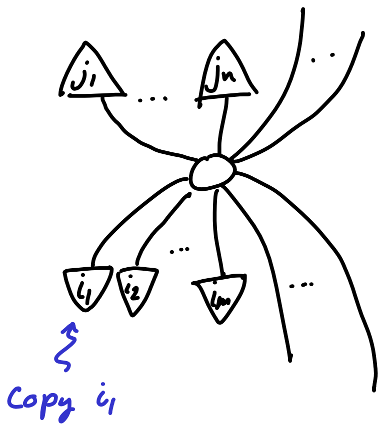
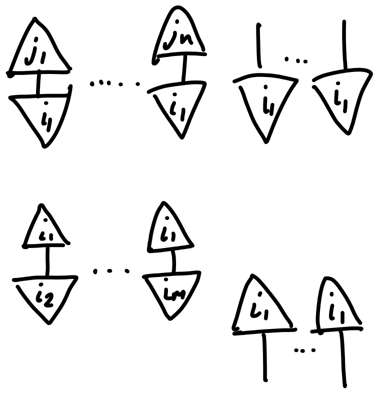
Quantum Processes & Computation

Model solutions, sheet 5
Oxford MT 2022

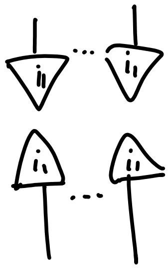
Ex 5.1



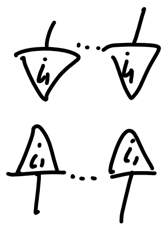
$=$



$$= \delta_{i_1}^{i_2}\cdots\delta_{i_1}^{i_m}\,\delta_{i_1}^{j_1}\cdots\delta_{i_1}^{j_n}$$

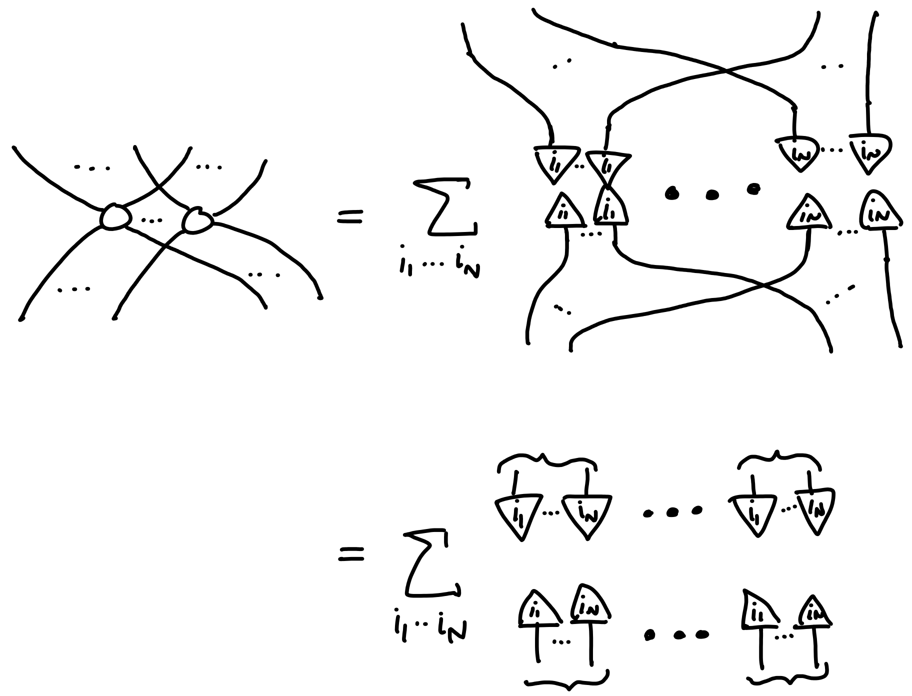


$$= \delta_{i_1\cdots i_m}^{j_1\cdots j_n}$$

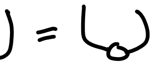


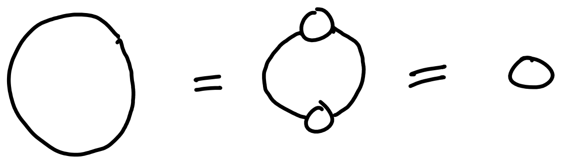
Ex 5.2

This follows just from expanding each of the spiders:

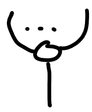


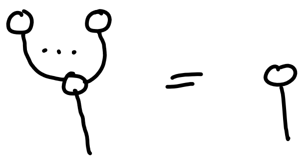
Ex 5.3

(i) Using $\cap$ =  & $\cup$ = , we have:

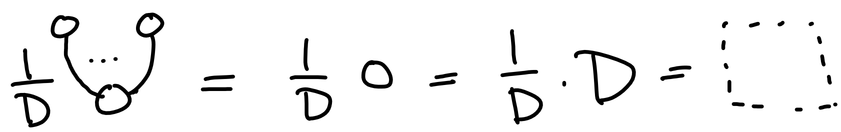


(ii) We just showed $\circ = \bigcirc = D$. There are 3 cases:

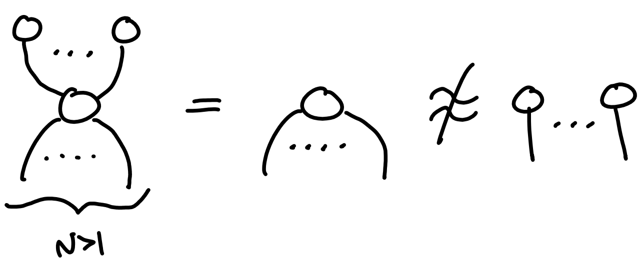
(1) 1-input spiders  are causal:



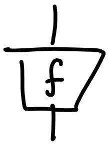
(2) 0-input spiders  can be renormalised to be causal:

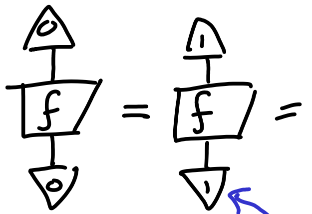
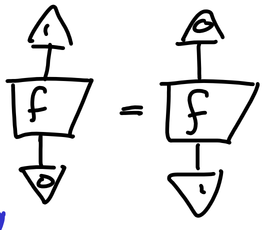


(3) Spiders with >1 input are <u>never</u> causal:

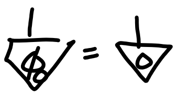


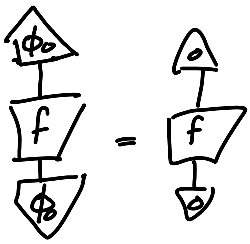
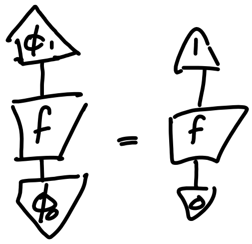
Ex 5.4

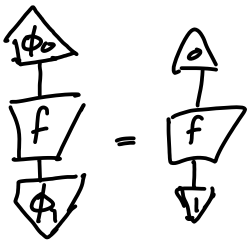
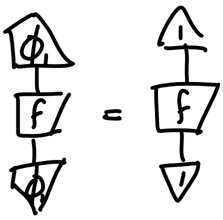
 $\longleftrightarrow \begin{pmatrix} \frac{2}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{2}{3} \end{pmatrix}$

