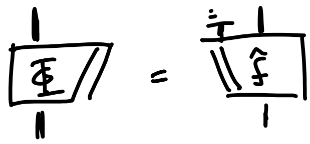
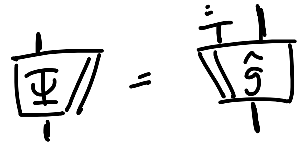
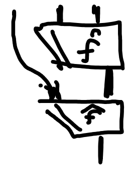
Quantum Processes & Computation

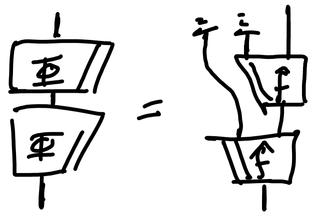
Model solutions, sheet 4

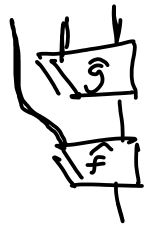
Oxford MT 2022

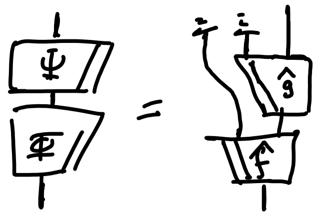
Ex 4.1

Let  and .

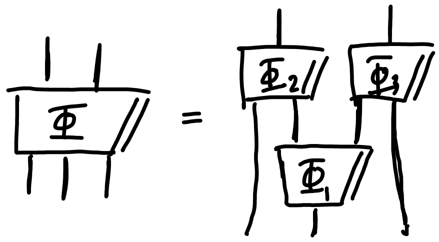
Then $\Phi \circ \Phi$ purifies as , since:



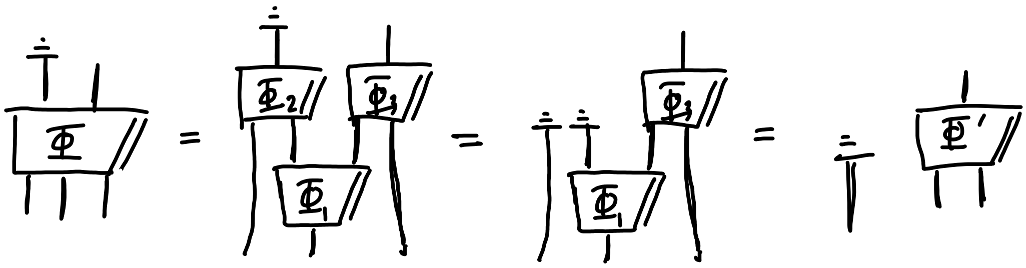
and similarly, $\Psi \circ \Phi$ purifies as , since:

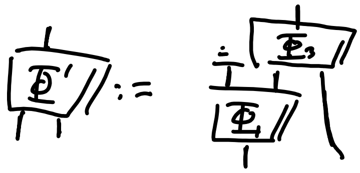


Ex 4.2

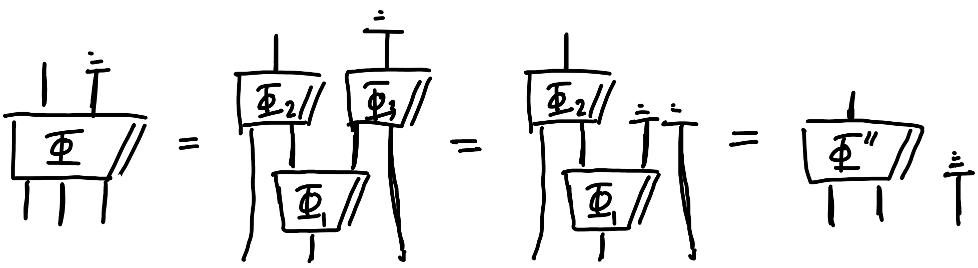
Let  for causal q.maps Φ_1, Φ_2, Φ_3.

