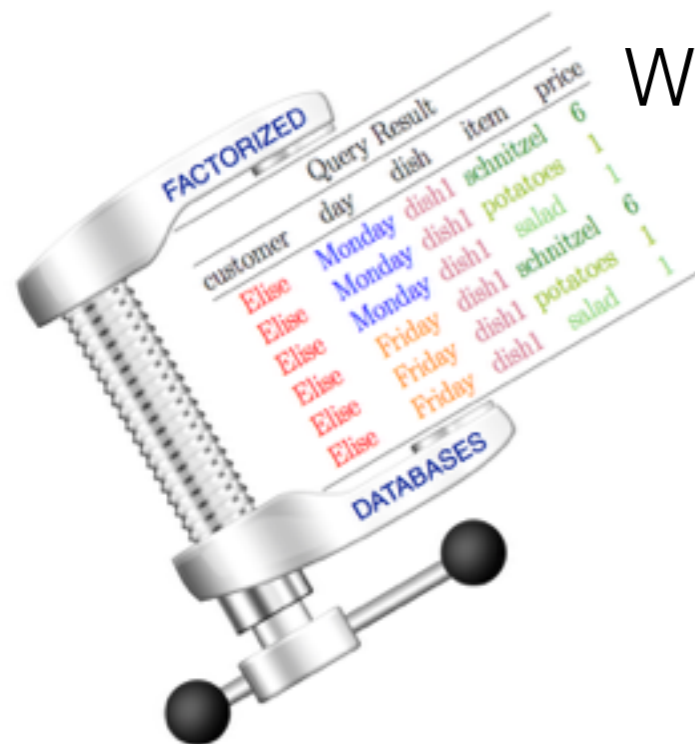


# Covers of Query Results

Ahmet Kara

Dan Olteanu



Workshop on Aggregate Queries  
Lille, June 2017



# The Database of a Fast Food Restaurant

## Orders

Customer	Day	Dish
elise	monday	burger
elise	friday	burger
steve	friday	hotdog
joe	monday	hotdog
joe	tuesday	hotdog
joe	friday	baguette

## Dishes

Dish	Item
burger	patty
burger	onion
burger	bun
hotdog	bun
hotdog	onion
hotdog	sausage

## Items

Item	Price
patty	6
onion	2
bun	2
sausage	4
cheese	3

# The Database of a Fast Food Restaurant

**Orders**

Customer	Day	Dish
elise	monday	burger
elise	friday	burger
steve	friday	hotdog
joe	monday	hotdog
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joe	friday	baguette

**Dishes**

Dish	Item
burger	patty
burger	onion
burger	bun
hotdog	bun
hotdog	onion
hotdog	sausage

**Items**

Item	Price
patty	6
onion	2
bun	2
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$$Q(\text{Customer, Day, Dish, Item, Price}) = \text{Orders} \bowtie \text{Dishes} \bowtie \text{Items}$$

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elise	monday	burger	bun	2
elise	friday	burger	patty	6
elise	friday	burger	onion	2
elise	friday	burger	bun	2

⋮ ⋮ ⋮ ⋮ ⋮

# Redundancies in the Query Result

**Orders**

Customer	Day	Dish
elise	monday	burger
elise	friday	burger
steve	friday	hotdog
joe	monday	hotdog
joe	tuesday	hotdog
joe	friday	baguette

**Dishes**

Dish	Item
burger	patty
burger	onion
burger	bun
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**Items**

Item	Price
patty	6
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elise	friday	burger	patty	6
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⋮ ⋮ ⋮ ⋮ ⋮

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elise	friday	burger	onion	2
elise	friday	burger	bun	2

⋮ ⋮ ⋮ ⋮ ⋮

# Searching for a Core of the Query Result

$Q(\text{Customer, Day, Dish, Item, Price}) = \text{Orders} \bowtie \text{Dishes} \bowtie \text{Items}$

Customer	Day	Dish	Item	Price
elise	monday	burger	patty	6
elise	monday	burger	onion	2
elise	monday	burger	bun	2
elise	friday	burger	patty	6
elise	friday	burger	onion	2
elise	friday	burger	bun	2

⋮ ⋮ ⋮ ⋮ ⋮

# The Hypergraph of (a part of) the Query Result

elise	monday	burger
-------	--------	--------

elise	friday	burger
-------	--------	--------

$$Q(\text{Customer, Day, Dish, Item, Price}) = \text{Orders} \bowtie \text{Dishes} \bowtie \text{Items}$$

Customer	Day	Dish	Item	Price
elise	monday	burger	patty	6
elise	monday	burger	onion	2
elise	monday	burger	bun	2
elise	friday	burger	patty	6
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elise	friday	burger	bun	2

⋮ ⋮ ⋮ ⋮ ⋮

# The Hypergraph of (a part of) the Query Result

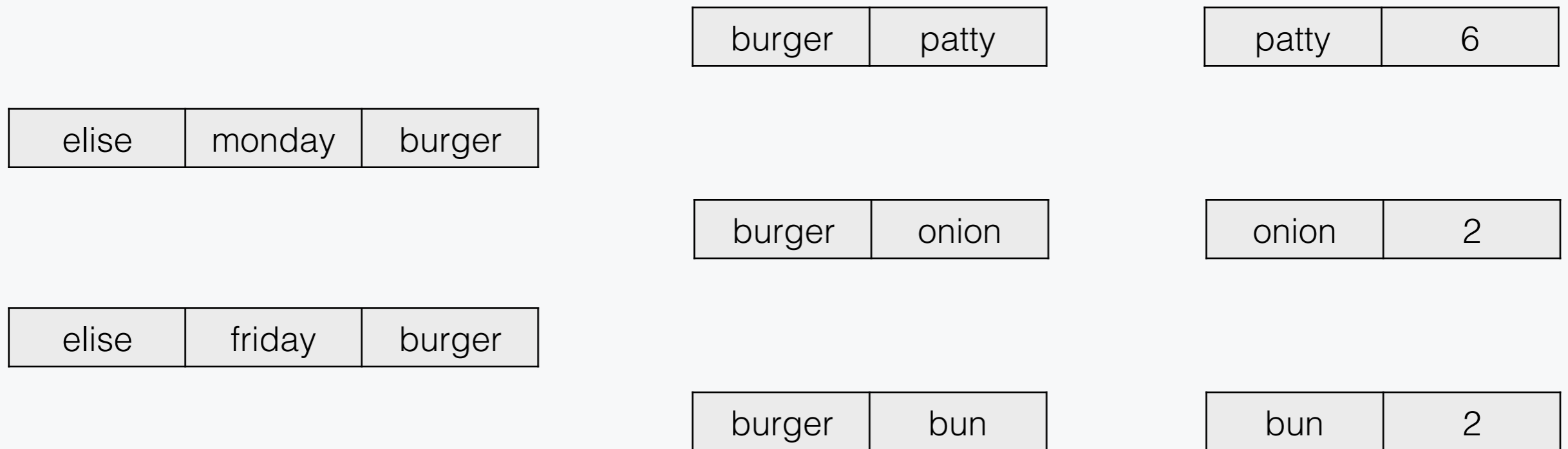


$$Q(\text{Customer, Day, Dish, Item, Price}) = \text{Orders} \bowtie \text{Dishes} \bowtie \text{Items}$$

Customer	Day	Dish	Item	Price
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elise	monday	burger	bun	2
elise	friday	burger	patty	6
elise	friday	burger	onion	2
elise	friday	burger	bun	2
⋮	⋮	⋮	⋮	⋮



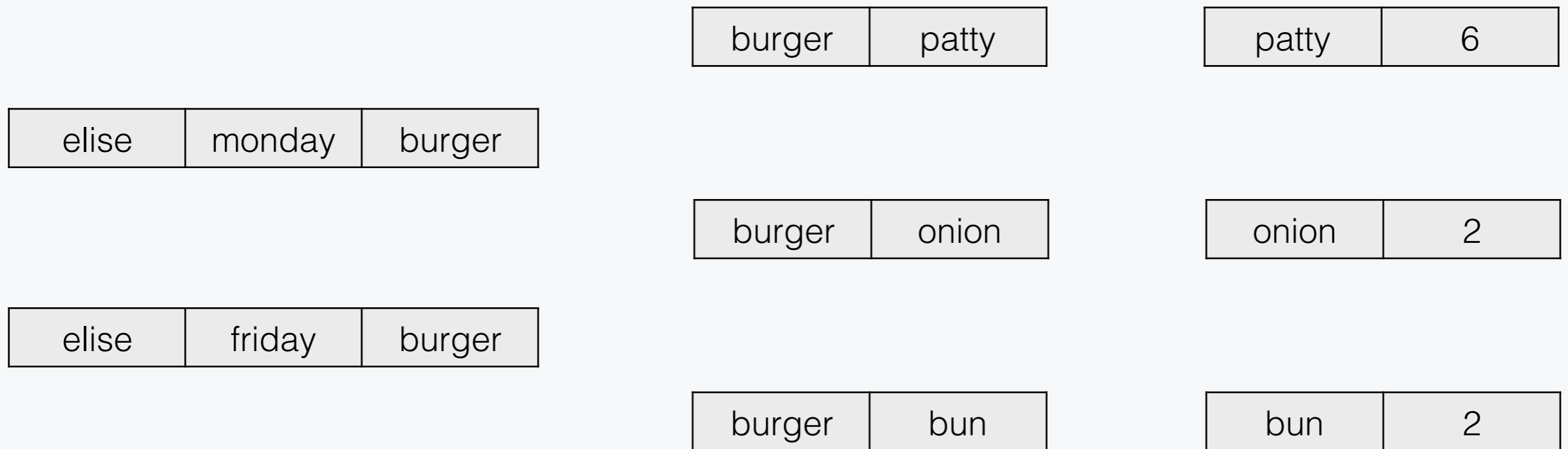
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elise	friday	burger	onion	2
elise	friday	burger	bun	2
⋮	⋮	⋮	⋮	⋮

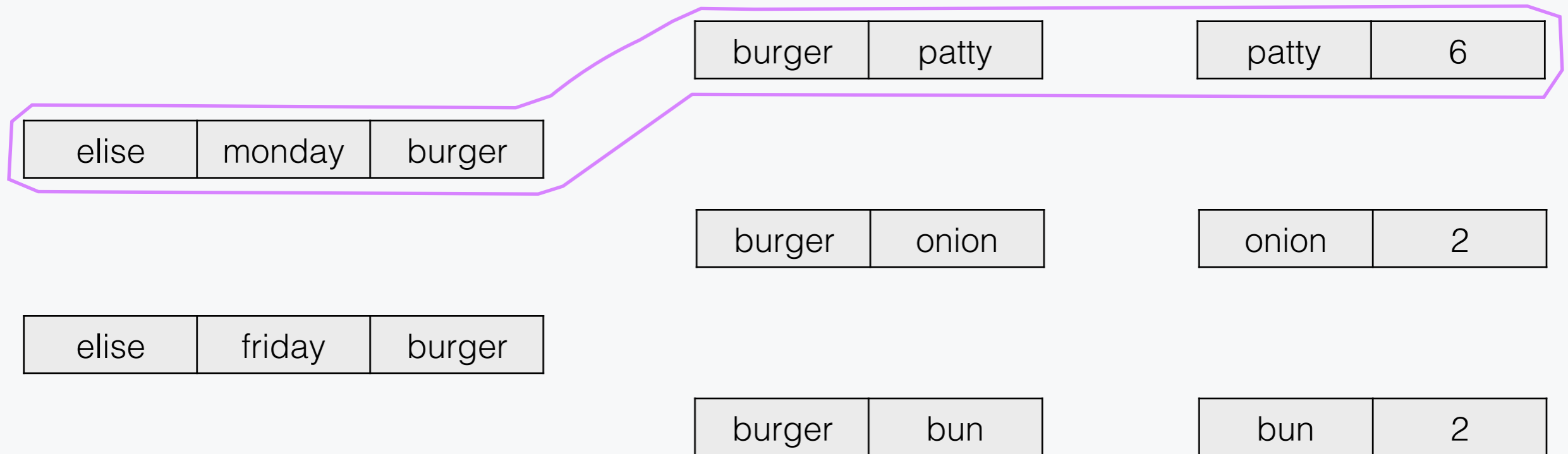
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elise	friday	burger	onion	2
elise	friday	burger	bun	2
⋮	⋮	⋮	⋮	⋮

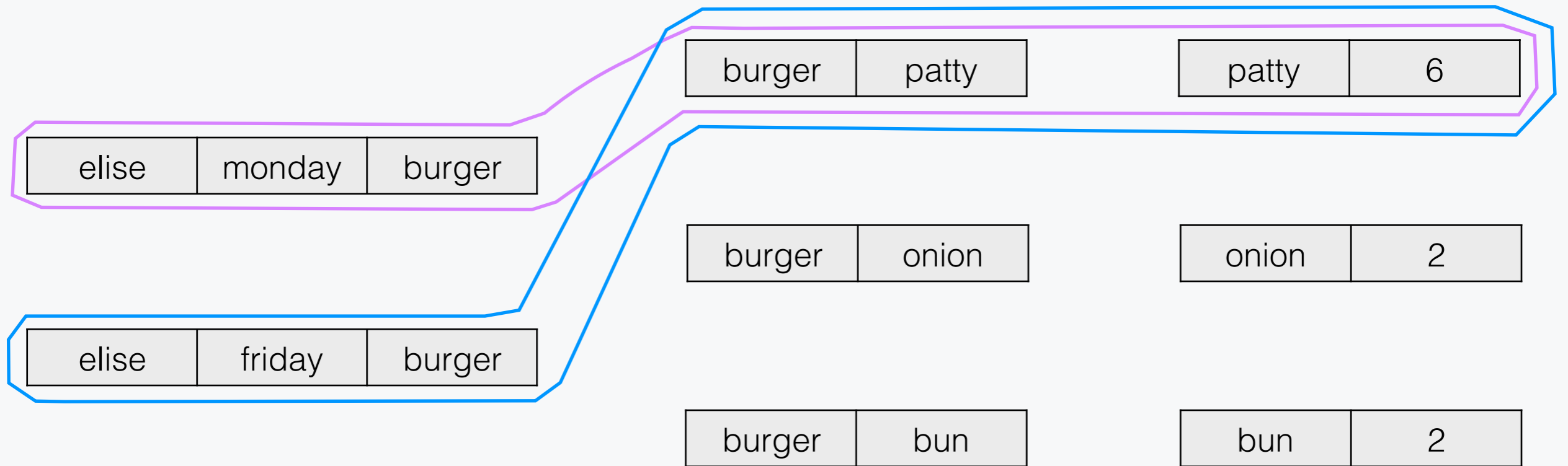
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$$Q(\text{Customer, Day, Dish, Item, Price}) = \text{Orders} \bowtie \text{Dishes} \bowtie \text{Items}$$

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elise	friday	burger	onion	2
elise	friday	burger	bun	2
⋮	⋮	⋮	⋮	⋮

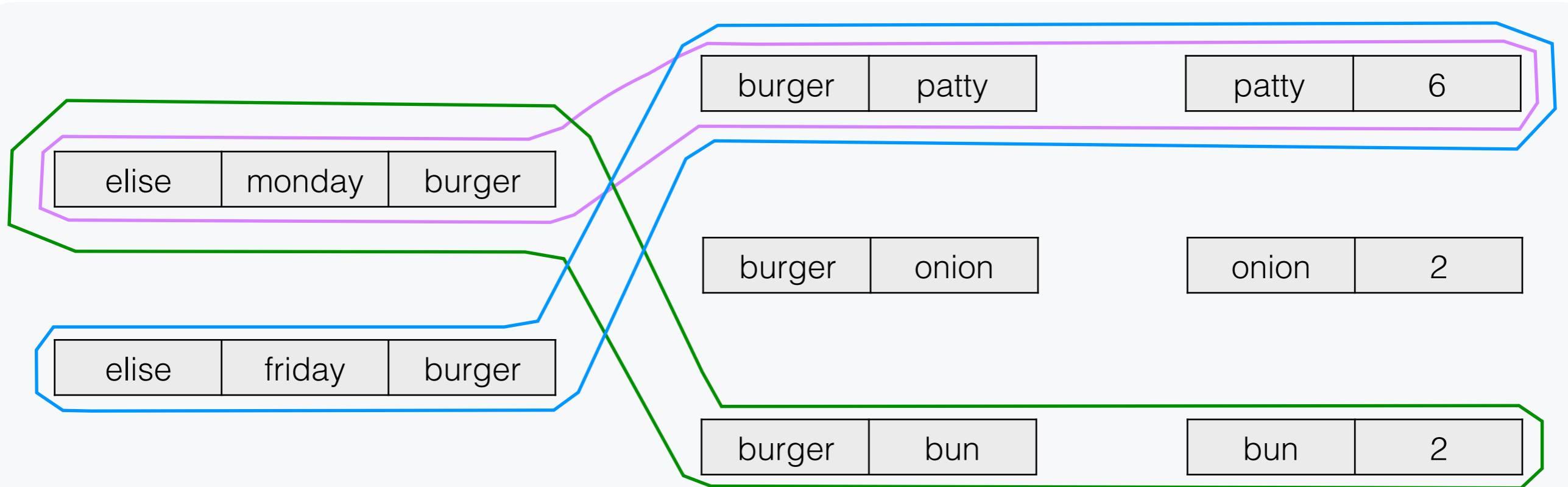
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elise	friday	burger	patty	6
elise	friday	burger	onion	2
elise	friday	burger	bun	2
⋮	⋮	⋮	⋮	⋮

