams of quantum maps and encode/decode:



Quantum processes

Causality generalises to cq-maps:



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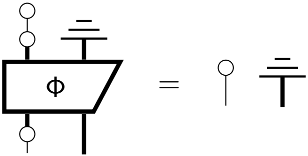
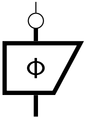


quantum processes := causal **cq-maps**

Special case: quantum measurements

A *measurement* is any **quantum process** from a quantum system to a classical one:



$\xleftrightarrow{\cong}$ POVMs

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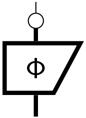
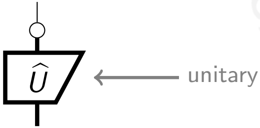


$\overset{\cong}{\longleftrightarrow}$ POVMs

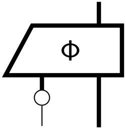
Special case:

ONB-measurement $:=$



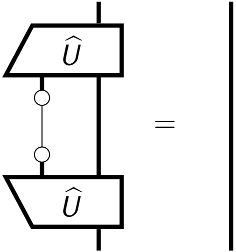
Special case: controlled-operations

A **quantum process** with a classical input is a *controlled operation*:



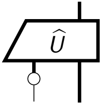
Special case: controlled-operations

A *controlled isometry* furthermore satisfies:



Special case: controlled-operations

Suppose we can use a single $\widehat{U}$ to build a *controlled isometry*:



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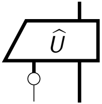
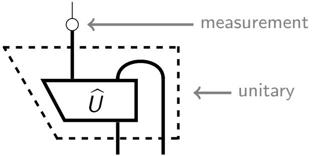
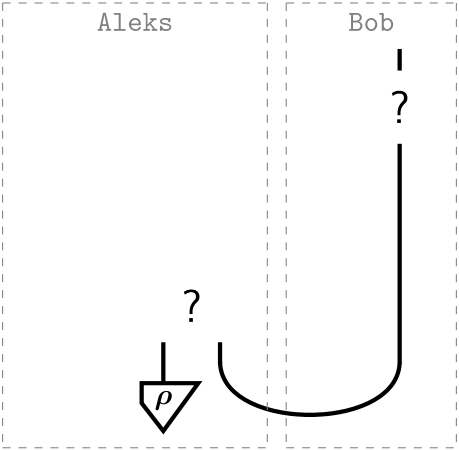


...and an ONB measurement:



Quantum teleportation: take 2

...then teleportation is a snap!



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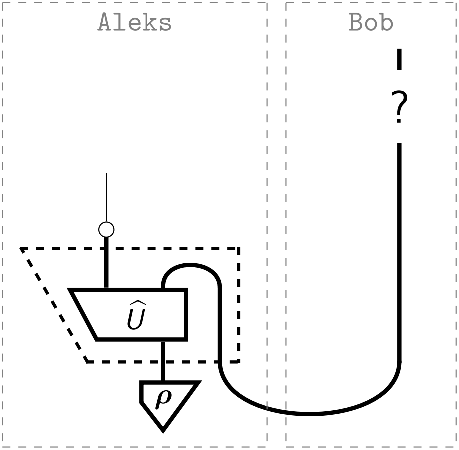

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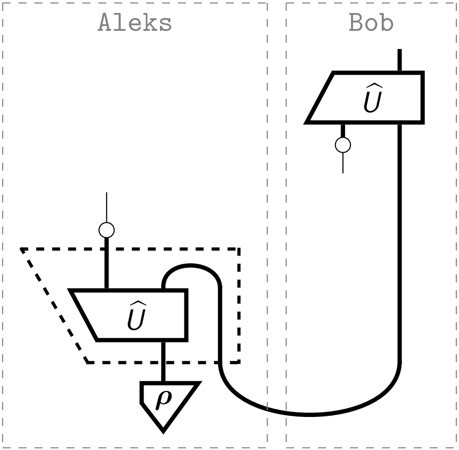

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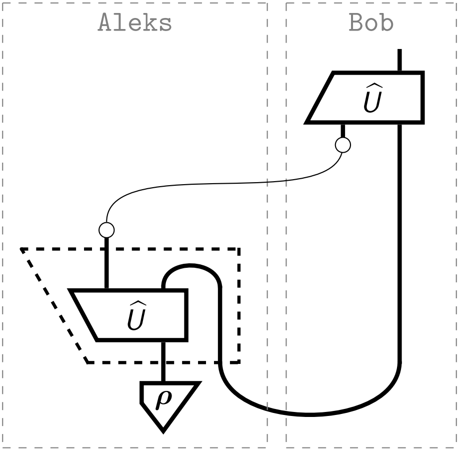