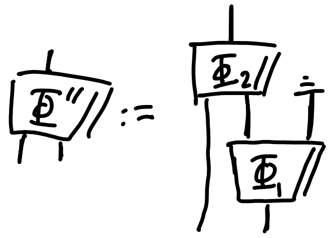
Then:



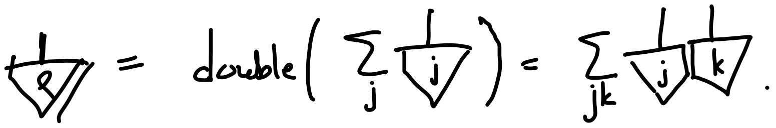
where  .

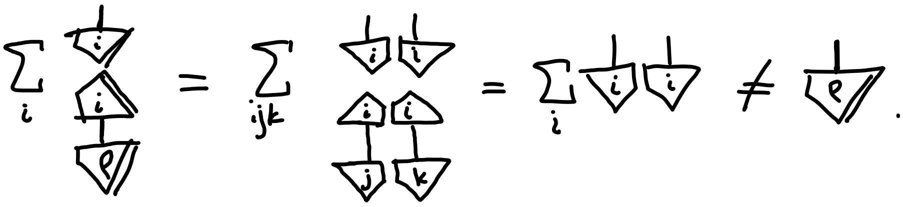
and:



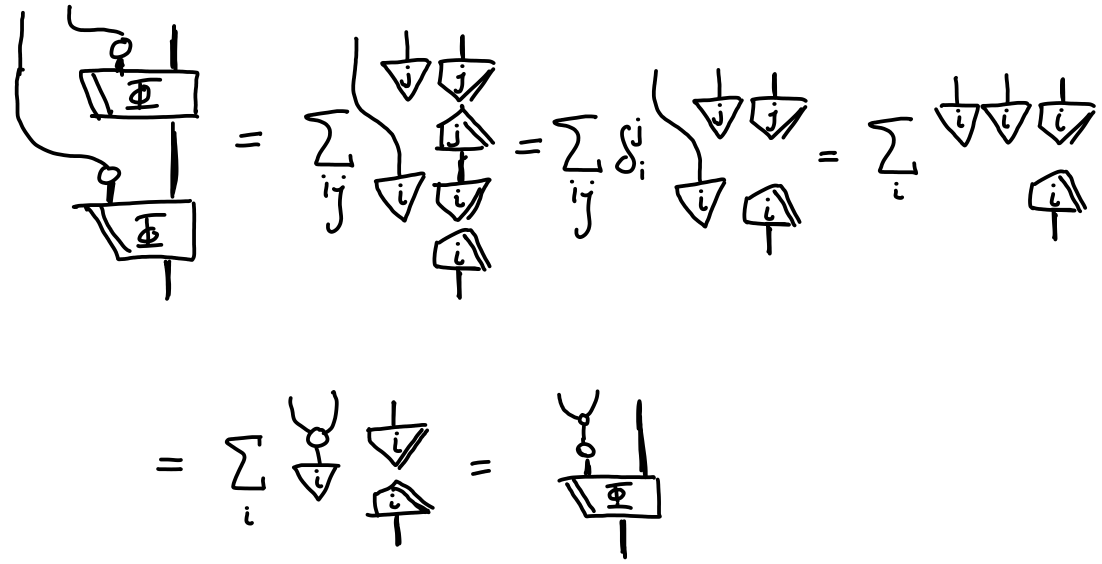
where  .

Ex 4.3

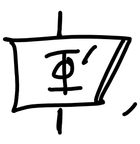
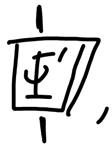
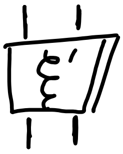
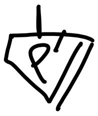
Let 