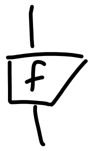
Means  $= \frac{2}{3}$ and  $= \frac{1}{3}$

z-basis

But if we change to the basis  , 

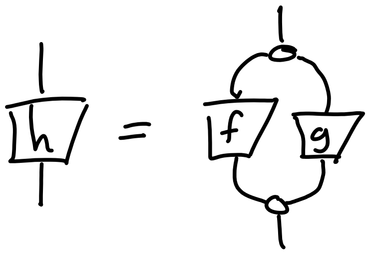
 $= \frac{2}{3}$  $= -\frac{1}{3}$

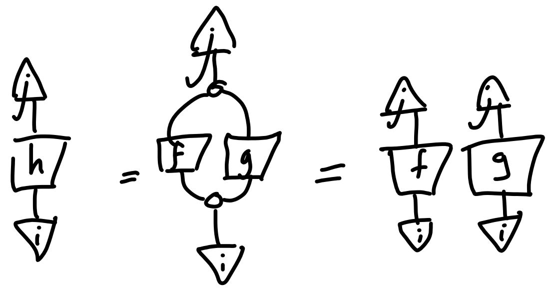
 $= -\frac{1}{3}$  $= \frac{2}{3}$

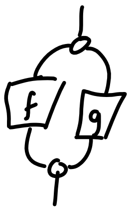
So  $\longleftrightarrow \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{pmatrix}$

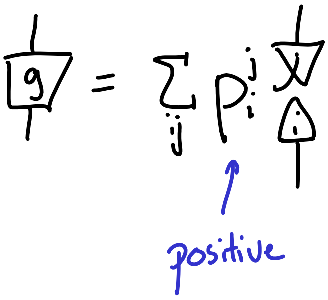
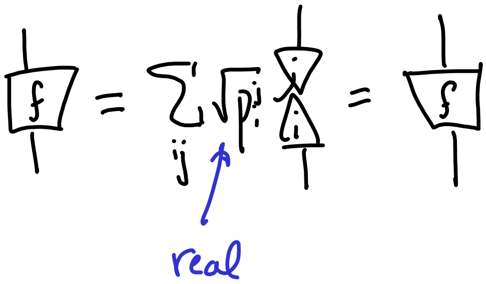
w.r.t. $\{$  $\}_i$

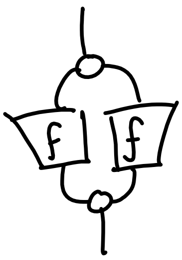
Ex 5.5

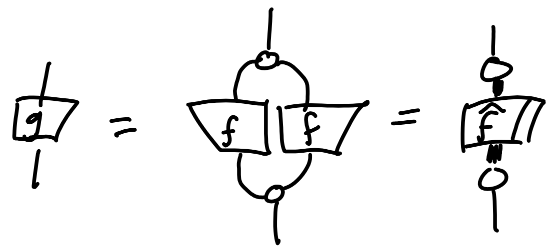
(i) Let  .

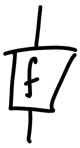
$h^j_i :=$  $= f^j_i g^j_i$

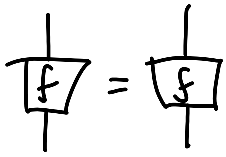
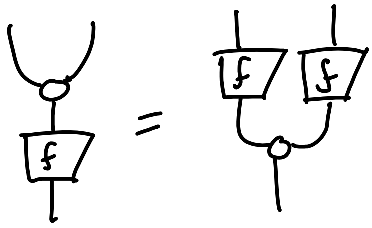
Hence:  has matrix $\begin{pmatrix} h^1_1 & \cdots & h^1_D \\ \vdots & \ddots & \vdots \\ h^D_1 & \cdots & h^D_D \end{pmatrix} = \begin{pmatrix} f^1_1 g^1_1 & \cdots & f^1_D g^1_D \\ \vdots & \ddots & \vdots \\ f^D_1 g^D_1 & \cdots & f^D_D g^D_D \end{pmatrix}$

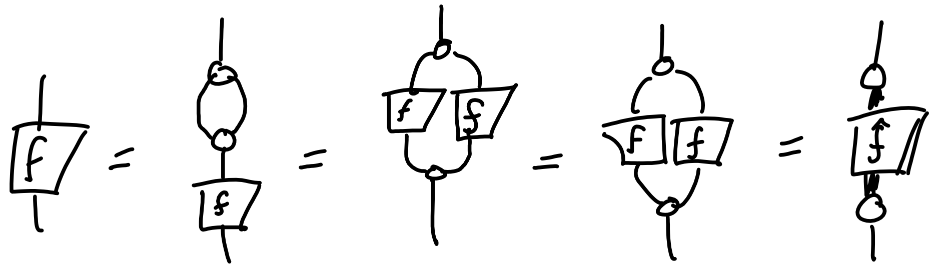
(ii) For  , let 

Then, by (i)  has matrix entries $\sqrt{p^j_i} \cdot \sqrt{p^j_i} = p^j_i$.

