Pseudospectra of Similarity Transformations

THEOREM. Let $A = SBS^{-1}$. Then $\Lambda_\varepsilon(A) \subseteq \Lambda_{\kappa(S)\varepsilon}(B)$.

Notation. $\kappa(X) \equiv \|X\| \|X^{-1}\|$.

Proof. Suppose $z \in \Lambda_\varepsilon(A)$. The result follows from the inequality

$$\varepsilon^{-1} \leq \|(zI - A)^{-1}\| = \|(zSS^{-1} - SBS^{-1})^{-1}\| \leq \|S\| \, \|S^{-1}\| \, \|(zI - B)^{-1}\|. \qquad \blacksquare$$

COROLLARY. If A is diagonalizable, $A = V\Lambda V^{-1}$, then $\Lambda_\varepsilon(A) \subseteq \Lambda(A) + \Delta_{\kappa(V)\varepsilon}$.

Notation. $\Delta_\delta \equiv \{z \in \mathbb{C} : |z| \leq \delta\}$ is the closed disk of radius δ.

Set addition is defined componentwise: $S_1 + S_2 = \{s_1 + s_2 : s_1 \in S_1, s_2 \in S_2\}$.

History. The theorem is given, for example, in [ET00]. The corollary is a version of the Bauer-Fike theorem [BF63], presented in the language of pseudospectra in [Tre99b].

Bibliography.

[BF60] F. L. Bauer and C. T. Fike. *Norms and exclusion theorems.* Numer. Math. **2** (1960), 137–141.

[ET00] M. Embree and L. N. Trefethen. *Generalizing eigenvalue theorems to pseudospectra.* To appear in SIAM J. Sci. Comput.

[Tre99b] L. N. Trefethen. *Spectra and pseudospectra: The behavior of non-normal matrices and operators.* In *The Graduate Student's Guide to Numerical Analysis*, M. Ainsworth, J. Levesley, and M. Marletta, eds., Springer-Verlag, Berlin, 1999.