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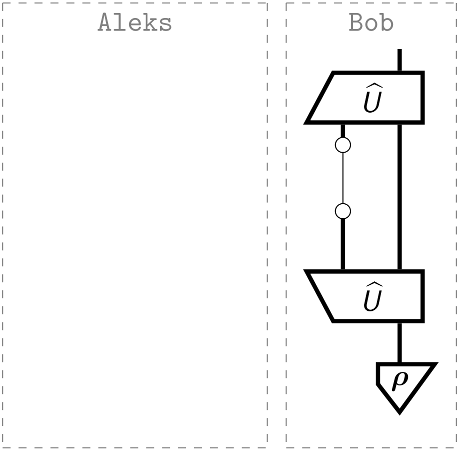

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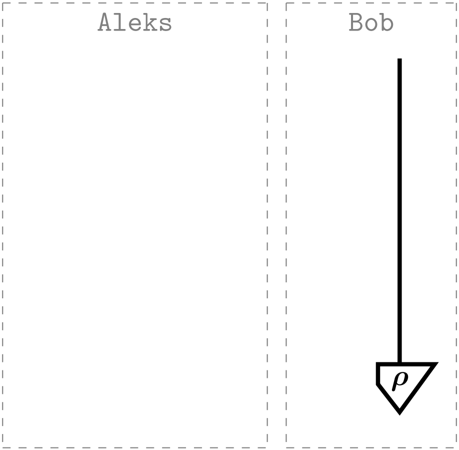
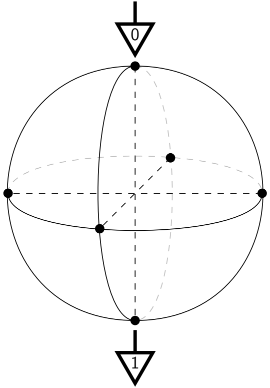


Complementary bases



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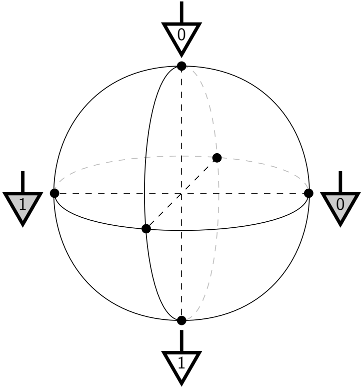

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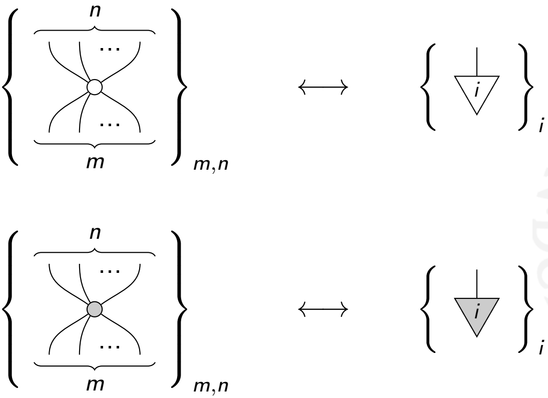
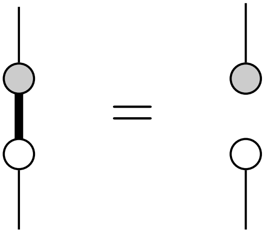


Complementarity



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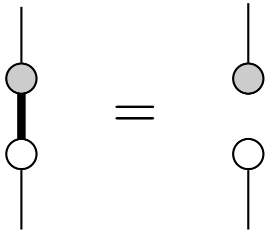
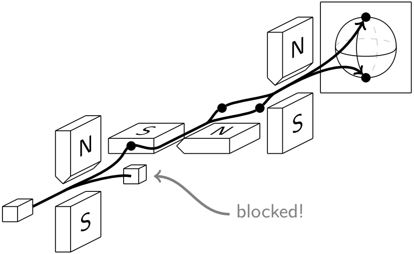
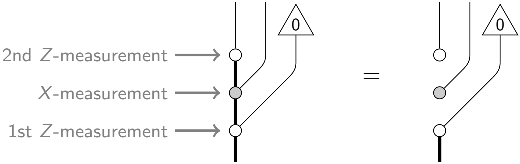


Interpretation:

(encode in ○) THEN (measure in ●) = (no data flow)