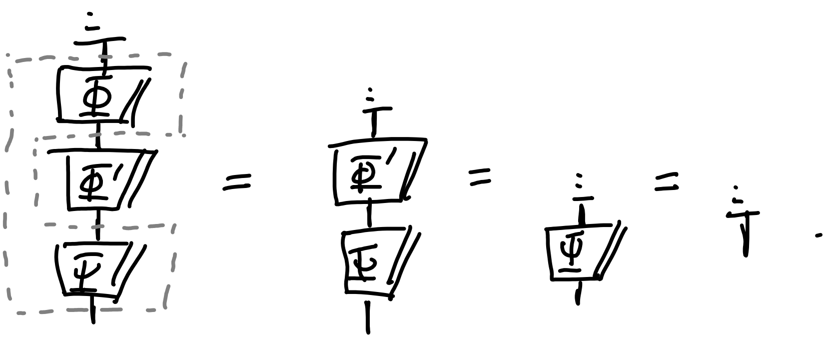
Then 

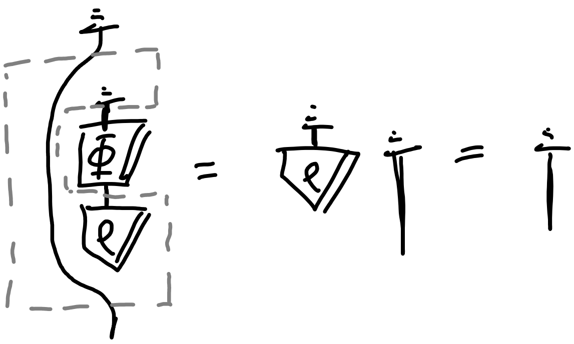
Ex 4.4

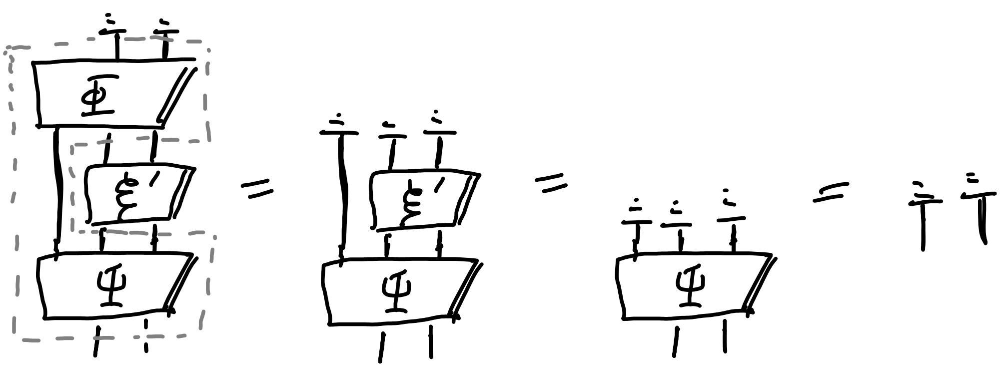


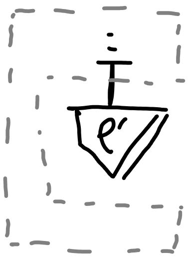
Ex 4.5

(a) Let , ,  and  be any causal processes, then:

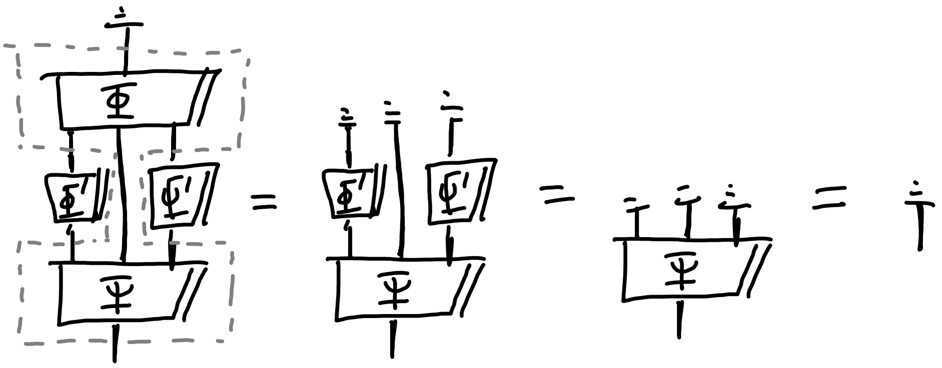
(i) 

(ii) 