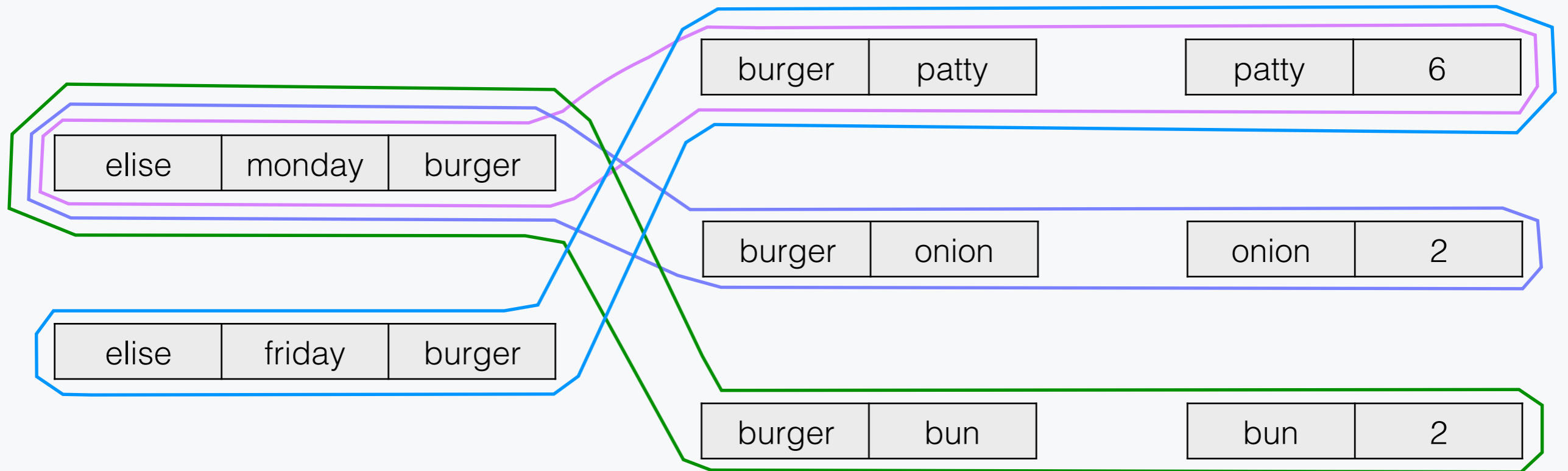
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elise	monday	burger	bun	2
elise	friday	burger	patty	6
elise	friday	burger	onion	2
elise	friday	burger	bun	2
⋮	⋮	⋮	⋮	⋮

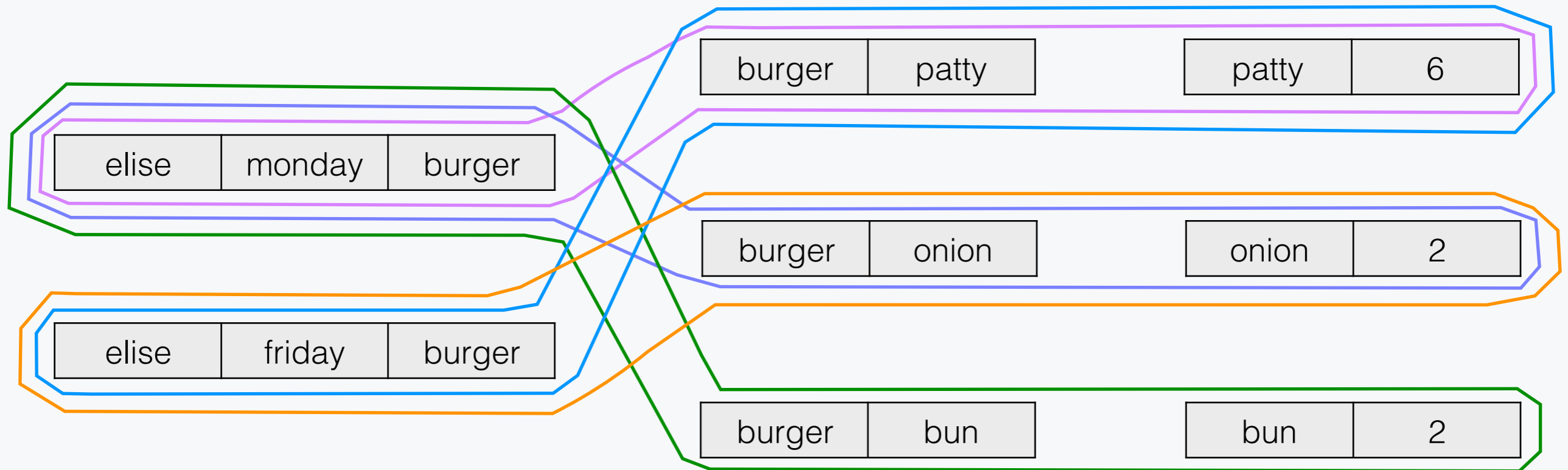
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elise	friday	burger	onion	2
elise	friday	burger	bun	2
⋮	⋮	⋮	⋮	⋮

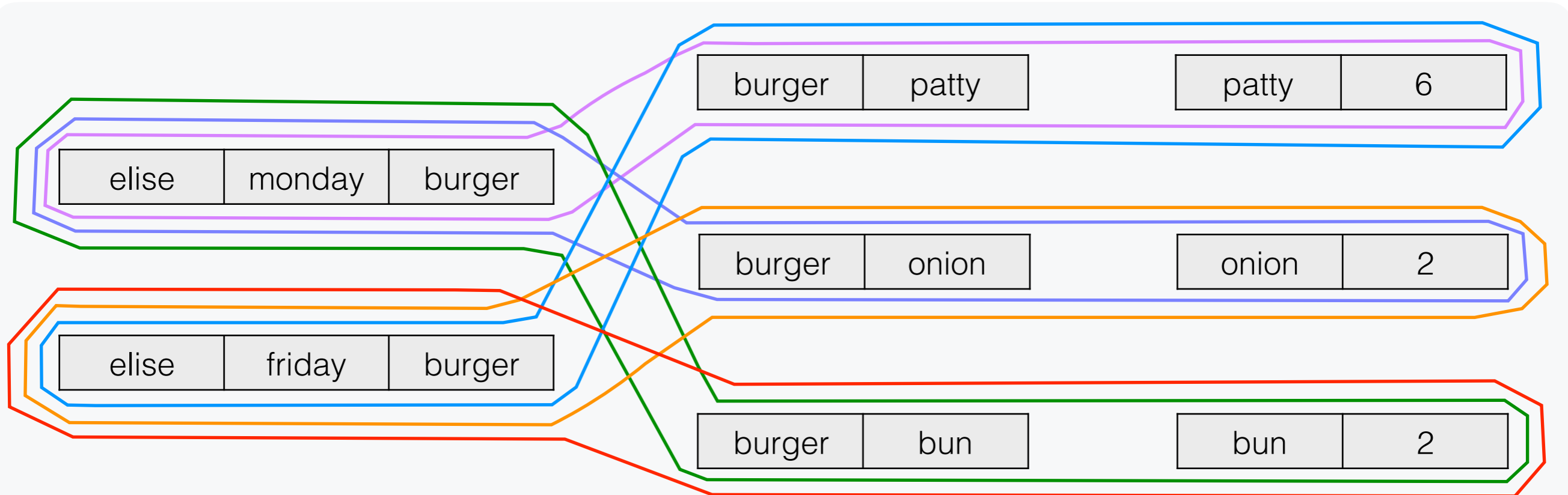
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$$Q(\text{Customer, Day, Dish, Item, Price}) = \text{Orders} \bowtie \text{Dishes} \bowtie \text{Items}$$

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elise	friday	burger	onion	2
elise	friday	burger	bun	2
⋮	⋮	⋮	⋮	⋮

# The Hypergraph of (a part of) the Query Result

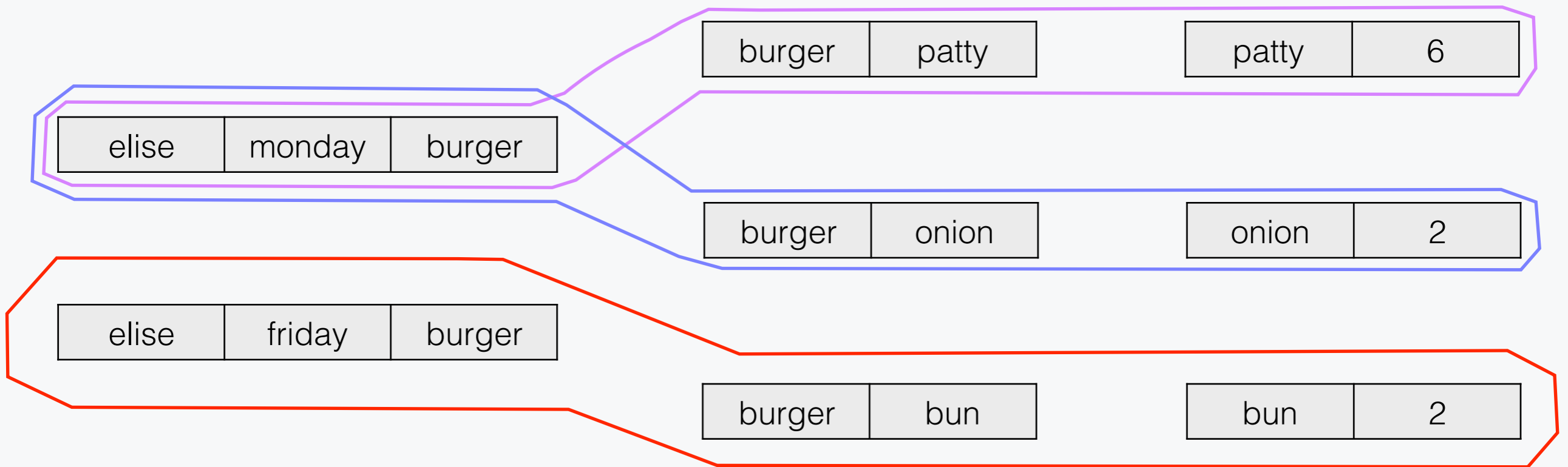


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elise	monday	burger	patty	6
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elise	monday	burger	bun	2
elise	friday	burger	patty	6
elise	friday	burger	onion	2
elise	friday	burger	bun	2
⋮	⋮	⋮	⋮	⋮



# A Minimal Edge Cover of the Hypergraph



$$Q(\text{Customer, Day, Dish, Item, Price}) = \text{Orders} \bowtie \text{Dishes} \bowtie \text{Items}$$

Customer	Day	Dish	Item	Price
elise	monday	burger	patty	6
elise	monday	burger	onion	2
elise	monday	burger	bun	2
elise	friday	burger	patty	6
elise	friday	burger	onion	2
elise	friday	burger	bun	2
⋮	⋮	⋮	⋮	⋮

## A Cover of (a part of) the Query Result

Customer	Day	Dish	Item	Price
elise	monday	burger	patty	6
elise	monday	burger	onion	2
elise	friday	burger	bun	2
⋮	⋮	⋮	⋮	⋮

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elise	friday	burger	patty	6
elise	friday	burger	onion	2
elise	friday	burger	bun	2
⋮	⋮	⋮	⋮	⋮

# Why Covers?

## Observation on Query Results

- Query results can contain redundancies in computation and representation

## Benefits of Covers

- A succinct **relational** representation system for query results
- The full query result can be enumerated from a cover with linearithmic time pre-computation and constant delay
- Allow for the succinct representation of the results of a wide range of computational problems
- Support subsequent operations:
  - aggregates
  - learning regression models
  - parallel computation

# Previous Work

## Foundations of Factorised Databases

- *Size Bounds for Factorised Representations of Query Results*  
[Olteanu and Závodný, TODS 2015]
- *FDB: A Query Engine for Factorised Relational Databases*  
[Bakibayev, Olteanu and Závodný, PVLDB, 2012]
- *Factorised Representations of Query Results: Size Bounds and Readability*  
[Olteanu and Závodný, ICDT 2012]

## Aggregation in Factorised Databases

- *Aggregation and Ordering in Factorised Databases*  
[Bakibayev, Kočiský, Olteanu, and Závodný, PVLDB 2013]

## Regression Models in Factorised Databases

- *Learning Linear regression Models over Factorised Joins*  
[Schleich, Olteanu and Ciucanu, SIGMOD 2016]
- *F: Regression Models over Factorised Views*  
[Olteanu and Schleich, PVLDB 2016]

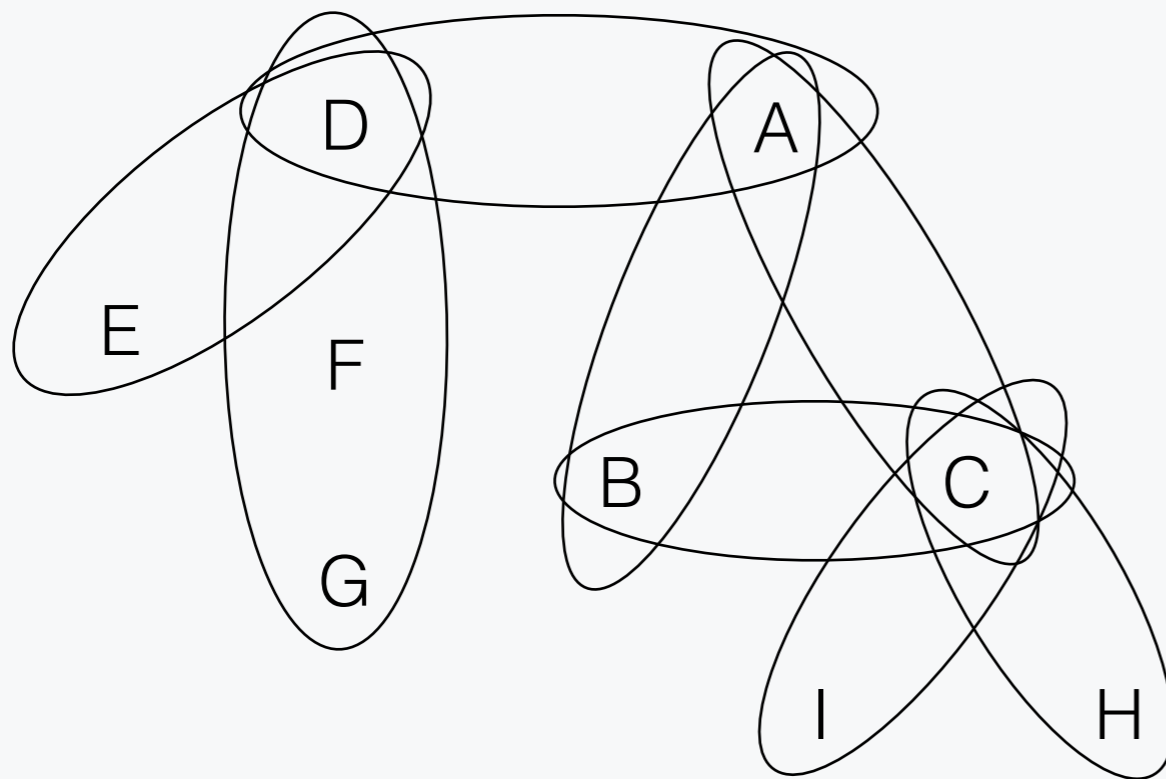
# Outline of the Talk

- Covers of Query Results
- Computation of Covers
- Covers of more general Computational Problems

# Hypertree Decompositions

$$Q(A, B, C, D, E, F, G, H, I) = R(A, B) \bowtie S(A, C) \bowtie T(B, C) \bowtie U(D, A) \bowtie \\ V(E, D) \bowtie W(D, F, G) \bowtie X(C, I) \bowtie Y(C, H)$$

