project no 033763

QICS

Foundational Structures for Quantum Information and Computation

Specific Targeted Research Project (STREP)

Thematic priority: Quantum Information Processing and Communications

EXECUTIVE SUMMARY

Period covered: Jan. 1st 2007 – Dec. 31 2007

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Start date of project: Jan. 1st 2007

Duration: 36 months

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An exciting and promising first year of QICS – executive account

`http://se10.comlab.ox.ac.uk:8080/FOCS/FP6STREPQICS_en.html`

It is our pleasure to report on a very successful and exciting first year of QICS research. The ultimate goal of QICS, as stated in the initial proposal, is to radically increase our understanding of the foundational structures of quantum informatics. The *method* is a cross-disciplinary endeavour, involving,

- *physicists* who are challenging the boundaries of nature's capabilities by studying novel quantum computational models such as measurement based quantum computational schemes and quantum cellular automata, mainly in Braunschweig and Innsbruck,
- *logicians* who adopt novel structural tools such as category theory, type systems and formal calculi to cast quantum behaviour, mainly in McGill, Oxford and York,
- *mathematicians* trying to achieve an understanding of quantum information by providing both qualitative and quantitative accounts on it, mainly in Bristol, McGill and York, and,
- *computer scientists* who bring in their know-how on high-level methods to cope with complex interactive and distributed situations, mainly in Grenoble, McGill, Oxford and Paris.

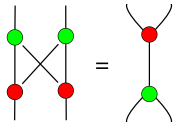
The first round of QICS hiring involved many permutations between these groups, resulting in successful interactions e.g. computer science methods and structures have now been successfully applied to measurement based quantum computational models – we discuss this in more detail below.

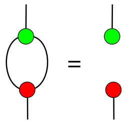
The workpackage on *structures and methods for measurement-based quantum computation* [**W1**] addressed fundamental questions, for example, on the relation of the entanglement of the resource state with the computational power of the scheme, which were still largely unanswered. More specifically, which resource states beyond the cluster state would allow universal quantum computation and which entanglement features would be responsible for that? And, which resource states would give no advantages over classical computation at all? This endeavour was highly successful. It has resulted in a much better understanding of graph states, a key resource for measurement based quantum computing, and exposed their scope for application. There is also a number of intriguing new applications and developments of graph state methods in statistical physics: problems involving statistical mechanics of classical spin systems, can be related to problems in quantum physics, relating a large class of classical spin models to quantum stabelizer states. Substantial progress has also been made on the clarification of which features of multi-partite entanglement are responsible for universality of resources in measurement based quantum computing. A detailed discussion of these results is in [**W1**]. We also witnessed the developments of high level calculi for measurement based quantum computing which now makes them subject to the activity in [**W4**]. Finally, adequate categorical structures have been crafted to reason, both diagrammatically and automated, about a variety of schemes and resources for measurement based quantum computing.

This brings us to the second workpackage, on *categorical semantics, logics and diagrammatic methods* [**W2**]. The main task for this year was to craft the appropriate structures to address problems in workpackage 1 on measurement based quantum computing and workpackage 3 on information flow in quantum informatics. This resulted in a both axiomatic, diagrammatic, and logical (cf. automation) account on:

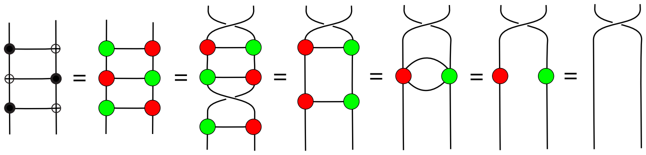
1. classical vs. quantum data, crucial for the applications in measurement-based quantum computational models which require classically controlled correction operations;

2. complementary observables, which enables abstract simulation of elementary gate computations; the key axioms of this structure are the well-known bialgebra equation and a (degenerated) Hopf law:



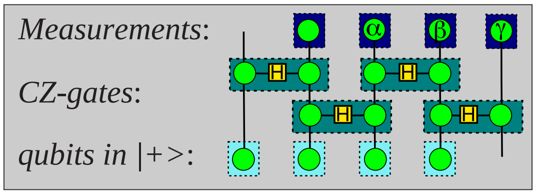


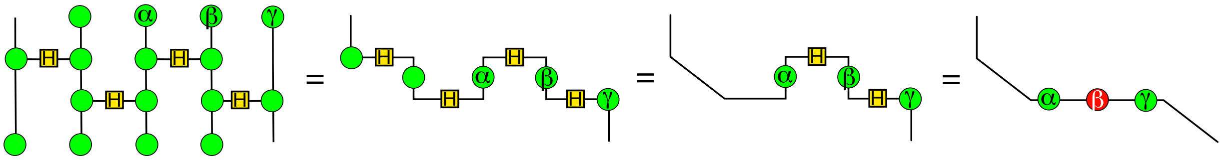
   which provides enough structural power for typical circuit computations:



3. all gates and multipartite entangled states of the Hilbert-space formalism in high-level terms, providing us with all the expressive power the full-blown Hilbert space quantum formalism.

Parts **2.** and **3.** together to verify measurement based quantum computational schemes e.g. realisation of arbitrary one-qubit unitaries in no more than three rewrites:





Other notable results in this workpackage are several concrete new categorical models which capture particular features of quantum informatics and important steps toward a categorical axiomatics for topological quantum computing. Details can be found in [**W2**].

The workpackage on *classical-quantum interaction and information flow* [**W3**] has as its main goal to delineate a notion of quantum information flow when quantum and classical systems are interacting. As compared to the purely classical counterpart to this, the situation is of course far more complicated here given that besides the flows between the quantum and the classical there are also the flows within the quantum itself subject to entanglement. In this workpackage we try to approach this involved problem from several angles.

- *Resource inequalities*. The discovery by the QICS team of the so-called mother and father protocols in this quantum information resource calculus is a fundamentally significant development – it conceptually unifies a wide variety of previously diverse quantum information processing results, such as characterisation of noisy channel capacities, entanglement distillation, quantum broadcasting and state merging and many more.

- *Quantum data processing*. Here several results were obtained and a key fundamental issue of quantum computation viz. the relationship of classical to quantum computational complexity, and the characterisation of ways in which the latter is an extension of the former.

- *Categorical operational semantics*. Here it was shown that only a tiny bit of structure, namely abstract counterparts to copying and uniform erasing, turns out to be sufficient to extract from an abstract family of quantum processes, a variety of classical processes such as reversible classical processes, deterministic- and non-deterministic processes, stochastic processes and even informatic order in terms of majorisation. We were also able to prove the no-cloning theorem based on purely topological (cf. information flow) principles.

- *Coalgebraic structures and methods.* They are the natural mathematical framework to accomodate non-deterministic branching. We were able to recast a range of important quantum informatic concepts coalgebraically, making them subject to a variety of high-level methods.

The QICS workpackage on *quantum automata, machines and calculi* [**W4**] is defining the state-of-the-art in this area. By studying several forms of abstract models of what quantum information processing devices can or should be, this workpackage has produced significant advances in understanding the structure, the mathematical and logical foundations, the operating principles and some of the computational properties of such devices, some of which are:

- a classically-controlled Turing machine which is significantly simpler than Deutsch's quantum Turing machine and which can be specialized into a pure measurement-based quantum Turing machine;
- a universal one-dimensional quantum cellular automata (QCA) capable of simulating all others has been described, and it has been proved that one-dimensional QCA always admit a two layered block representation and that their inverse is also a QCA; this last result came as a major surprise, since such a property does not hold for classical CA; a proof that every QCA can be put in the form of a tiling of more elementary, finite dimensional unitary evolutions, has also led to a most welcome, clear and robust definition of n-dimensional QCA, phrased in the traditional setting of Hilbert spaces;
- a language QML, has the significant advantage of a semantic domain directly built upon quantum objects and operations, but is restricted to first order; a translator from QML to quantum gate networks has recently been implemented;
- a hierarchy of denotational semantics have been defined for a simple quantum imperative language; remarkable progress in the study of semantics for languages giving access to quantum resources has been made by relying upon the abstract setting of dagger compact categories with biproducts.

Several other results on abstract models can be found in [**W4**]. In addition, a connection has been established between measurement-based quantum computations with graph states and the field of mathematical logic, showing that the computational power of graph states is reflected in the expressive power of classical formal logic languages defined on the underlying mathematical graphs, bringing us back to workpackage 1, thus closing the circle.

Bob Coecke, Oxford, February 10, 2008.
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References

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W2 S. Abramsky and B. Coecke (2008) Introduction to QICS deliverable D2 for the workpackage *Categorical semantics, logics and diagrammatic methods*. `Click here here for download.`

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