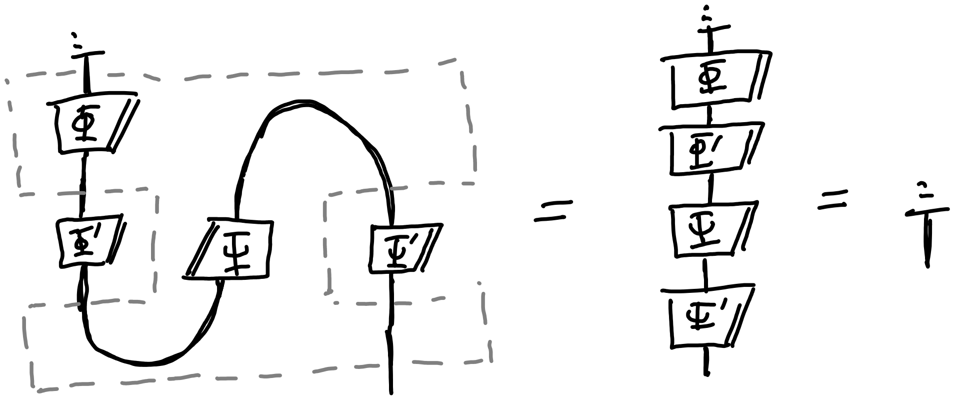
(iii) 

(iv)  = 1

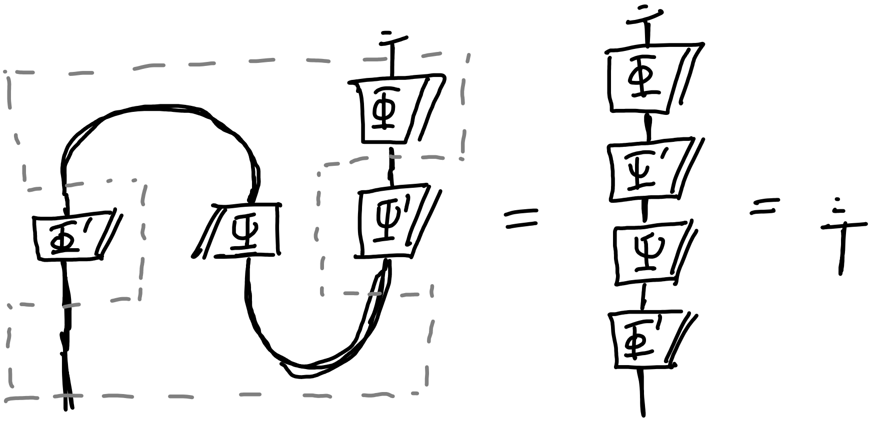
(v)



(vi)

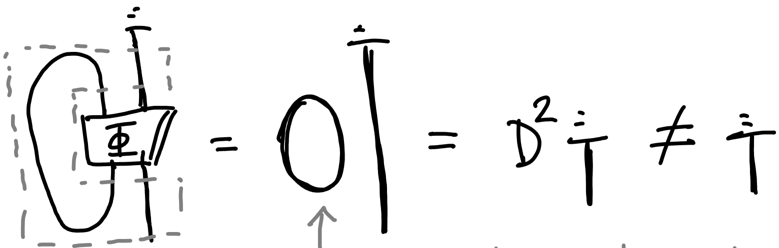


(vii)

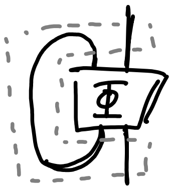


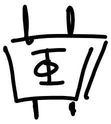
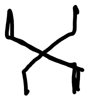
(b) Assume all wires are of type $\hat{A}$, where $D := \dim A \neq 1$. Then:

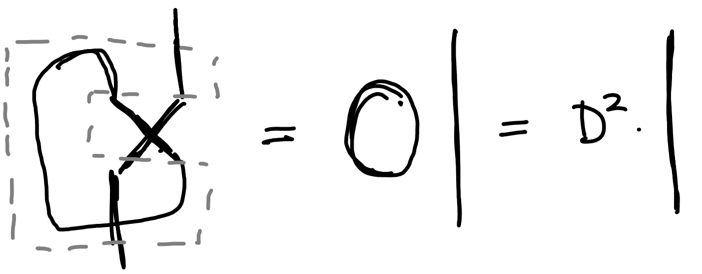
(i)  $:= \mid\ \mid$ is causal but:



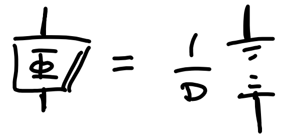
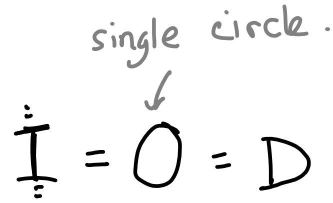
double circle = double (D) = D^2

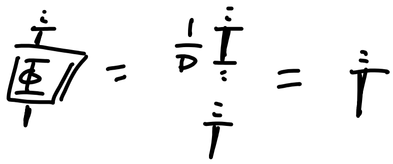
So:  is not causal.

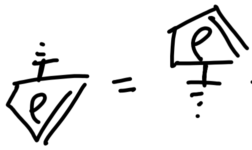
(ii) Similarly,  $:=$  is causal, but:

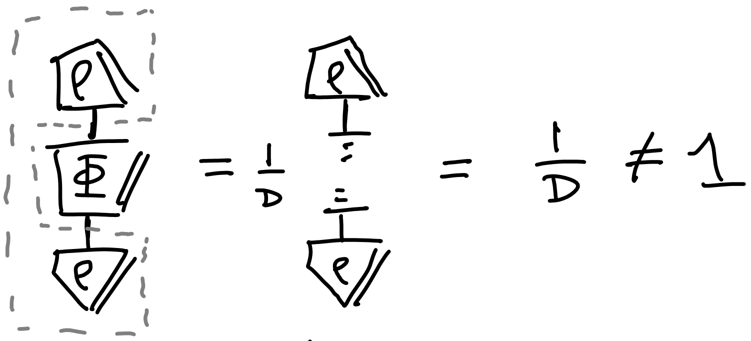


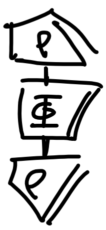
is not causal.

(iii)  is causal, because 

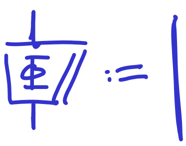
and hence 

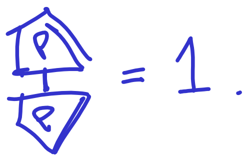
But  $= 1$, so:

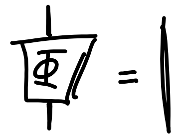


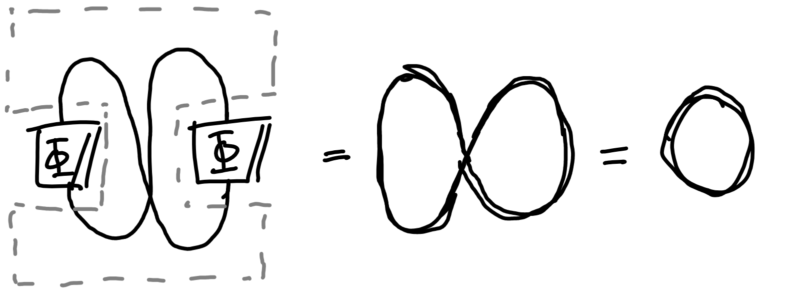
Hence  is not causal, because 1 is the only causal number.

* * *

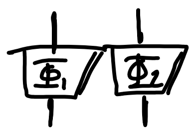
Note  does <u>not</u> give a counter-example, because ρ might be a pure state, in which case



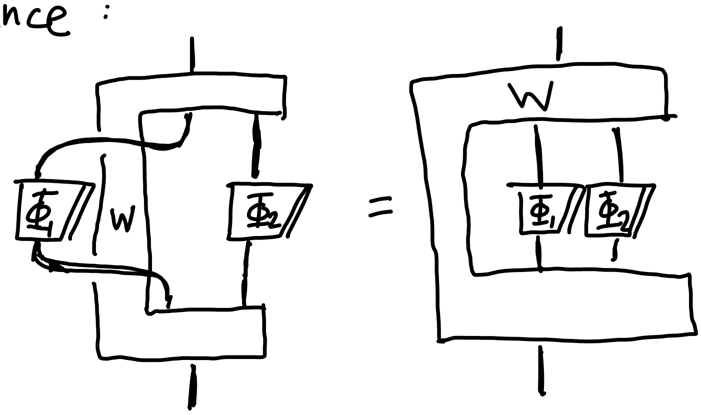
(iv)  is causal but

 $= D^2 \neq 1$

is not causal.