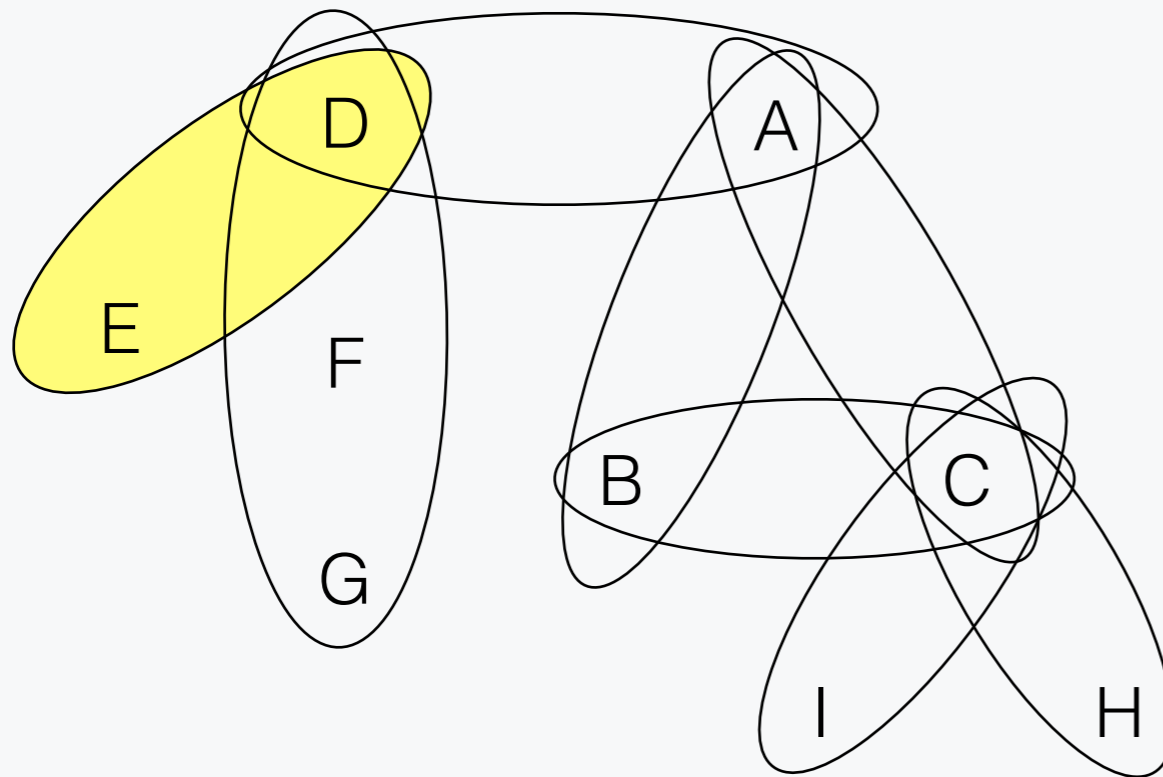
Hypergraph of Query  $Q$



# Hypertree Decompositions

$$Q(A, B, C, D, E, F, G, H, I) = R(A, B) \bowtie S(A, C) \bowtie T(B, C) \bowtie U(D, A) \bowtie V(E, D) \bowtie W(D, F, G) \bowtie X(C, I) \bowtie Y(C, H)$$

Hypergraph of Query  $Q$



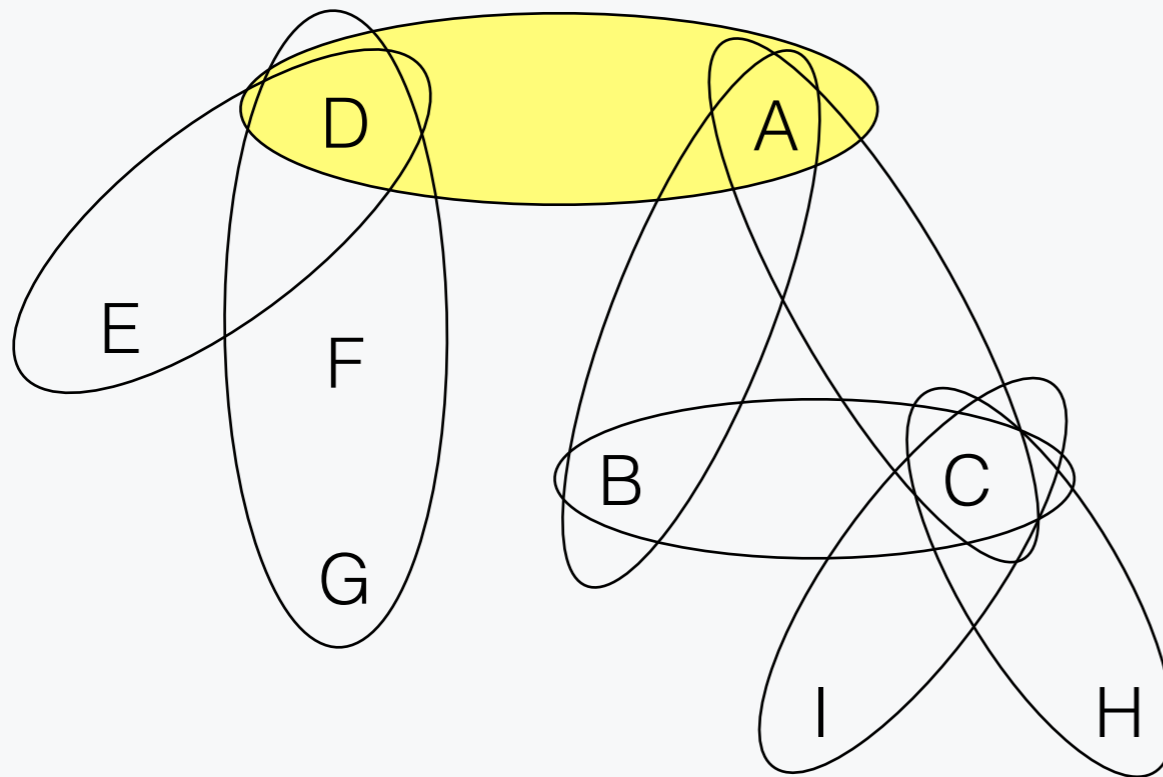
A Hypertree Decomposition of  $Q$



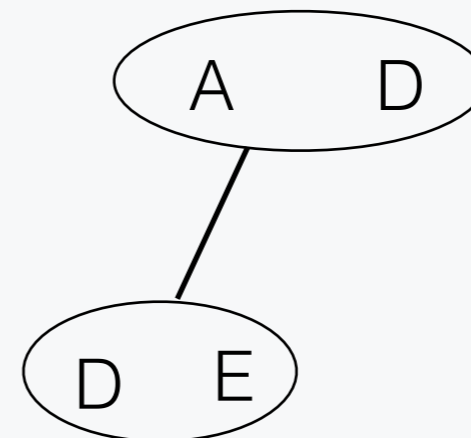
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Hypergraph of Query  $Q$



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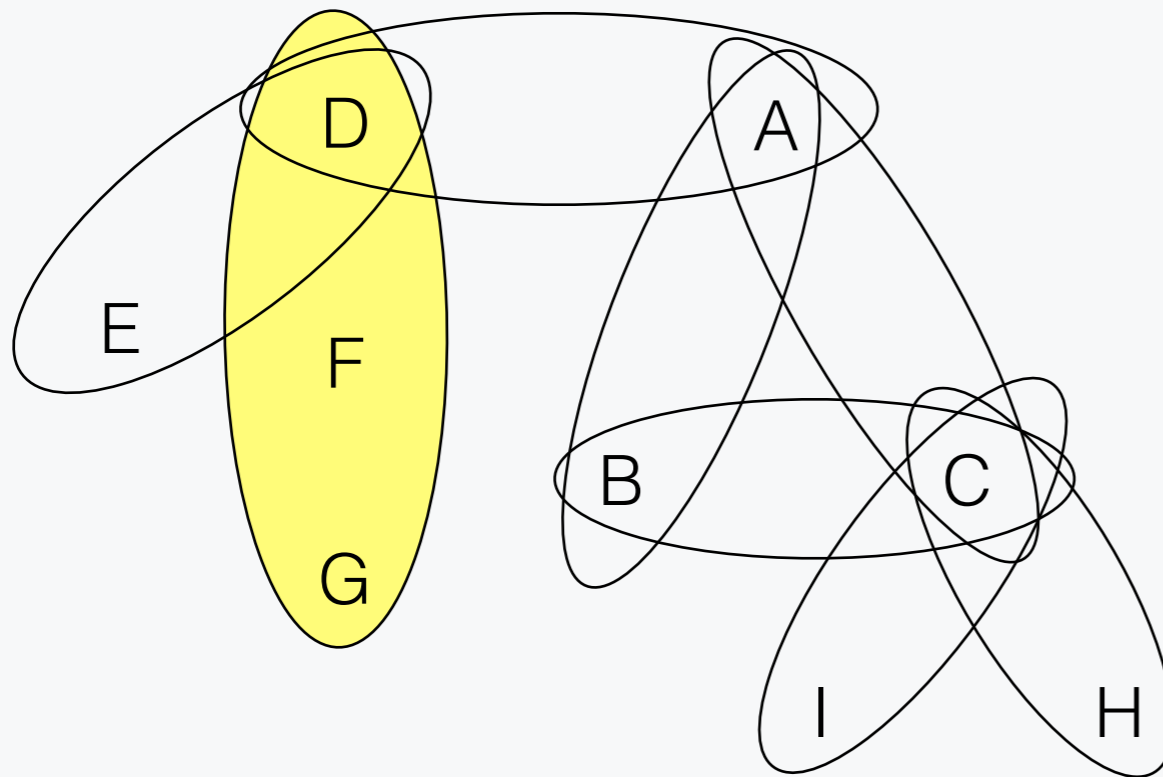




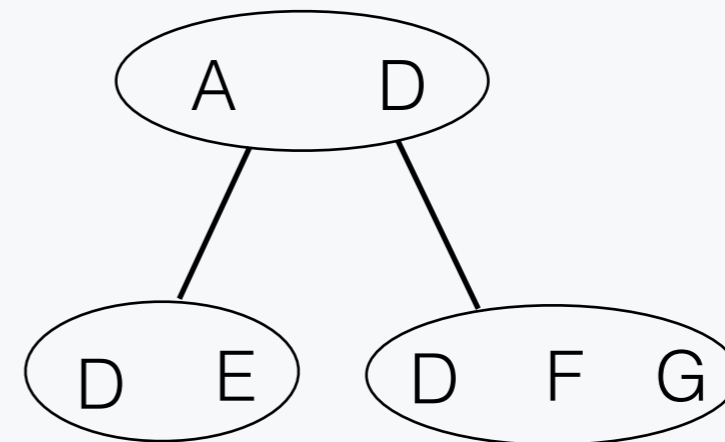
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Hypergraph of Query  $Q$



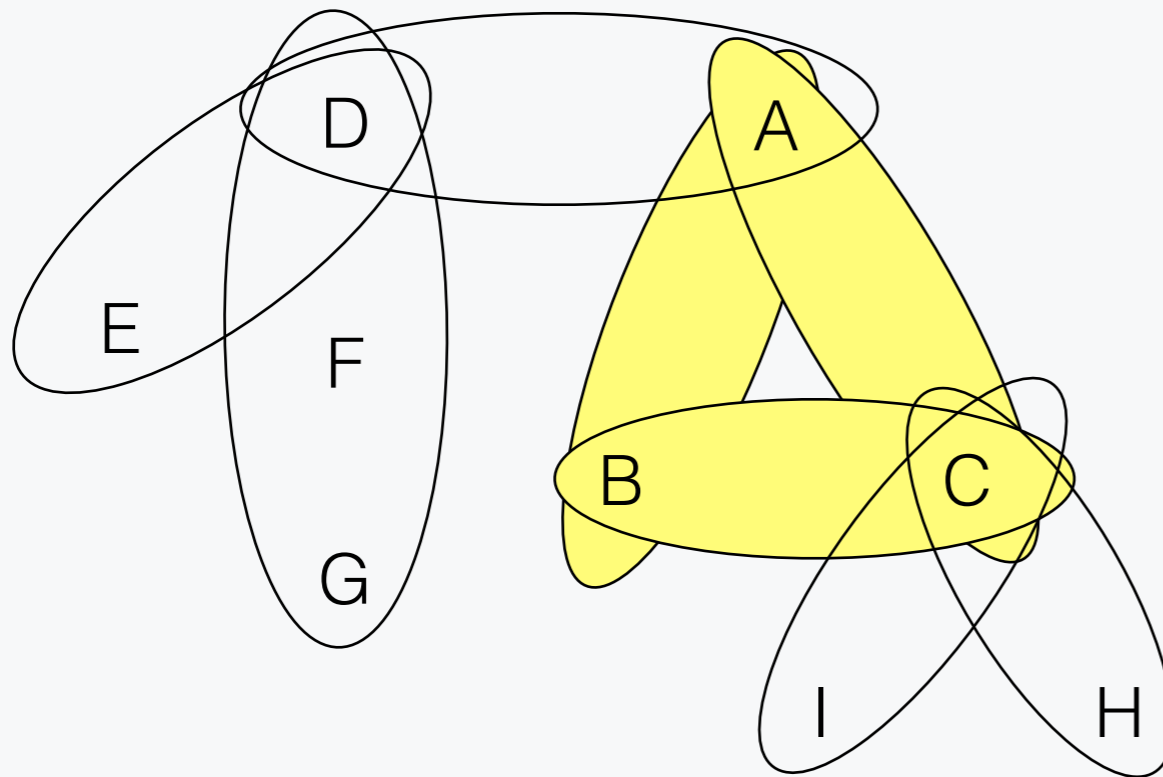
A Hypertree Decomposition of  $Q$



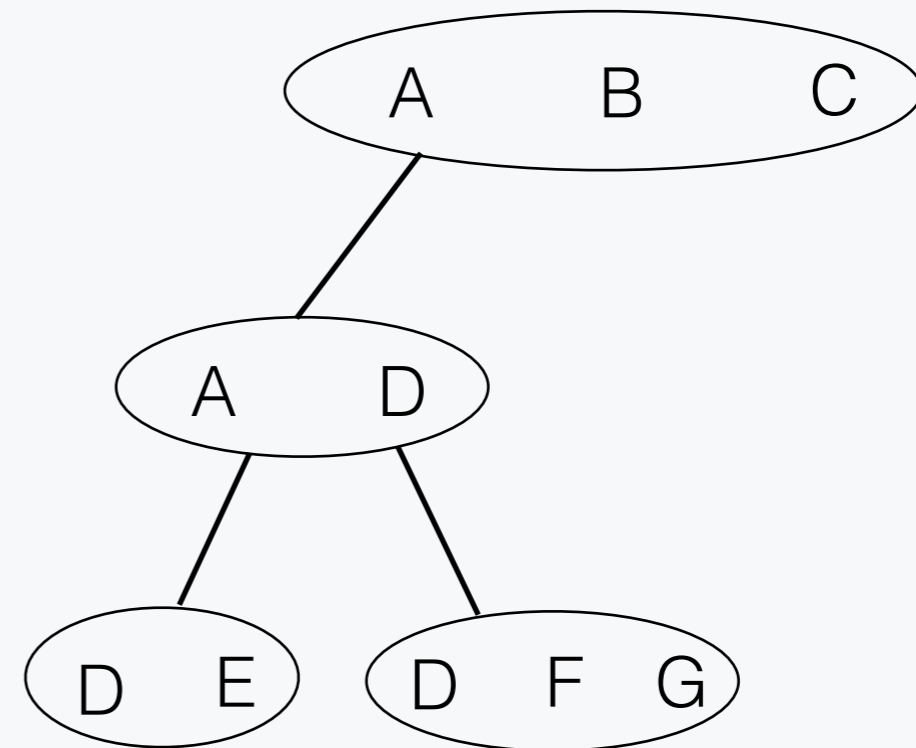
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Hypergraph of Query  $Q$