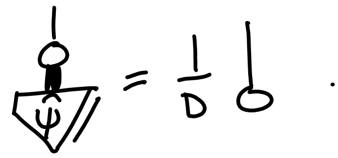
Hence  .

(iii) If  is a function-map, then

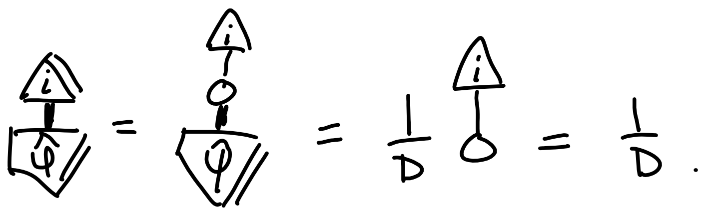
 and 

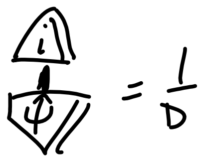
So: .

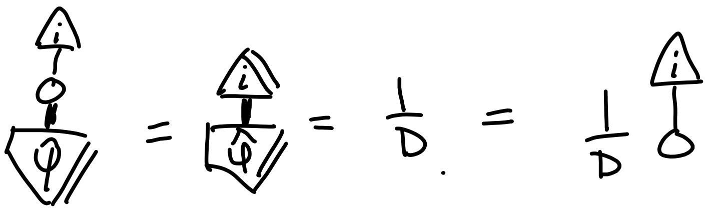
Ex 5.6

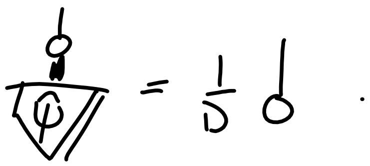
$\hat{\psi}$ is unbiased for $\circ$ iff 

For ($\Rightarrow$), assume $\hat{\psi}$ unbiased. Then:



For ($\Leftarrow$), assume  for all i. Then:



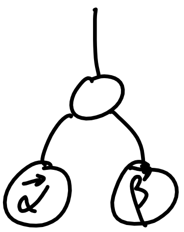
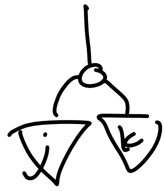
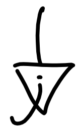
Hence 

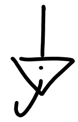
Ex 5.7

Unit.

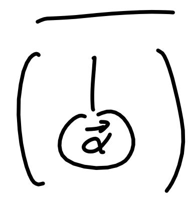
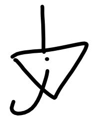
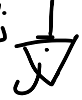
 $=$ double() $= \text{double}\left(\sum_j$  $\right) = \text{double}\left(\sum_j e^{i\alpha_j}$  $\right)$

for all $\alpha_j = 0$, i.e. $\vec{\alpha} = (0, \dots, 0)$.

 $= \sum_{jk} e^{i\alpha_j} e^{i\beta_k}$  $= \sum_{jk} \delta_j^k e^{i(\alpha_j + \beta_k)}$ 

$= \sum_j e^{i(\alpha_j + \beta_j)}$ 

So $\vec{\alpha} + \vec{\beta} = (\alpha_1 + \beta_1, \dots, \alpha_D + \beta_D)$

 $= \sum_j \overline{e^{i\alpha_j}}$  $= \sum_j e^{-i\alpha_j}$ .

So $-\vec{\alpha} = (-\alpha_1, \dots, -\alpha_D)$.