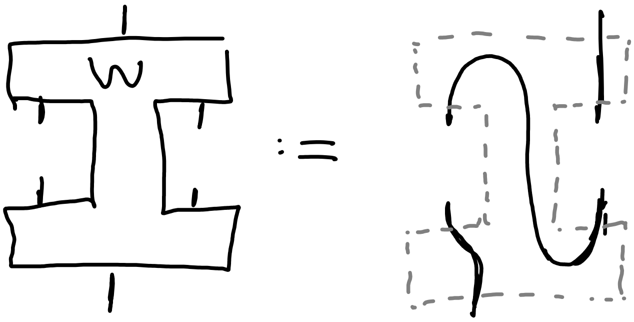
(c) Suppose W is second-order causal. Then, for any causal Φ_1, Φ_2,  is causal.

Hence:

 is causal.

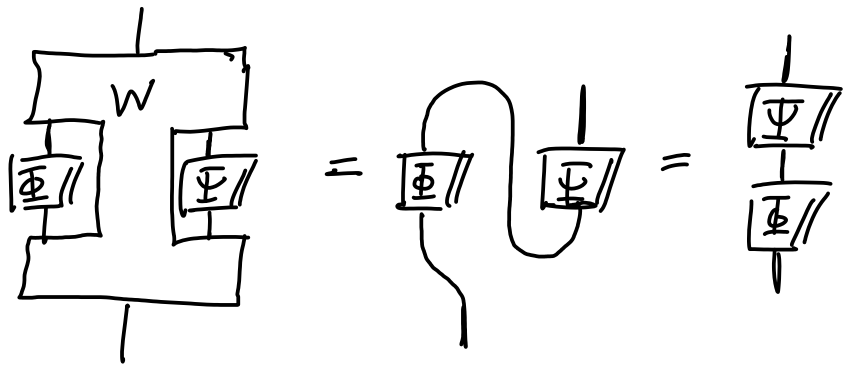
So W is second-order 2-causal.

For the converse, note that



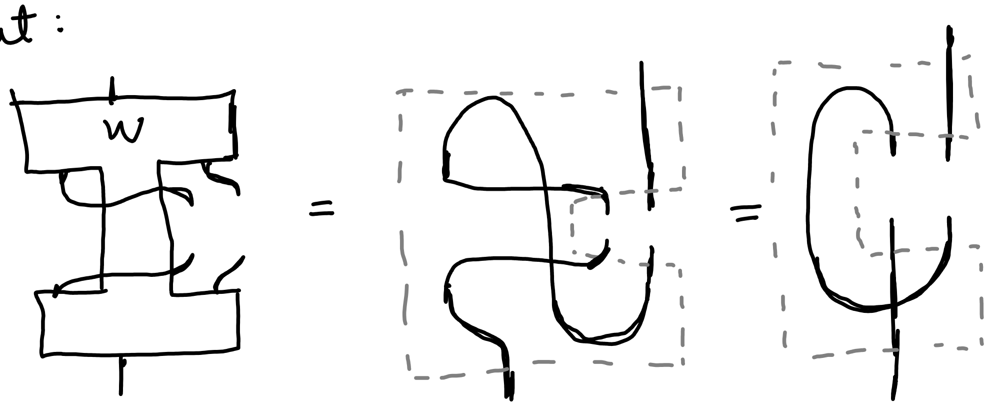
is second-order 2-causal,

i.e for any causal Φ, Ψ :



is causal.

But:



which, as shown in (b)(ii), is <u>not</u> second-order causal.