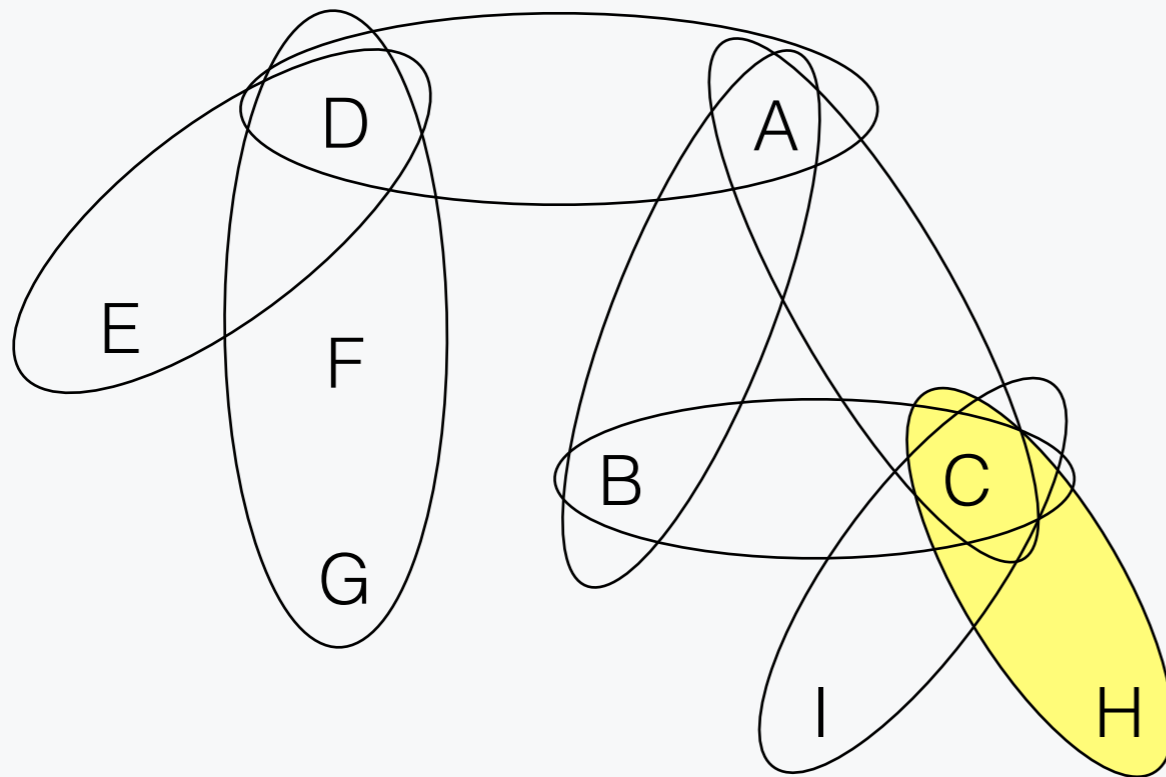
A Hypertree Decomposition of  $Q$



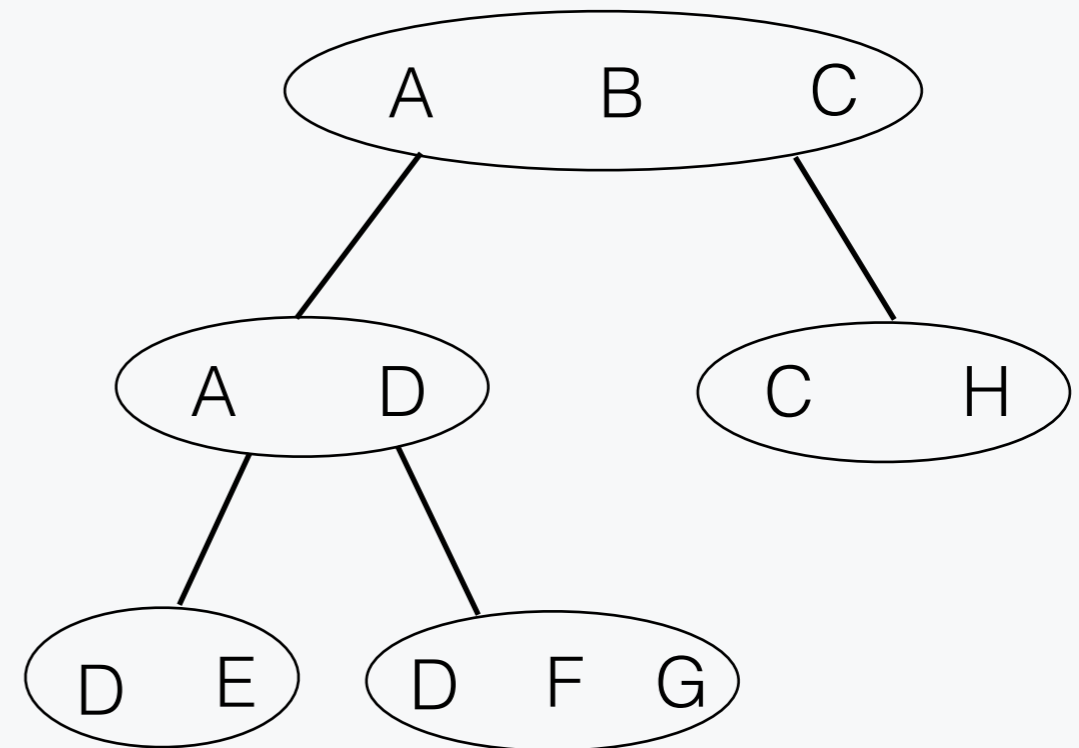
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Hypergraph of Query  $Q$



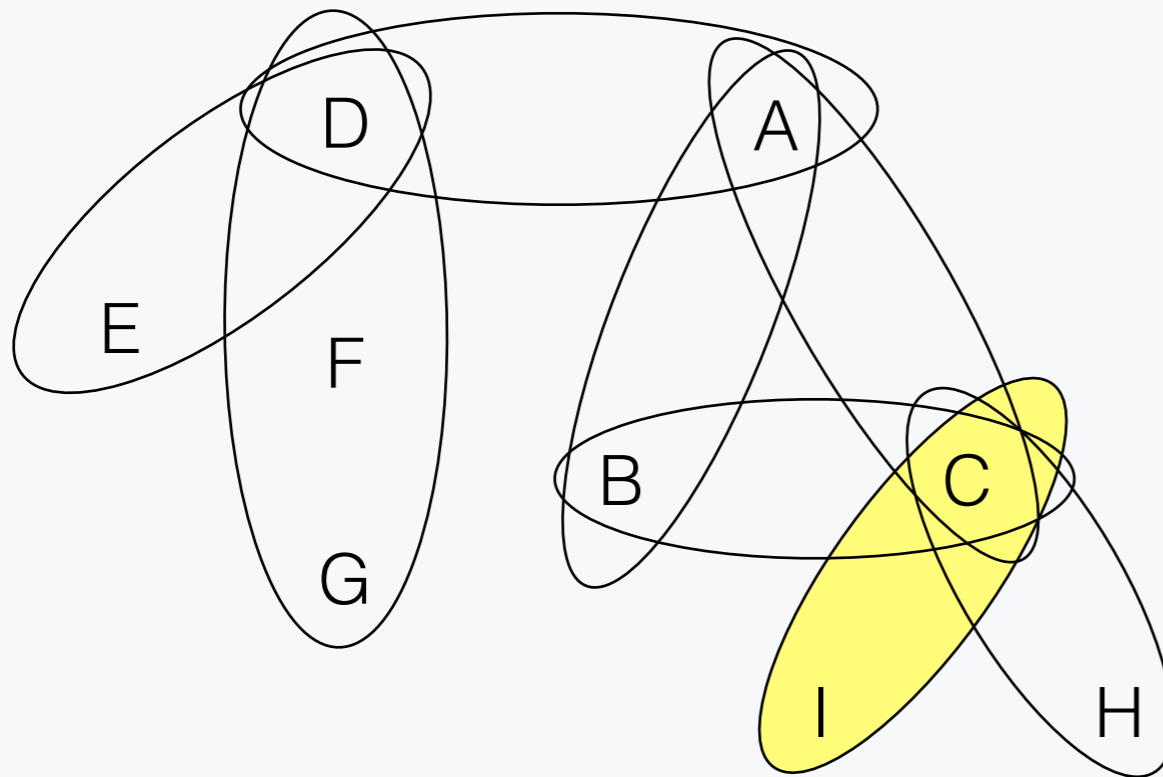
A Hypertree Decomposition of  $Q$



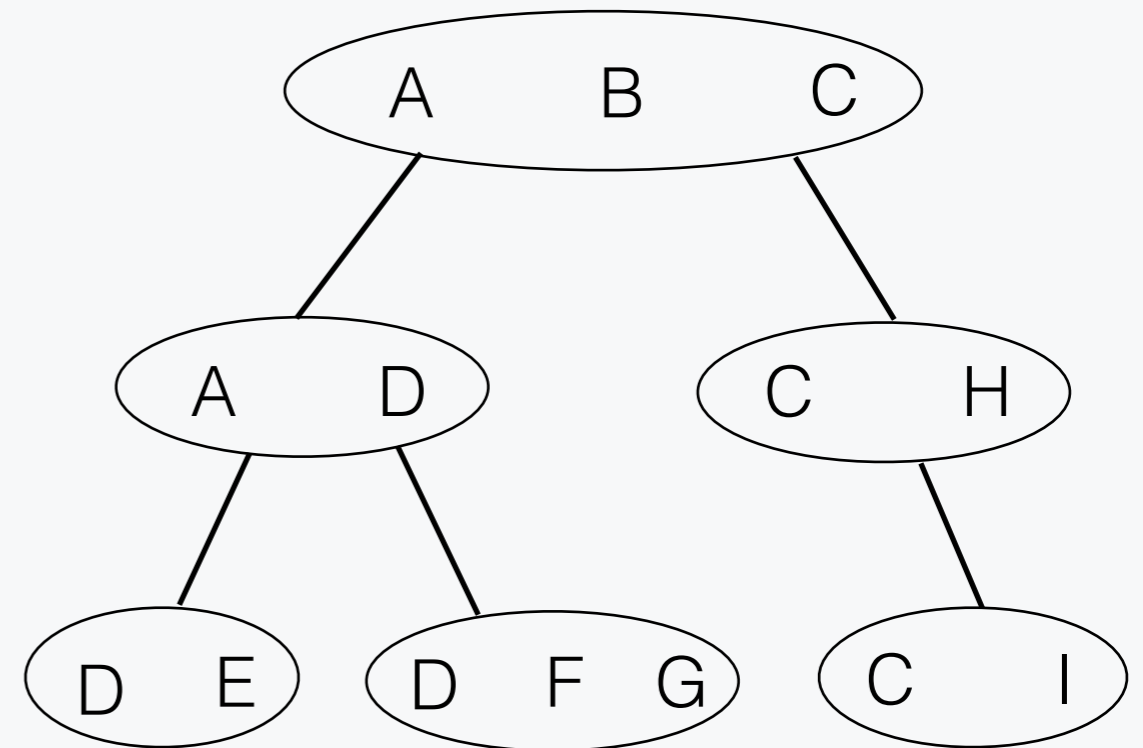
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Hypergraph of Query  $Q$



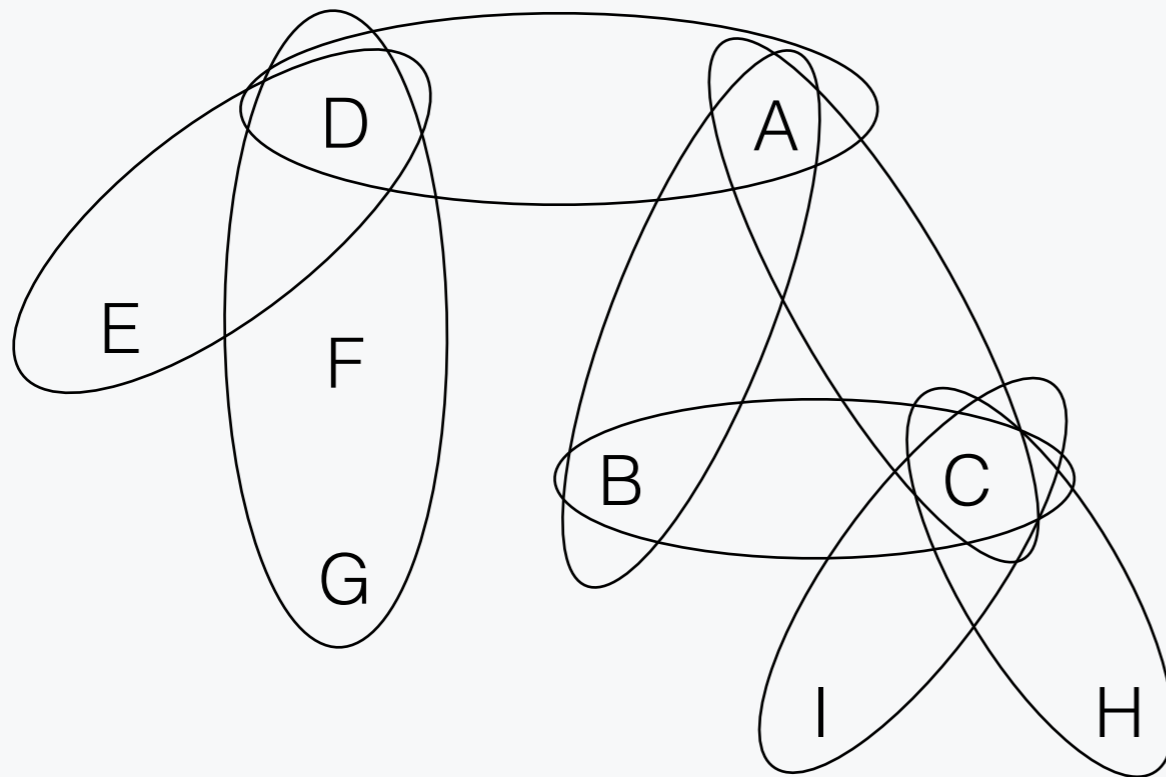
A Hypertree Decomposition of  $Q$



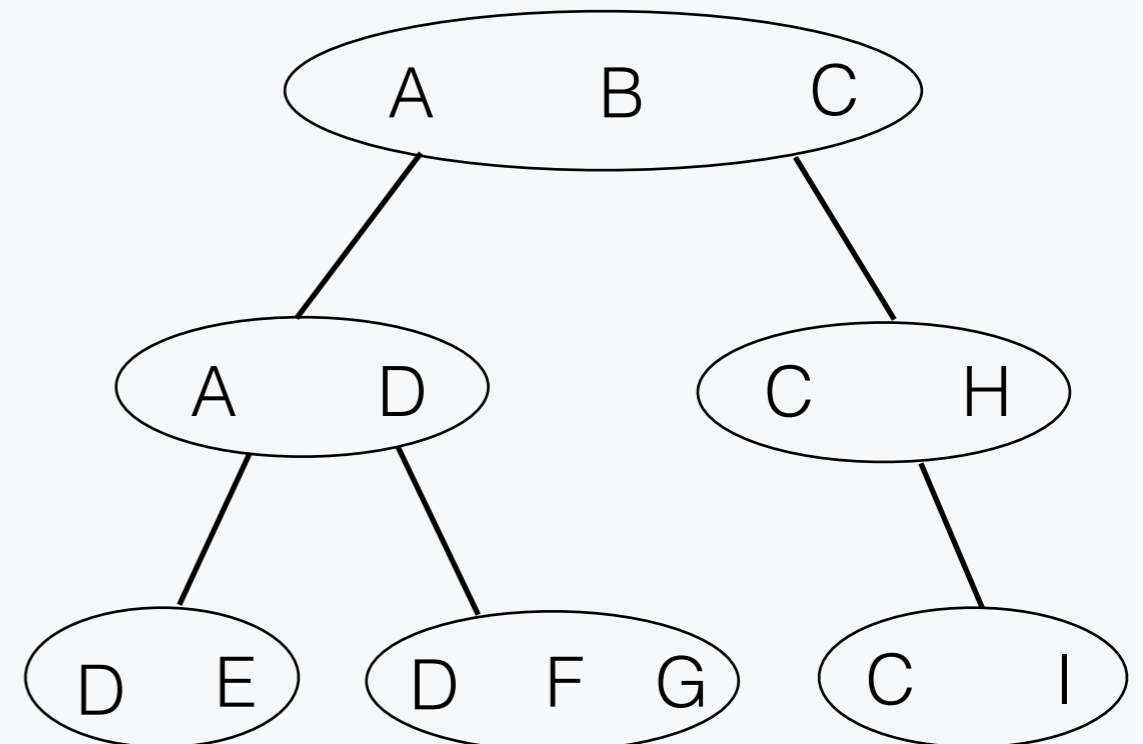
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Hypergraph of Query  $Q$