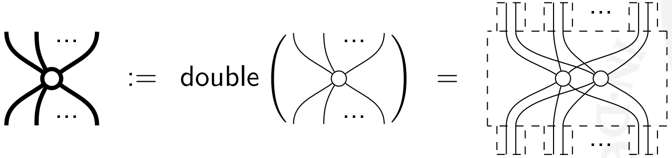
Consequence: Stern-Gerlach





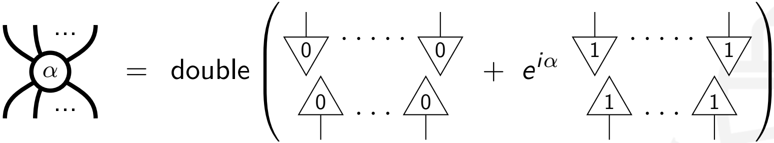
Quantum computation

Doubling a classical spider gives a *quantum spider*:



Universality

By decorating quantum spiders with *phases*:



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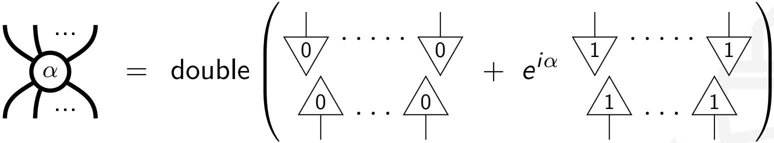
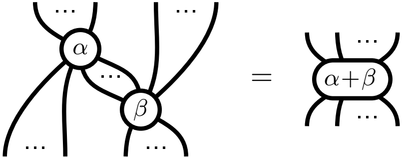


we have:



and spider-diagrams become **universal for quantum computation**!

Soundness and completeness

Restricting the phase group to $\mathbb{Z}_4 \cong \{0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}\} \subset U(1)$ gives *stabiliser QT*.

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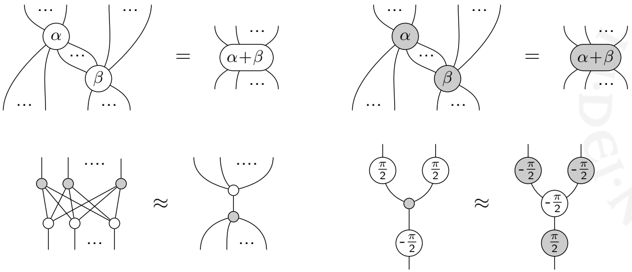
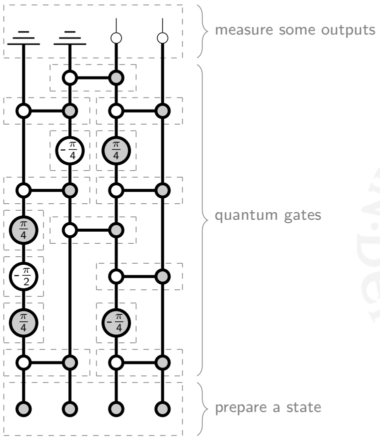
Sound and complete presentation via the **ZX-calculus**:

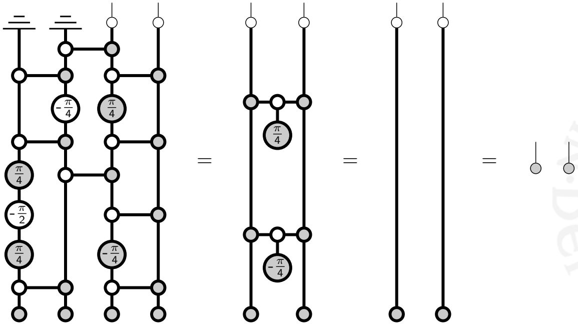


Outline

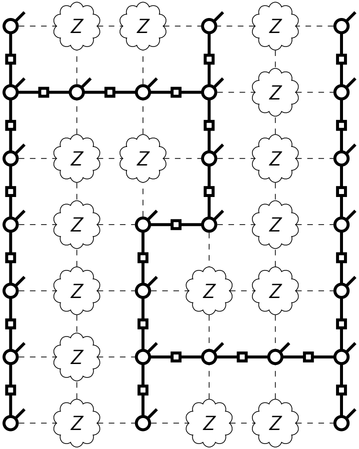
Quantum circuits and rewriting



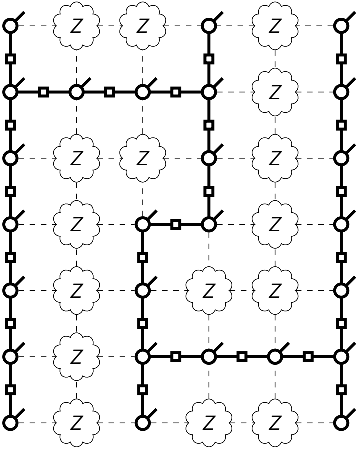
Quantum circuits and rewriting

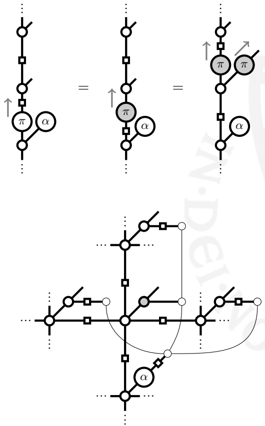


Measurement-based quantum computing



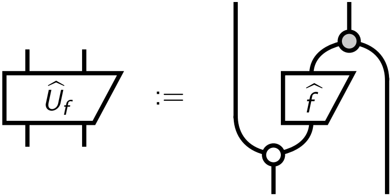
Measurement-based quantum computing





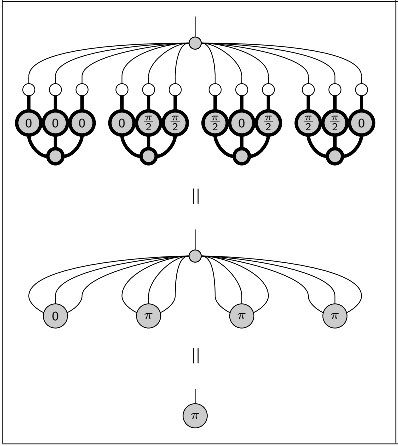
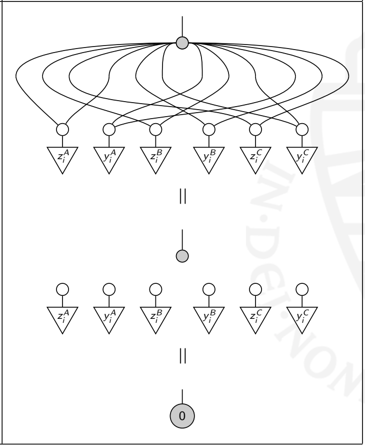
Quantum algorithms

Spiders can be used to build *quantum oracles*:



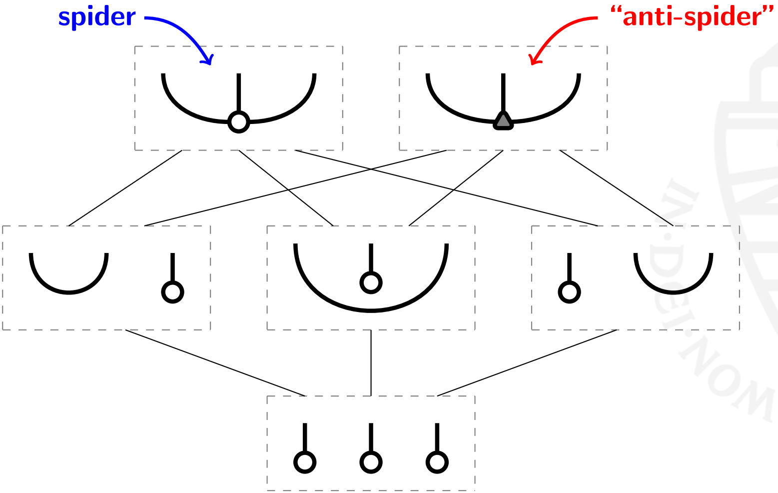
⇒ simple derivations of **Deutsch-Jozsa**, **quantum seach**, and **hidden subgroup** algorithms.

GHZ/Mermin non-locality

quantum theory	any local theory
	

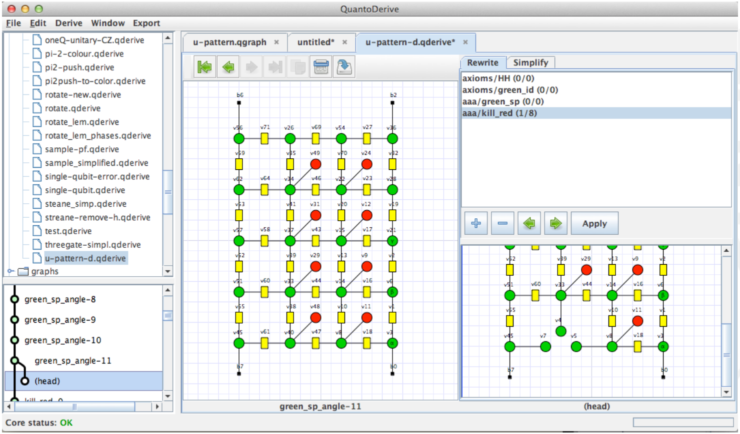
Multipartite entanglement

SLOCC-classification of 3 qubits:



Automation

Quantomatic:



Thanks! Joint work with Bob Coecke (book):

...and many more!

Abramsky, Backens, Duncan, Edwards, Gogioso, Hadzihasanovic, Heunen, Lal, Merry, Pavlovic, Perdrix, Quick, Selinger, Vicary, Zamdzhiev, ...

http://quantomatic.github.io