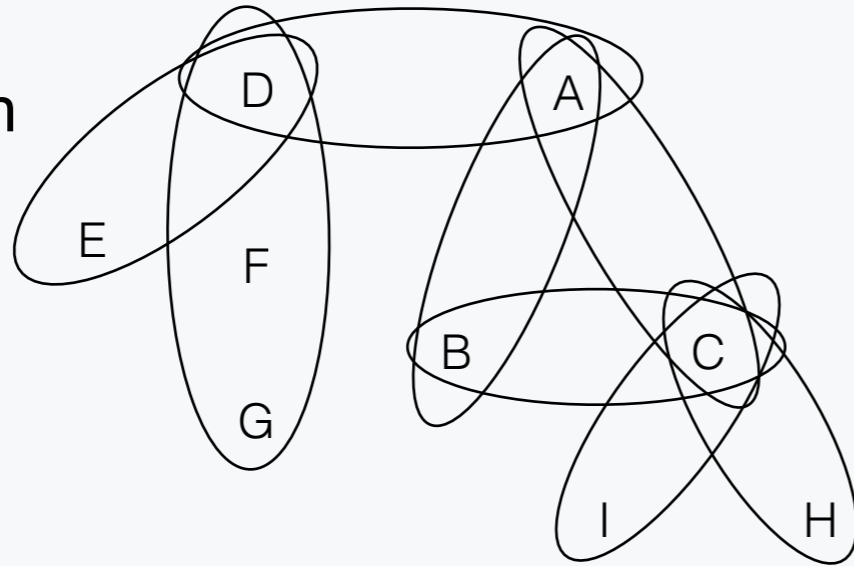
A Hypertree Decomposition of  $Q$



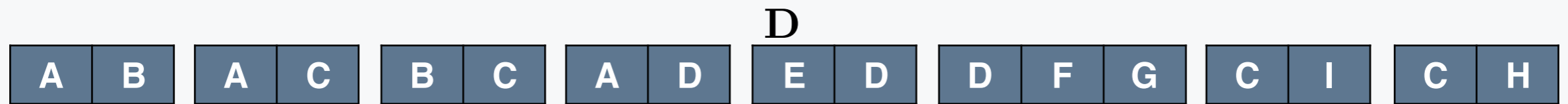
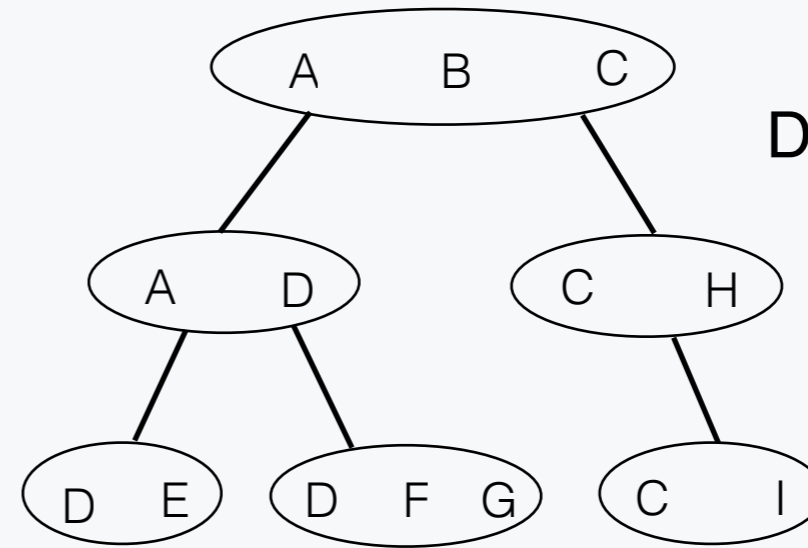
Coverage, Connectivity

# Result Preservation

Hypergraph  
of  $Q$ :



A Hypertree  
Decomposition  
of  $Q$



$Q(\mathbf{D})$

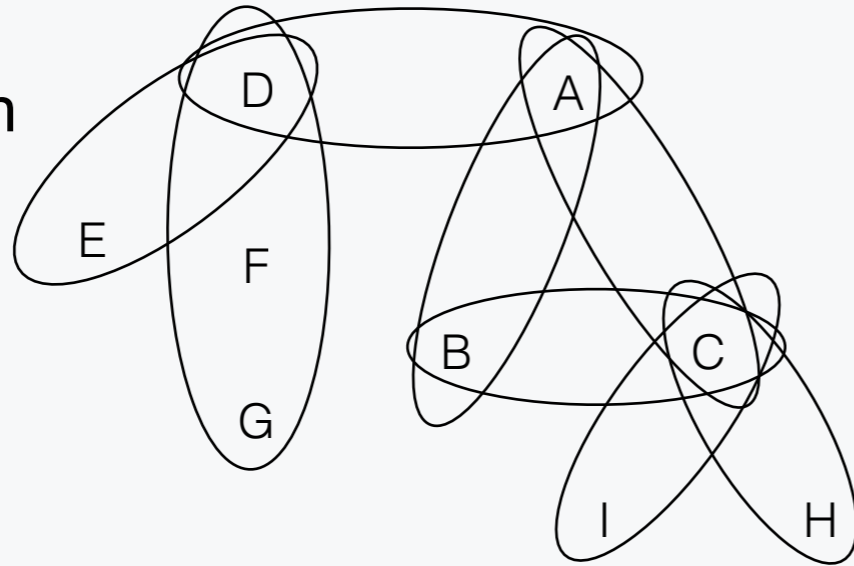


**C**

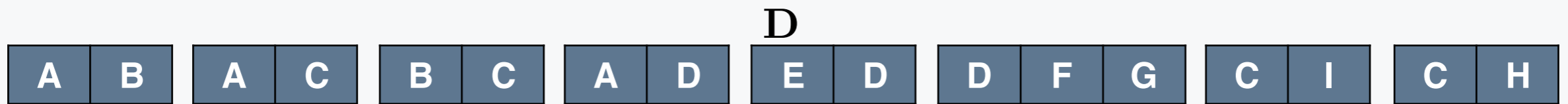
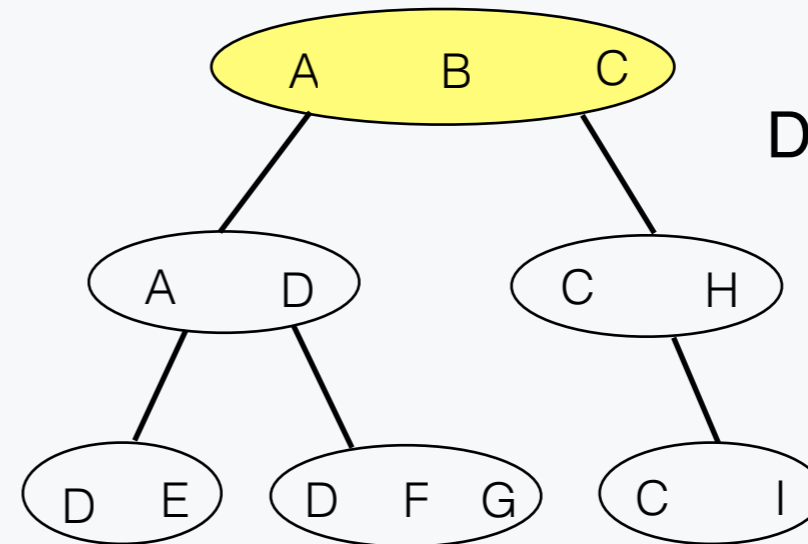


# Result Preservation

Hypergraph  
of  $Q$ :



A Hypertree  
Decomposition  
of  $Q$



$Q(\mathbf{D})$



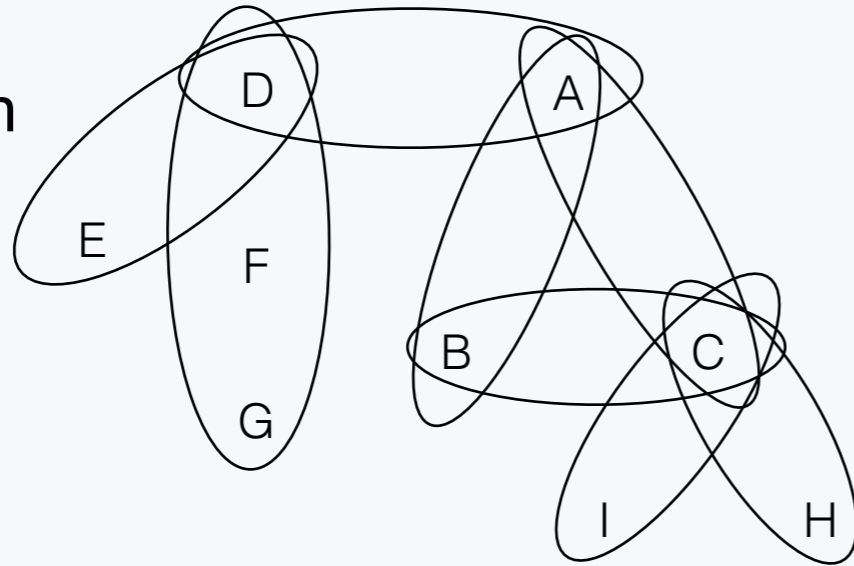
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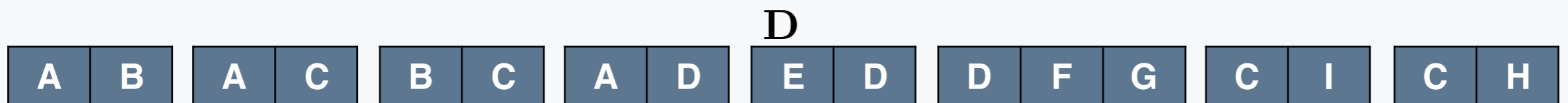
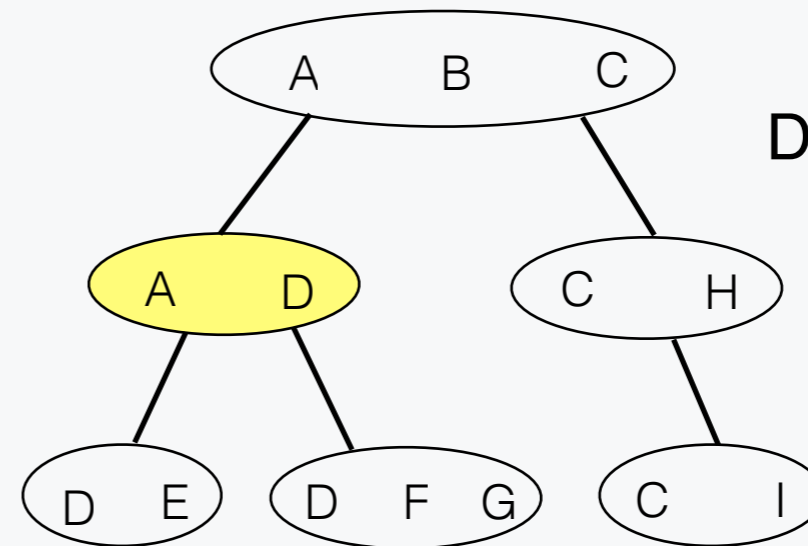
$$\pi_{\{A,B,C\}}Q(\mathbf{D}) = \pi_{\{A,B,C\}}\mathbf{C}$$

# Result Preservation

Hypergraph  
of  $Q$ :



A Hypertree  
Decomposition  
of  $Q$



$Q(\mathbf{D})$



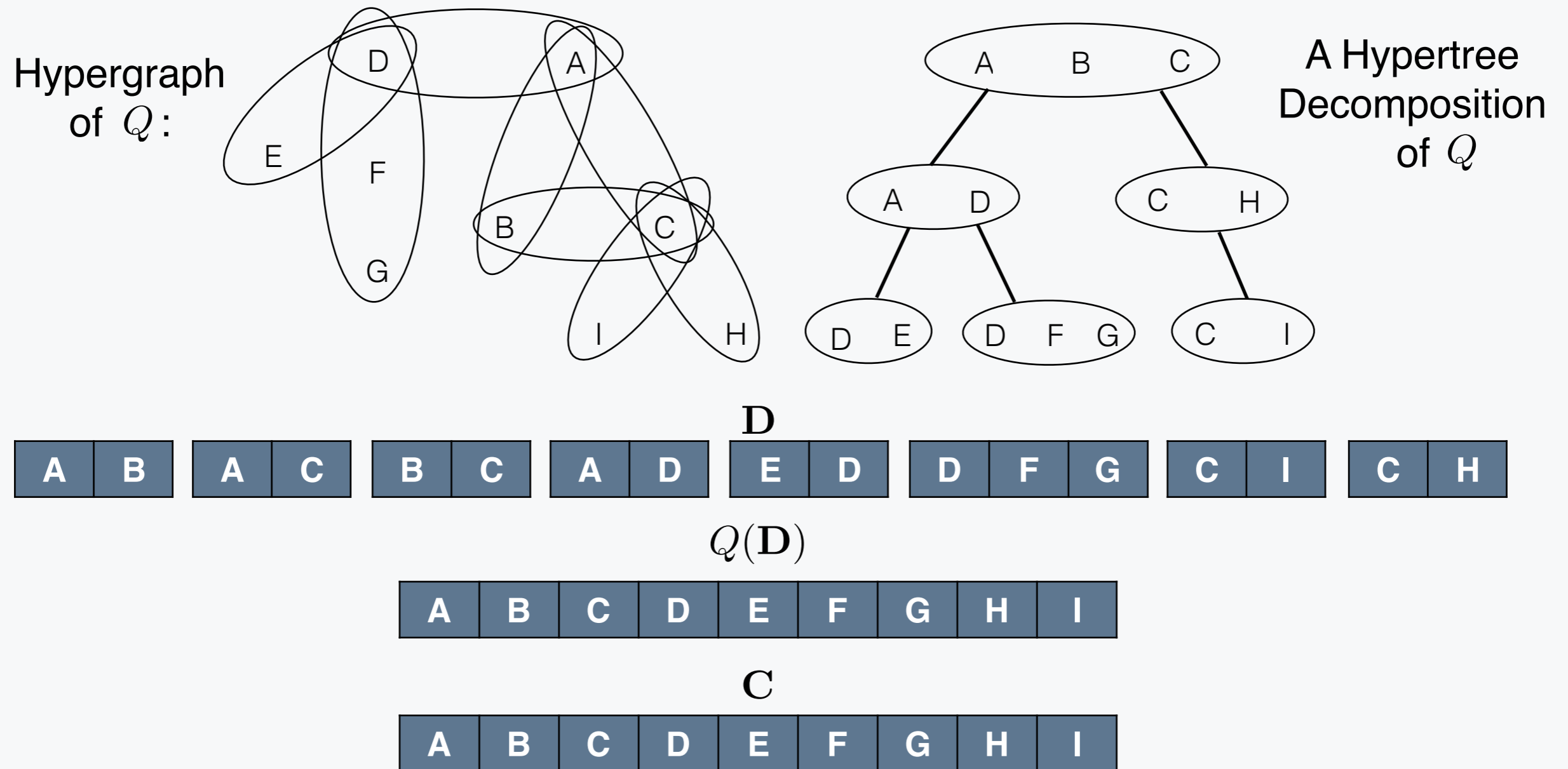
**C**



$$\pi_{\{A,D\}}Q(\mathbf{D}) = \pi_{\{A,D\}}\mathbf{C}$$



# Result Preservation



Given a query  $Q$ , a hypertree decomposition  $\mathcal{T}$  of  $Q$  and a database  $\mathbf{D}$ , a relation  $\mathbf{C}$  is result-preserving with respect to  $(Q, \mathcal{T}, \mathbf{D})$  if

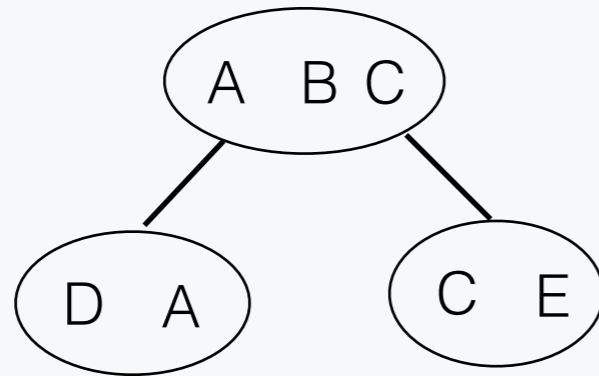
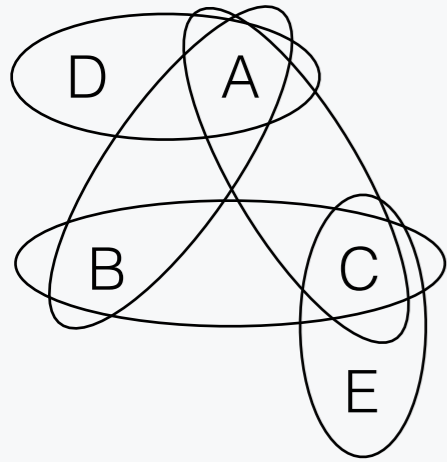
$$\pi_B \mathbf{C} = \pi_B Q(\mathbf{C}) \text{ for each bag } B \text{ of } \mathcal{T}.$$

# Covers of Query Results

Given a query  $Q$ , a hypertree decomposition  $\mathcal{T}$  of  $Q$  and a database  $\mathbf{D}$ ,  
a cover of the query result  $Q(\mathbf{D})$  over  $\mathcal{T}$  is a  
**minimal result-preserving** relation with respect to  $(Q, \mathcal{T}, \mathbf{D})$ .

# Covers and Non-Covers

$$Q(A, B, C, D, E) = R \bowtie S \bowtie T \bowtie U \bowtie V$$



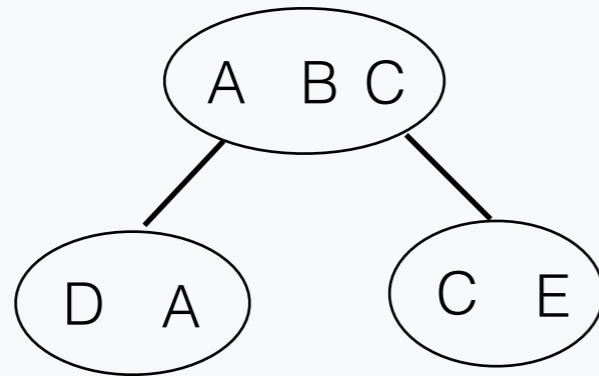
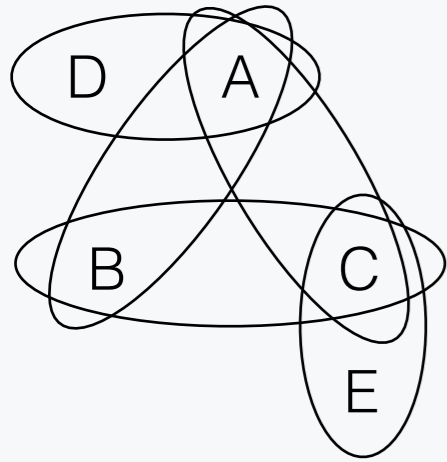
R		S		T		U		V	
D	A	A	B	A	C	B	C	C	E
d <sub>1</sub>	a <sub>1</sub>	a <sub>1</sub>	b <sub>1</sub>	a <sub>1</sub>	c <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>	c <sub>1</sub>	e <sub>1</sub>
d <sub>2</sub>	a <sub>1</sub>	a <sub>1</sub>	b <sub>2</sub>	a <sub>1</sub>	c <sub>2</sub>	b <sub>2</sub>	c <sub>1</sub>	c <sub>1</sub>	e <sub>2</sub>
d <sub>3</sub>	a <sub>1</sub>	a <sub>1</sub>	b <sub>3</sub>	a <sub>2</sub>	c <sub>2</sub>	b <sub>3</sub>	c <sub>2</sub>	c <sub>2</sub>	e <sub>1</sub>
d <sub>1</sub>	a <sub>2</sub>	a <sub>2</sub>	b <sub>4</sub>	a <sub>1</sub>	c <sub>3</sub>	b <sub>4</sub>	c <sub>2</sub>	c <sub>2</sub>	e <sub>2</sub>
d <sub>2</sub>	a <sub>2</sub>	a <sub>2</sub>	b <sub>1</sub>					c <sub>3</sub>	e <sub>1</sub>

$Q(\mathbf{D})$

D	A	B	C	E
d <sub>1</sub>	a <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>	e <sub>1</sub>
d <sub>2</sub>	a <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>	e <sub>1</sub>
d <sub>3</sub>	a <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>	e <sub>1</sub>
d <sub>1</sub>	a <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>	e <sub>2</sub>
d <sub>2</sub>	a <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>	e <sub>1</sub>
d <sub>1</sub>	a <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>	e <sub>1</sub>
⋮	⋮	⋮	⋮	⋮

# Covers and Non-Covers

$$Q(A, B, C, D, E) = R \bowtie S \bowtie T \bowtie U \bowtie V$$



R	
D	A
d <sub>1</sub>	a <sub>1</sub>
d <sub>2</sub>	a <sub>1</sub>
d <sub>3</sub>	a <sub>1</sub>
d <sub>1</sub>	a <sub>2</sub>
d <sub>2</sub>	a <sub>2</sub>

S	
A	B
a <sub>1</sub>	b <sub>1</sub>
a <sub>1</sub>	b <sub>2</sub>
a <sub>1</sub>	b <sub>3</sub>
a <sub>2</sub>	b <sub>4</sub>
a <sub>2</sub>	b <sub>1</sub>

T	
A	C
a <sub>1</sub>	c <sub>1</sub>
a <sub>1</sub>	c <sub>2</sub>
a <sub>2</sub>	c <sub>2</sub>
a <sub>1</sub>	c <sub>3</sub>

U	
B	C
b <sub>1</sub>	c <sub>1</sub>
b <sub>2</sub>	c <sub>1</sub>
b <sub>3</sub>	c <sub>2</sub>
b <sub>4</sub>	c <sub>2</sub>

V	
C	E
c <sub>1</sub>	e <sub>1</sub>
c <sub>1</sub>	e <sub>2</sub>
c <sub>2</sub>	e <sub>1</sub>
c <sub>2</sub>	e <sub>2</sub>
c <sub>3</sub>	e <sub>1</sub>

$$\pi_{\{D,A\}}Q(\mathbf{D})$$

D	A
d <sub>1</sub>	a <sub>1</sub>
d <sub>2</sub>	a <sub>1</sub>
d <sub>3</sub>	a <sub>1</sub>
d <sub>1</sub>	a <sub>2</sub>
d <sub>2</sub>	a <sub>2</sub>

$$\pi_{\{A,B,C\}}Q(\mathbf{D})$$

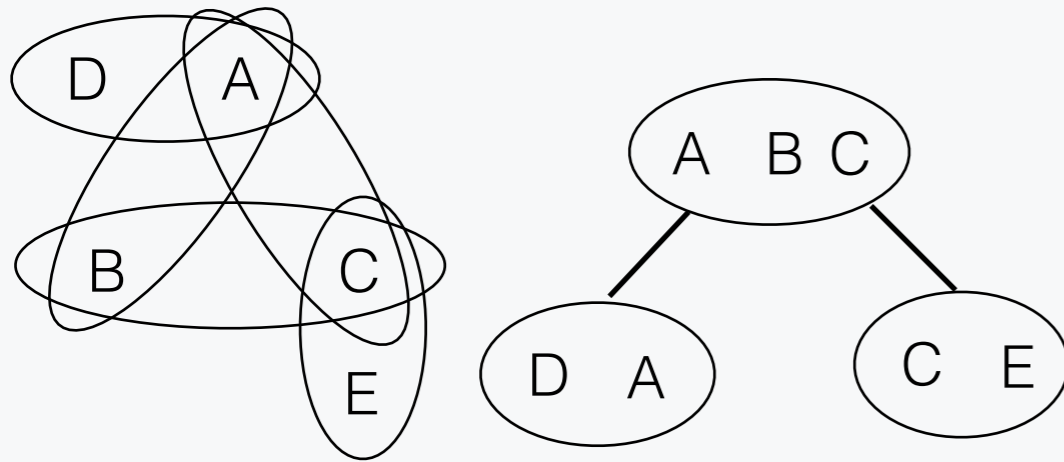
A	B	C
a <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>
a <sub>2</sub>	b <sub>2</sub>	c <sub>1</sub>
a <sub>1</sub>	b <sub>3</sub>	c <sub>2</sub>
a <sub>2</sub>	b <sub>4</sub>	c <sub>2</sub>

$$\pi_{\{C,E\}}Q(\mathbf{D})$$

C	E
c <sub>1</sub>	e <sub>1</sub>
c <sub>1</sub>	e <sub>2</sub>
c <sub>2</sub>	e <sub>1</sub>
c <sub>2</sub>	e <sub>2</sub>

# Covers and Non-Covers

$$Q(A, B, C, D, E) = R \bowtie S \bowtie T \bowtie U \bowtie V$$



R		S		T		U		V	
D	A	A	B	A	C	B	C	C	E
d <sub>1</sub>	a <sub>1</sub>	a <sub>1</sub>	b <sub>1</sub>	a <sub>1</sub>	c <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>	c <sub>1</sub>	e <sub>1</sub>
d <sub>2</sub>	a <sub>1</sub>	a <sub>1</sub>	b <sub>2</sub>	a <sub>1</sub>	c <sub>2</sub>	b <sub>2</sub>	c <sub>1</sub>	c <sub>1</sub>	e <sub>2</sub>
d <sub>3</sub>	a <sub>1</sub>	a <sub>1</sub>	b <sub>3</sub>	a <sub>2</sub>	c <sub>2</sub>	b <sub>3</sub>	c <sub>2</sub>	c <sub>2</sub>	e <sub>1</sub>
d <sub>1</sub>	a <sub>2</sub>	a <sub>2</sub>	b <sub>4</sub>	a <sub>1</sub>	c <sub>3</sub>	b <sub>4</sub>	c <sub>2</sub>	c <sub>2</sub>	e <sub>2</sub>
d <sub>2</sub>	a <sub>2</sub>	a <sub>2</sub>	b <sub>1</sub>					c <sub>3</sub>	e <sub>1</sub>

not result-preserving

D	A	B	C	E
d <sub>1</sub>	a <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>	e <sub>1</sub>
d <sub>2</sub>	a <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>	e <sub>1</sub>
d <sub>3</sub>	a <sub>1</sub>	b <sub>3</sub>	c <sub>2</sub>	e <sub>1</sub>
d <sub>1</sub>	a <sub>2</sub>	b <sub>4</sub>	c <sub>2</sub>	e <sub>1</sub>
d <sub>2</sub>	a <sub>2</sub>	b <sub>4</sub>	c <sub>2</sub>	e <sub>1</sub>

$\pi_{\{D,A\}}Q(\mathbf{D})$

D	A
d <sub>1</sub>	a <sub>1</sub>
d <sub>2</sub>	a <sub>1</sub>
d <sub>3</sub>	a <sub>1</sub>
d <sub>1</sub>	a <sub>2</sub>
d <sub>2</sub>	a <sub>2</sub>

$\pi_{\{A,B,C\}}Q(\mathbf{D})$

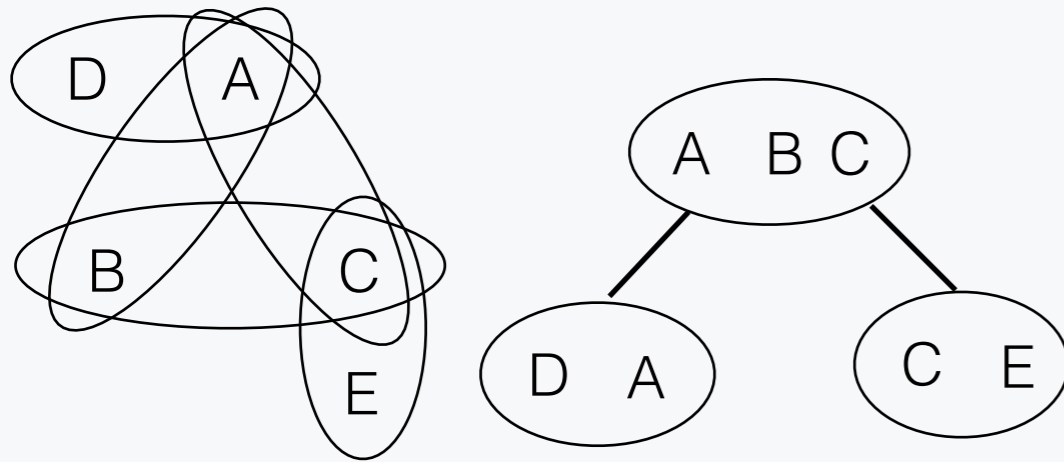
A	B	C
a <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>
a <sub>2</sub>	b <sub>2</sub>	c <sub>1</sub>
a <sub>1</sub>	b <sub>3</sub>	c <sub>2</sub>
a <sub>2</sub>	b <sub>4</sub>	c <sub>2</sub>

$\pi_{\{C,E\}}Q(\mathbf{D})$

C	E
c <sub>1</sub>	e <sub>1</sub>
c <sub>1</sub>	e <sub>2</sub>
c <sub>2</sub>	e <sub>1</sub>
c <sub>2</sub>	e <sub>2</sub>

# Covers and Non-Covers

$$Q(A, B, C, D, E) = R \bowtie S \bowtie T \bowtie U \bowtie V$$



R		S		T		U		V	
D	A	A	B	A	C	B	C	C	E
d <sub>1</sub>	a <sub>1</sub>	a <sub>1</sub>	b <sub>1</sub>	a <sub>1</sub>	c <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>	c <sub>1</sub>	e <sub>1</sub>
d <sub>2</sub>	a <sub>1</sub>	a <sub>1</sub>	b <sub>2</sub>	a <sub>1</sub>	c <sub>2</sub>	b <sub>2</sub>	c <sub>1</sub>	c <sub>1</sub>	e <sub>2</sub>
d <sub>3</sub>	a <sub>1</sub>	a <sub>1</sub>	b <sub>3</sub>	a <sub>2</sub>	c <sub>2</sub>	b <sub>3</sub>	c <sub>2</sub>	c <sub>2</sub>	e <sub>1</sub>
d <sub>1</sub>	a <sub>2</sub>	a <sub>2</sub>	b <sub>4</sub>	a <sub>1</sub>	c <sub>3</sub>	b <sub>4</sub>	c <sub>2</sub>	c <sub>2</sub>	e <sub>2</sub>
d <sub>2</sub>	a <sub>2</sub>	a <sub>2</sub>	b <sub>1</sub>					c <sub>3</sub>	e <sub>1</sub>

not minimal

D	A	B	C	E
d <sub>1</sub>	a <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>	e <sub>1</sub>
d <sub>2</sub>	a <sub>1</sub>	b <sub>2</sub>	c <sub>1</sub>	e <sub>2</sub>
d <sub>3</sub>	a <sub>1</sub>	b <sub>3</sub>	c <sub>2</sub>	e <sub>1</sub>
d <sub>1</sub>	a <sub>2</sub>	b <sub>4</sub>	c <sub>2</sub>	e <sub>2</sub>
d <sub>2</sub>	a <sub>2</sub>	b <sub>4</sub>	c <sub>2</sub>	e <sub>2</sub>
d <sub>2</sub>	a <sub>2</sub>	b <sub>4</sub>	c <sub>2</sub>	e <sub>1</sub>

$\pi_{\{D,A\}}Q(\mathbf{D})$

D	A
d <sub>1</sub>	a <sub>1</sub>
d <sub>2</sub>	a <sub>1</sub>
d <sub>3</sub>	a <sub>1</sub>
d <sub>1</sub>	a <sub>2</sub>
d <sub>2</sub>	a <sub>2</sub>

$\pi_{\{A,B,C\}}Q(\mathbf{D})$

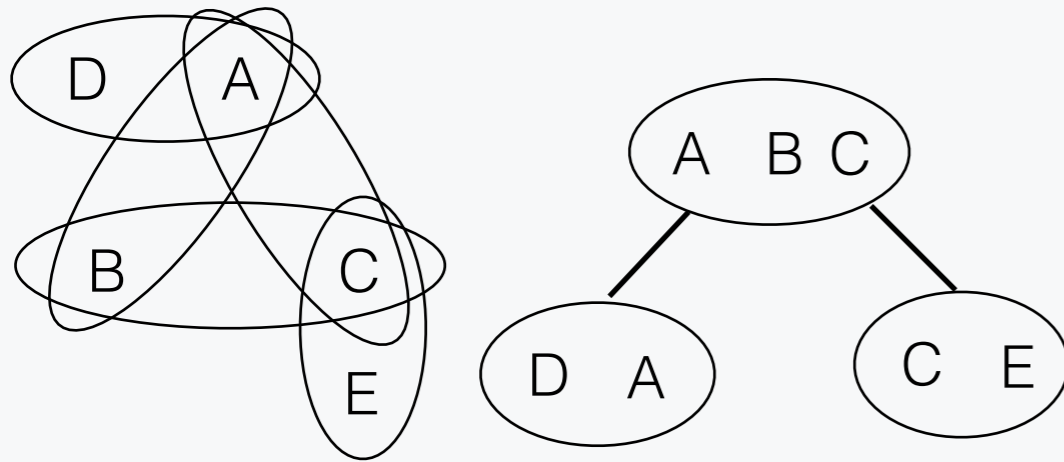
A	B	C
a <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>
a <sub>2</sub>	b <sub>2</sub>	c <sub>1</sub>
a <sub>1</sub>	b <sub>3</sub>	c <sub>2</sub>
a <sub>2</sub>	b <sub>4</sub>	c <sub>2</sub>

$\pi_{\{C,E\}}Q(\mathbf{D})$

C	E
c <sub>1</sub>	e <sub>1</sub>
c <sub>1</sub>	e <sub>2</sub>
c <sub>2</sub>	e <sub>1</sub>
c <sub>2</sub>	e <sub>2</sub>

# Covers and Non-Covers

$$Q(A, B, C, D, E) = R \bowtie S \bowtie T \bowtie U \bowtie V$$



R		S		T		U		V	
D	A	A	B	A	C	B	C	C	E
d <sub>1</sub>	a <sub>1</sub>	a <sub>1</sub>	b <sub>1</sub>	a <sub>1</sub>	c <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>	c <sub>1</sub>	e <sub>1</sub>
d <sub>2</sub>	a <sub>1</sub>	a <sub>1</sub>	b <sub>2</sub>	a <sub>1</sub>	c <sub>2</sub>	b <sub>2</sub>	c <sub>1</sub>	c <sub>1</sub>	e <sub>2</sub>
d <sub>3</sub>	a <sub>1</sub>	a <sub>1</sub>	b <sub>3</sub>	a <sub>2</sub>	c <sub>2</sub>	b <sub>3</sub>	c <sub>2</sub>	c <sub>2</sub>	e <sub>1</sub>
d <sub>1</sub>	a <sub>2</sub>	a <sub>2</sub>	b <sub>4</sub>	a <sub>1</sub>	c <sub>3</sub>	b <sub>4</sub>	c <sub>2</sub>	c <sub>2</sub>	e <sub>2</sub>
d <sub>2</sub>	a <sub>2</sub>	a <sub>2</sub>	b <sub>1</sub>					c <sub>3</sub>	e <sub>1</sub>

a cover

D	A	B	C	E
d <sub>1</sub>	a <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>	e <sub>1</sub>
d <sub>2</sub>	a <sub>1</sub>	b <sub>2</sub>	c <sub>1</sub>	e <sub>2</sub>
d <sub>3</sub>	a <sub>1</sub>	b <sub>3</sub>	c <sub>2</sub>	e <sub>1</sub>
d <sub>1</sub>	a <sub>2</sub>	b <sub>4</sub>	c <sub>2</sub>	e <sub>2</sub>
d <sub>2</sub>	a <sub>2</sub>	b <sub>4</sub>	c <sub>2</sub>	e <sub>2</sub>

$\pi_{\{D,A\}}Q(\mathbf{D})$     $\pi_{\{A,B,C\}}Q(\mathbf{D})$     $\pi_{\{C,E\}}Q(\mathbf{D})$

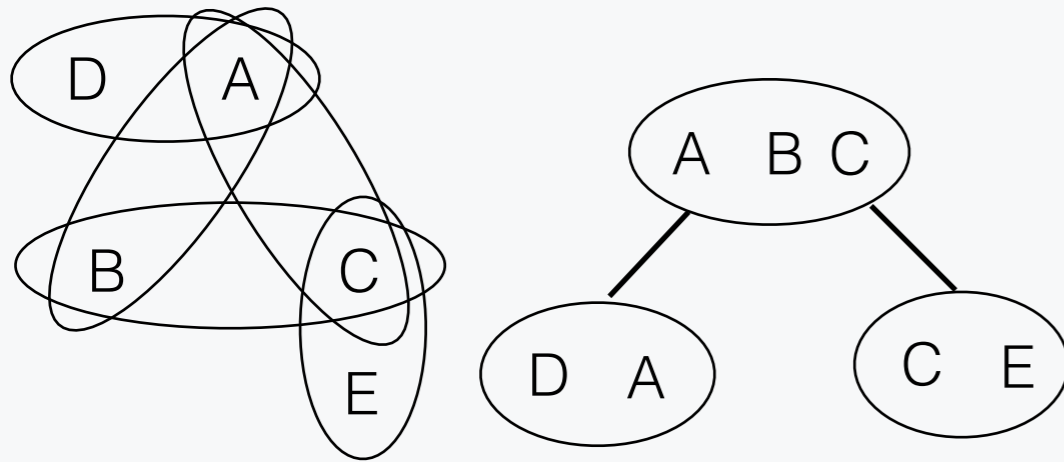
D	A
d <sub>1</sub>	a <sub>1</sub>
d <sub>2</sub>	a <sub>1</sub>
d <sub>3</sub>	a <sub>1</sub>
d <sub>1</sub>	a <sub>2</sub>
d <sub>2</sub>	a <sub>2</sub>

A	B	C
a <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>
a <sub>2</sub>	b <sub>2</sub>	c <sub>1</sub>
a <sub>1</sub>	b <sub>3</sub>	c <sub>2</sub>
a <sub>2</sub>	b <sub>4</sub>	c <sub>2</sub>

C	E
c <sub>1</sub>	e <sub>1</sub>
c <sub>1</sub>	e <sub>2</sub>
c <sub>2</sub>	e <sub>1</sub>
c <sub>2</sub>	e <sub>2</sub>

# Covers and Non-Covers

$$Q(A, B, C, D, E) = R \bowtie S \bowtie T \bowtie U \bowtie V$$



R		S		T		U		V	
D	A	A	B	A	C	B	C	C	E
d <sub>1</sub>	a <sub>1</sub>	a <sub>1</sub>	b <sub>1</sub>	a <sub>1</sub>	c <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>	c <sub>1</sub>	e <sub>1</sub>
d <sub>2</sub>	a <sub>1</sub>	a <sub>1</sub>	b <sub>2</sub>	a <sub>1</sub>	c <sub>2</sub>	b <sub>2</sub>	c <sub>1</sub>	c <sub>1</sub>	e <sub>2</sub>
d <sub>3</sub>	a <sub>1</sub>	a <sub>1</sub>	b <sub>3</sub>	a <sub>2</sub>	c <sub>2</sub>	b <sub>3</sub>	c <sub>2</sub>	c <sub>2</sub>	e <sub>1</sub>
d <sub>1</sub>	a <sub>2</sub>	a <sub>2</sub>	b <sub>4</sub>	a <sub>1</sub>	c <sub>3</sub>	b <sub>4</sub>	c <sub>2</sub>	c <sub>2</sub>	e <sub>2</sub>
d <sub>2</sub>	a <sub>2</sub>	a <sub>2</sub>	b <sub>1</sub>					c <sub>3</sub>	e <sub>1</sub>

another cover

D	A	B	C	E
d <sub>1</sub>	a <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>	e <sub>1</sub>
d <sub>2</sub>	a <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>	e <sub>1</sub>
d <sub>3</sub>	a <sub>1</sub>	b <sub>2</sub>	c <sub>1</sub>	e <sub>2</sub>
d <sub>3</sub>	a <sub>1</sub>	b <sub>3</sub>	c <sub>2</sub>	e <sub>1</sub>
d <sub>1</sub>	a <sub>2</sub>	b <sub>4</sub>	c <sub>2</sub>	e <sub>2</sub>
d <sub>2</sub>	a <sub>2</sub>	b <sub>4</sub>	c <sub>2</sub>	e <sub>2</sub>

$\pi_{\{D,A\}}Q(\mathbf{D})$     $\pi_{\{A,B,C\}}Q(\mathbf{D})$     $\pi_{\{C,E\}}Q(\mathbf{D})$

D	A
d <sub>1</sub>	a <sub>1</sub>
d <sub>2</sub>	a <sub>1</sub>
d <sub>3</sub>	a <sub>1</sub>
d <sub>1</sub>	a <sub>2</sub>
d <sub>2</sub>	a <sub>2</sub>

A	B	C
a <sub>1</sub>	b <sub>1</sub>	c <sub>1</sub>
a <sub>2</sub>	b <sub>2</sub>	c <sub>1</sub>
a <sub>1</sub>	b <sub>3</sub>	c <sub>2</sub>
a <sub>2</sub>	b <sub>4</sub>	c <sub>2</sub>

C	E
c <sub>1</sub>	e <sub>1</sub>
c <sub>1</sub>	e <sub>2</sub>
c <sub>2</sub>	e <sub>1</sub>
c <sub>2</sub>	e <sub>2</sub>



# Insights on Covers

Let  $Q$  be a query,  $\mathcal{T}$  a hypertree decomposition of  $Q$ ,  $\mathbf{D}$  a database  
and

$\mathbf{C}$  a cover of  $Q(\mathbf{D})$  over  $\mathcal{T}$ .

Covers are included in Query Result

$\mathbf{C}$  is a subset of  $Q(\mathbf{D})$ .

Covers allow Recovery of Query Result

$$\bowtie_{B \in \mathcal{S}(\mathcal{T})} \pi_B \mathbf{C} = Q(\mathbf{D}).$$

Characterization by Minimal Edge Covers in Hypergraph of Query Result

$\mathbf{C}$  corresponds to a minimal edge cover in the hypergraph of the join  
of the projections of  $Q(\mathbf{D})$  onto the bags of  $\mathcal{T}$ .

# Insights on Covers

Let  $Q$  be a query,  $\mathcal{T}$  a hypertree decomposition of  $Q$ ,  $\mathbf{D}$  a database  
and  
 $\mathbf{C}$  a cover of  $Q(\mathbf{D})$  over  $\mathcal{T}$ .

## Size of Covers

The size of  $\mathbf{C}$  is  $\mathcal{O}(|\mathbf{D}|^{\text{fhtw}(\mathcal{T})})$ .

There are arbitrarily large databases  $\mathbf{D}$  such that each cover of  
 $Q(\mathbf{D})$  has size  $\Omega(|\mathbf{D}|^{\text{fhtw}(\mathcal{T})})$ .

## Constant-delay Enumeration from Covers

$Q(\mathbf{D})$  can be enumerated from  $\mathbf{C}$  with  $\tilde{\mathcal{O}}(|\mathbf{C}|)$  pre-computation  
time and constant delay.

# Covers of Results of Acyclic Queries

## Representation System: Acyclic Query + Input Database

- Input database has asymptotically the same size as a minimum-sized cover of the query result.
- Enumeration of the query result from the input database has the same time complexity as covers [Bagan et al., 07].

So, why use a cover instead of the input database  
in case of acyclic queries?

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- Input database has asymptotically the same size as a minimum-sized cover of the query result.
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So, why use a cover instead of the input database in case of acyclic queries?

## Practical Benefits of Covers

- Covers provide cache and access locality.
- A cover is a sample of the query result.
- Subsequent processing may require a single relation (e.g, machine learning over joins).

# Computation of Covers

Two possible ways to compute covers in worst-case optimal time:

## Monolithic Approach

- Given a query, a hypertree decomposition and a database, construct a cover by considering all joins at once.

## Compositional Approach

- Given a query, a hypertree decomposition and a database, construct relations for the bags of the decomposition.
- Turn bag relations into a globally consistent database.
- Use a cover-join plan to compute the overall cover by computing partial covers.

# Computing a Cover of our Fast Food Database

## Orders

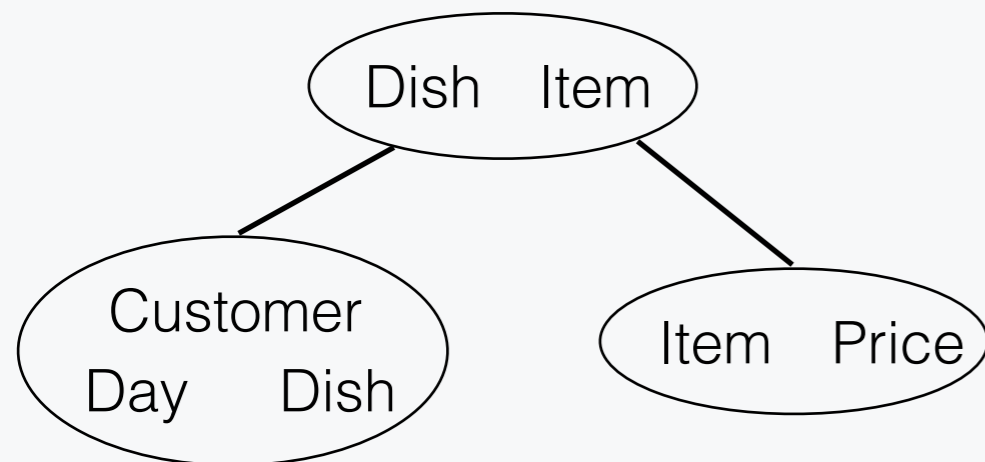
Customer	Day	Dish
elise	monday	burger
elise	friday	burger
steve	friday	hotdog
joe	monday	hotdog
joe	tuesday	hotdog

## Dishes

Dish	Item
burger	patty
burger	onion
burger	bun
hotdog	bun
hotdog	onion
hotdog	sausage

## Items

Item	Price
patty	6
onion	2
bun	2
sausage	4



# Computing a Cover of our Fast Food Database

**Orders**

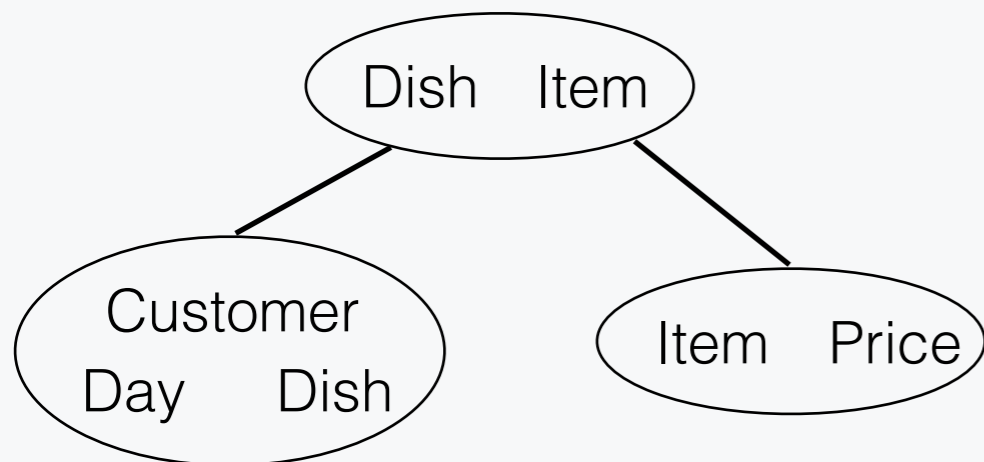
Customer	Day	Dish
elise	monday	burger
elise	friday	burger
steve	friday	hotdog
joe	monday	hotdog
joe	tuesday	hotdog

**Dishes**

Dish	Item
burger	patty
burger	onion
burger	bun
hotdog	bun
hotdog	onion
hotdog	sausage

**Items**

Item	Price
patty	6
onion	2
bun	2
sausage	4



## Valid cover-join plans

$(\text{Orders} \bowtie \text{Dishes}) \bowtie \text{Items}$

$\text{Orders} \bowtie (\text{Dishes} \bowtie \text{Items})$

## A non-valid plan

$(\text{Orders} \bowtie \text{Items}) \bowtie \text{Dishes}$

# Computing a Cover of our Fast Food Database

**Orders**

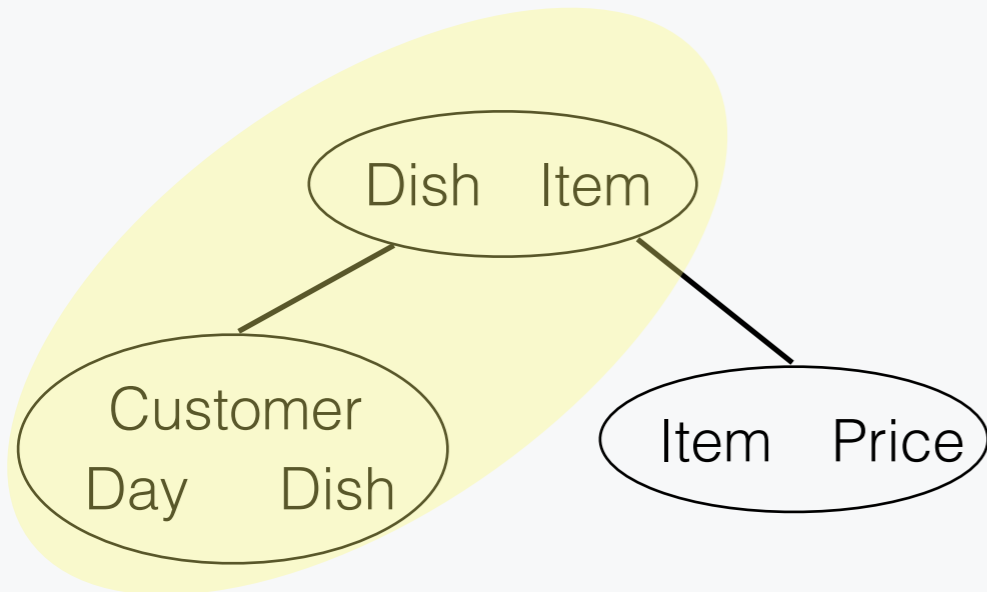
Customer	Day	Dish
elise	monday	burger
elise	friday	burger
steve	friday	hotdog
joe	monday	hotdog
joe	tuesday	hotdog

**Dishes**

Dish	Item
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burger	onion
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hotdog	bun
hotdog	onion
hotdog	sausage

**Items**

Item	Price
patty	6
onion	2
bun	2
sausage	4



## Valid cover-join plans

→ (Orders ⋈ Dishes) ⋈ Items  
 Orders ⋈ (Dishes ⋈ Items)

## A non-valid plan

(Orders ⋈ Items) ⋈ Dishes



# Computing a Cover of our Fast Food Database

**Orders**

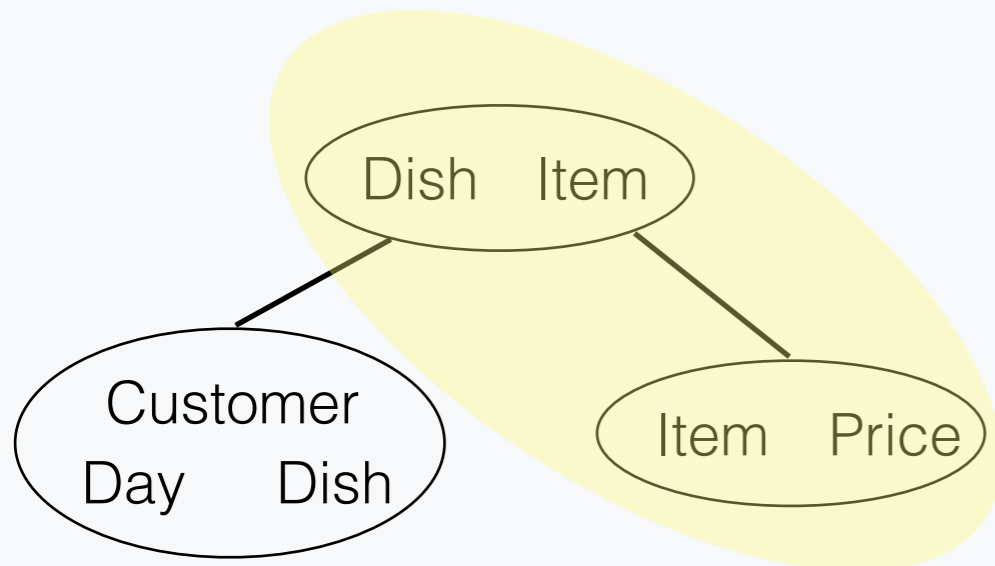
Customer	Day	Dish
elise	monday	burger
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**Items**

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## Valid cover-join plans

$(\text{Orders} \bowtie \text{Dishes}) \bowtie \text{Items}$

→  $\text{Orders} \bowtie (\text{Dishes} \bowtie \text{Items})$

## A non-valid plan

$(\text{Orders} \bowtie \text{Items}) \bowtie \text{Dishes}$

# Computing a Cover of our Fast Food Database

**Orders**

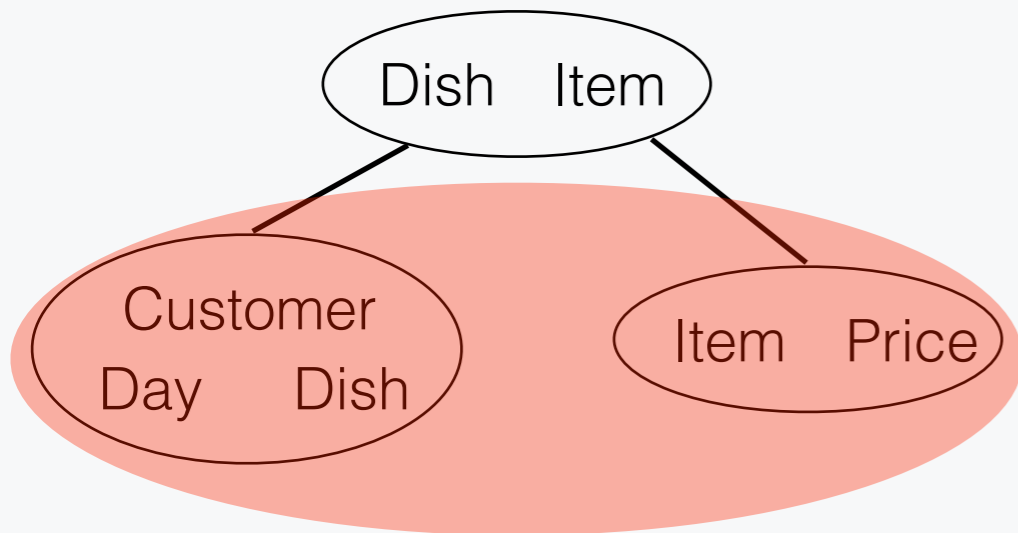
Customer	Day	Dish
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## Valid cover-join plans

$(\text{Orders} \bowtie \text{Dishes}) \bowtie \text{Items}$

$\text{Orders} \bowtie (\text{Dishes} \bowtie \text{Items})$

## A non-valid plan

$\rightarrow (\text{Orders} \bowtie \text{Items}) \bowtie \text{Dishes}$

# Computing a Cover of our Fast Food Database

**Orders**

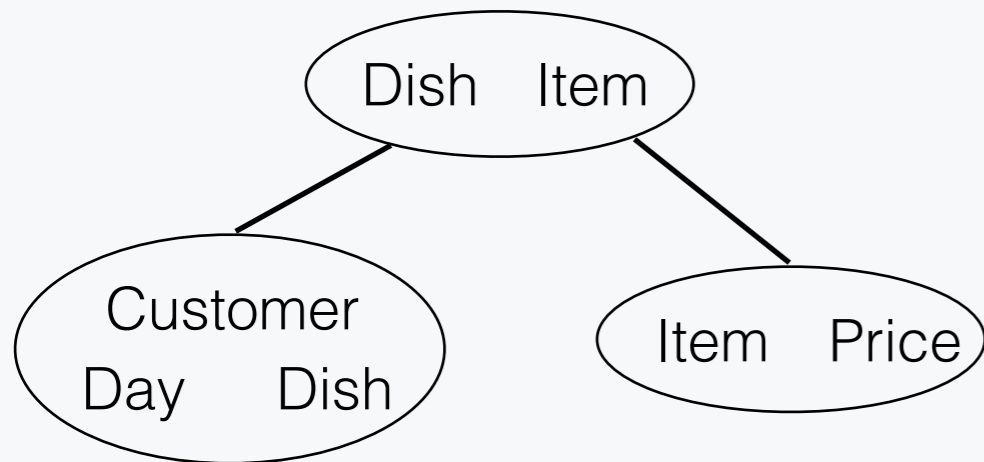
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## Valid cover-join plans

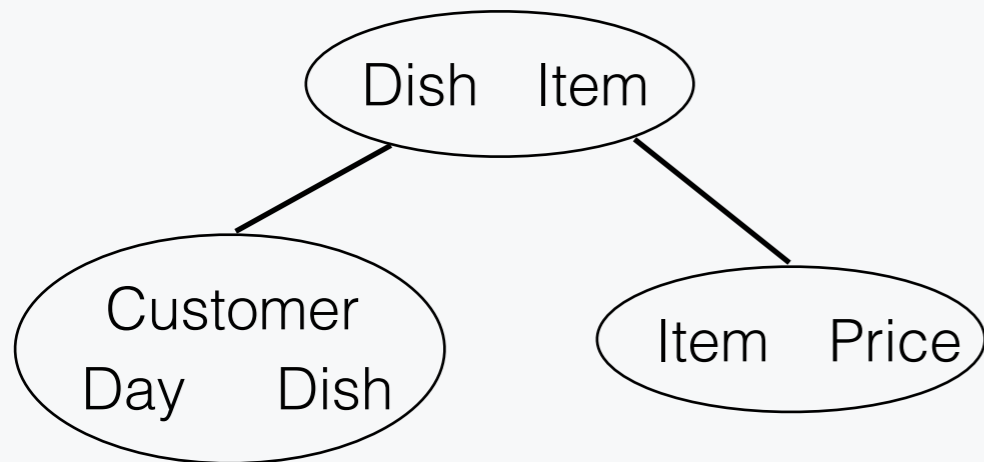
→ (Orders ⋈ Dishes) ⋈ Items  
 Orders ⋈ (Dishes ⋈ Items)

## A non-valid plan

(Orders ⋈ Items) ⋈ Dishes

# Computing a Cover of our Fast Food Database

Orders			Dishes		Items	
Customer	Day	Dish	Dish	Item	Item	Price
elise	monday	burger	burger	patty	patty	6
elise	friday	burger	burger	onion	onion	2
steve	friday	hotdog	burger	bun	bun	2
joe	monday	hotdog	hotdog	bun	bun	2
joe	tuesday	hotdog	hotdog	onion	sausage	4



## Valid cover-join plans

→ (Orders ⋈ Dishes) ⋈ Items  
 Orders ⋈ (Dishes ⋈ Items)

## A non-valid plan

(Orders ⋈ Items) ⋈ Dishes

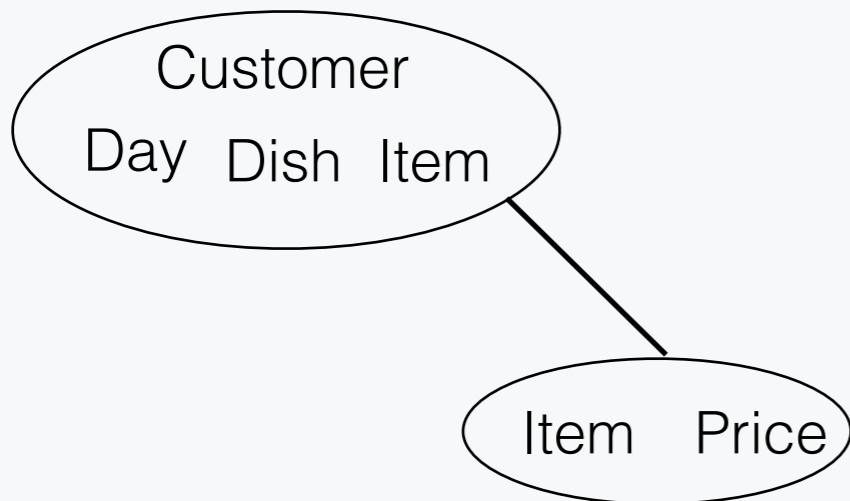
# Computing a Cover of our Fast Food Database

Orders  $\bowtie$  Dishes

Customer	Day	Dish	Item
elise	monday	burger	patty
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joe	monday	hotdog	onion
joe	tuesday	hotdog	sausage

Items

Item	Price
patty	6
onion	2
bun	2
sausage	4



Valid cover-join plans

→ (Orders  $\bowtie$  Dishes)  $\bowtie$  Items

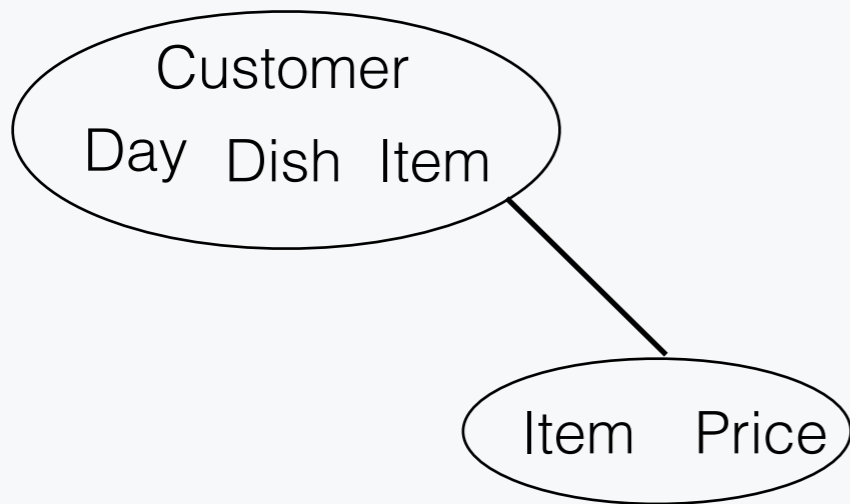
Orders  $\bowtie$  (Dishes  $\bowtie$  Items)

A non-valid plan

(Orders  $\bowtie$  Items)  $\bowtie$  Dishes

# Computing a Cover of our Fast Food Database

Orders $\bowtie$ Dishes				Items	
Customer	Day	Dish	Item	Item	Price
elise	monday	burger	patty	patty	6
elise	friday	burger	onion	onion	2
elise	friday	burger	bun	bun	2
steve	friday	hotdog	bun	sausage	4
joe	monday	hotdog	onion		
joe	tuesday	hotdog	sausage		



Valid cover-join plans

→ (Orders  $\bowtie$  Dishes)  $\bowtie$  Items

Orders  $\bowtie$  (Dishes  $\bowtie$  Items)

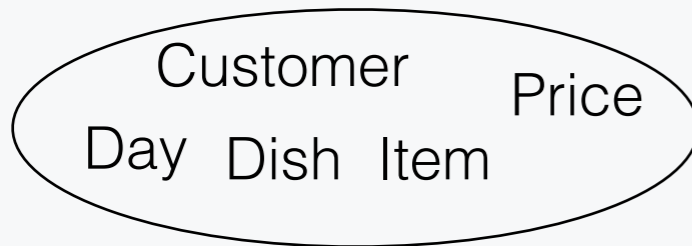
A non-valid plan

(Orders  $\bowtie$  Items)  $\bowtie$  Dishes

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(Orders  $\bowtie$  Dishes)  $\bowtie$  Items

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joe	tuesday	hotdog	sausage	4



Valid cover-join plans

→ (Orders  $\bowtie$  Dishes)  $\bowtie$  Items

Orders  $\bowtie$  (Dishes  $\bowtie$  Items)

A non-valid plan

(Orders  $\bowtie$  Items)  $\bowtie$  Dishes

# Computation Time for Covers

(Orders  $\bowtie$  Dishes)  $\bowtie$  Items

Customer	Day	Dish	Item	Price
elise	monday	burger	patty	6
elise	friday	burger	onion	2
elise	friday	burger	bun	2
steve	friday	hotdog	bun	2
joe	monday	hotdog	onion	2
joe	tuesday	hotdog	sausage	4

## Computation Time for Covers

Given a query  $Q$ , a hypertree decomposition  $\mathcal{T}$  of  $Q$  and a database  $\mathbf{D}$ , any cover-join plan for  $Q$  over  $\mathcal{T}$  computes a cover of  $Q(\mathbf{D})$  over  $\mathcal{T}$  in time  $\tilde{O}(|\mathbf{D}|^{\text{fhtw}(\mathcal{T})})$ .



# Covering the Results of Functional Aggregate Queries (FAQs)

## Example

- $\varphi$  counts for each tuple  $(a,b,d)$ , the number of tuples in the result containing this tuple:

$$\varphi(a, b, d) = \sum_{c,e,f,g,h} \psi_1(a, b, c) \cdot \psi_2(b, d, e) \cdot \psi_3(d, e, f) \cdot \psi_4(f, h) \cdot \psi_5(e, g)$$

where

$$\psi_1(a, b, c) = 1 \Leftrightarrow (a, b, c) \in R_1 \text{ and so on ...}$$

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where

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- push aggregates past join:

$$\varphi(a, b, d) = \underbrace{\left( \sum_c \psi_1(a, b, c) \right)}_{\psi_6(a,b)} \cdot \sum_e \left( \psi_2(b, d, e) \cdot \underbrace{\sum_f \left( \psi_3(d, e, f) \cdot \underbrace{\sum_h \psi_4(f, h)}_{\psi_7(f)} \right)}_{\psi_9(d,e)} \right) \cdot \underbrace{\sum_g \psi_5(e, g)}_{\psi_8(e)}$$

$\underbrace{\hspace{15em}}_{\psi_{10}(b,d)}$

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## Example

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- Compute a cover of the join of the relational encodings of  $\psi_6(a, b)$  and  $\psi_{10}(b, d)$

# Functional Aggregate Queries [Khamis et al., PODS 2016]

FAQs encompass many problems in

- Constraint Satisfaction,
- Logic,
- Databases,
- Matrix Operations,
- Probabilistic Graphical Models,
- Machine Learning,
- ...

Computation Time of an FAQ result

$$\tilde{O}(|\mathbf{D}|^{\text{faqw}(\varphi)} + \text{output size})$$

Computation Time of an FAQ-Cover

$$\tilde{O}(|\mathbf{D}|^{\text{faqw}(\varphi)})$$

# Summary

## Motivation

- Query results entail redundancies in representation and computation
- Design of a representation system for query results which reduces such redundancies

## Covers

- One-relational, lossless, succinct representation of query results
- Allow for constant-delay enumeration
- Can be applied to a wide range of computational problems

## On-going work

- Usage of covers in distributed databases to reduce communication complexity